Question: How does the mass of the hanging object (m_2) affect the angle of the ramp (θ) at which both objects have acceleration of $0\frac{m}{s^2}$?

Hypothesis: The relationship between θ and m_2 will not be linear, however the relationship between $\frac{\sin\theta}{1+\sin\theta}$ and m_2 will be linear. The reciprocal of the slope of the graph of $\frac{\sin\theta}{1+\sin\theta}$ vs. m_2 will be the total mass $(m_1 + m_2)$.

Strategy:

- The hanging mass in a modified Atwood's machine was varied by hanging various numbers of washers tied to a string.
- Books and meter sticks were used to lift one side of the ramp until both objects had an acceleration of $0\frac{m}{s^2}$ (no app was used; we adjusted the ramp until both objects stopped moving).
- The total mass was kept constant by moving unused washers ride on the cart so that the sum of m_1 and m_2 stayed the same.
- The angle of the ramp was measured using the Measure app on an iPhone, using the floor to calibrate 0°
- For acceleration to be zero, the force pulling m_1 down along the ramp ($m_1 g \sin\theta$) must be equal to the force pulling m_2 downwards ($m_2 g$).
    - The free body diagrams in Figure 2 show the forces on the individual masses in the modified Atwood's machine.

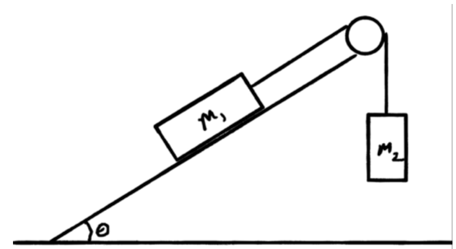

Figure 1

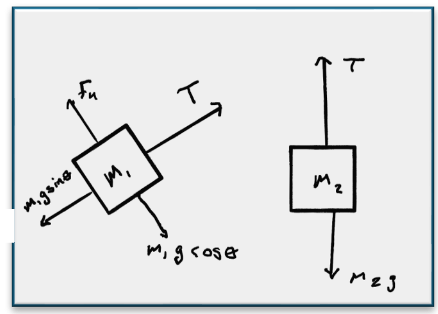

Figure 2

One can reach the equation $\frac{\sin\theta}{1+\sin\theta} = \frac{1}{482}m_2$ with the equations presented below.

$$m_1 g \sin\theta = m_2 g$$

$$m_1 g \sin\theta + m_2 g \sin\theta = m_2 g + m_2 g \sin\theta$$

Divide by g on both sides, then factor:

$$m_2(1 + \sin\theta) = \sin\theta\,(m_1 + m_2)$$

$$\frac{m_2}{m_1+m_2} = \frac{\sin\theta}{(1+\sin\theta)}$$

Since $m_1 + m_2$ always equals 482 grams,

$$\frac{\sin\theta}{1+\sin\theta} = \frac{1}{482}m_2$$

Data:

Total mass of the system:

$282g\ (m\ of\ car) + 200g\ (m\ of\ washers)$

= 482g

= 0.482kg

m_2 (g)	m_1 (g)	θ (°)
0	482	0
50	432	4
100	382	15
150	332	25
200	282	45

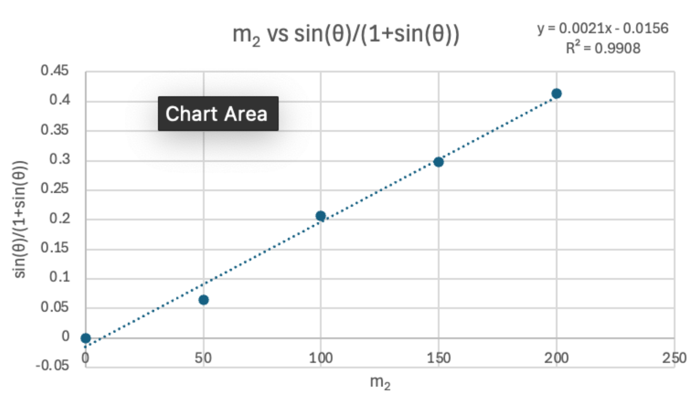

Figure 3

Analysis:

As shown in the graph, we had a linear slope which was 0.0021, meaning our experimental value for $\frac{\sin\theta}{1+\sin\theta}/m_2$ was 0.0021. Compared with our theoretical value for the slope, which was $\frac{1}{482}$, or about 0.00207. Calculating the percent error, we get a 1.22% error, which is very good. This means that our original hypothesis was proven to be true. The reason for our percent error could have been due to friction, which would have affected the experiment by decreasing the value of $m_1 g \sin\theta$ compared to what it is, since we don't account for the weight of the string. We used a lot of string to support the weight of the 200g worth of washers that we were hanging. This makes sense, as our experimental value for $\frac{m_2}{m_1+m_2}$ would be lower, meaning that we would get a higher slope to compensate for that (0.0021 > 0.00207).

Overall, our experiment was highly accurate, and we were able to correctly find that, on an inclined plane, the graph of $\frac{\sin\theta}{1+\sin\theta}$ vs. m_2 is linear with a slope of $\frac{1}{m_1+m_2}$.