Optimizing Train Headway Times using Fluid Dynamics and Simulations

Grant Proposal

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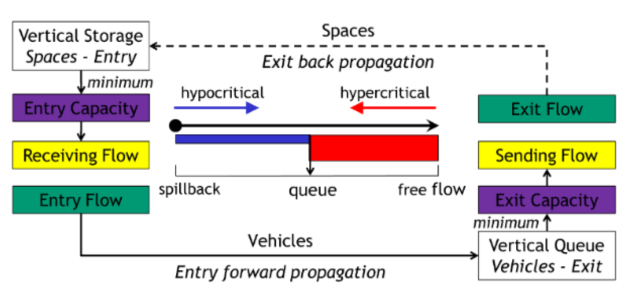
Abstract (RQ) or Executive Summary (Eng)

Subway systems are critical for the functionality of population dense urban areas and provide efficient transportation for millions of passengers daily. However, optimizing subway systems, particularly train headways, continues to remain a challenge. Current models for headway optimization depend on fixed boarding times, often 1-3 minutes, for the boarding time for passengers. The proposed system introduces an innovative approach where passengers are modeled as boarding the trains through the use of fluid dynamics. The aim is to provide accurate representation of boarding times which influence headway optimization. The proposed research has 3 aims: to develop a fluid dynamics-based model in order to simulate boarding times, to create a model with the functionality to determine optimal headway times and integrate the two models. The research utilizes SIMULINK to test models across different scenarios, in order to determine the model can be applied to almost any situation. This study has the potential to advance subway simulation methodologies and improve urban transport design.

Optimizing Train Headway Times using Fluid Dynamics and Simulations

Subways are the recreational and economic backbone of many cities. They are a key reason why densely populated regions are functional, to allow people to move within cities. Subways offer unparalleled abilities to move lots of people in incredibly short amounts of time (Singhania & Marinov, 2017). However, subway systems are not without flaws. One of the largest issues with subway systems is that when constructing and planning one, it needs to be as close as possible to perfect. Building subway systems requires lots of time, energy, and money; this puts a large emphasis on doing it right the first time. This problem is where subway simulations come into play. They allow for tests to be done on proposed subway systems which, can give insight into how a subway system would function when implemented (Li et al., 2021). Simulations allow for more informed design decisions when designing them, which may save hundreds of thousands of hours for its riders over the course of the subways life (Özgür Yalçınkaya & Bayhan, 2009).

One overlooked aspect of subway system simulation is the time it takes to board a train. The time is variable and dependent on the number of people getting on the train or leaving the train. The boarding time directly impacts the time it takes for trains to leave the station. While this might seem small, across hundreds of stops, it has lasting effects (Yildirim & Aydın, 2022). The way many simulations handle this time is by adding a flat number, ranging from 1 to 3 minutes for people to board depending, on the simulation



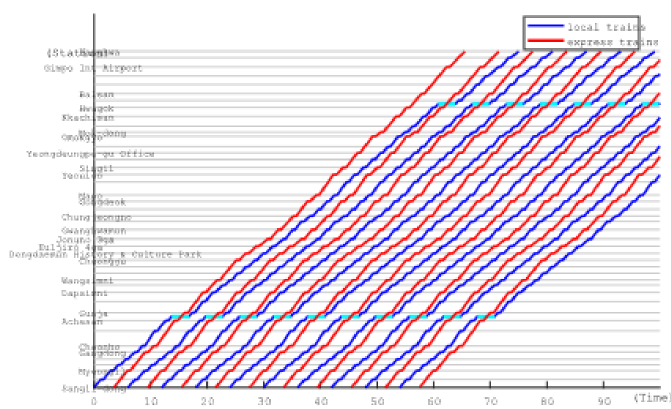
(Assis & Milani, 2004; Gentile, 2015; Peng et al., 2025). However, this

Figure 1: Fluid treatment in system (Gentile, 2015)

leads to inaccuracies in the data gathered by the simulation. The proposed solution is to use fluid dynamics to model people leaving and entering the trains from the station. While fluid dynamics has been researched before when it comes to subway system simulations, it was used to model train and station capacity, (Gentile, 2017; Gentile, 2015). By treating crowds of people as a fluid, similar to figure 1 (Gentile 2015) , and the subway car and platform as areas of high pressure and low pressure, we can apply fluid dynamics to estimate the time it would take for people to board a train. The proposed model's flow rate would fluctuate with the number of people inside the train and on the platform. The changing flow rate would dynamically change the time it takes for a train to leave the station, which we believe will have a significant impact on the optimal headways for trains, especially due to the stark differences in people at these stations during different periods of the day, which directly contributes to the time it takes to load these trains.

We can gain a more accurate perspective on subway systems by implementing this model on top of other models. The model that is being combined with fluid dynamics time model is one designed for finding optimal headways. Headways are the time between trains and are a core component in passenger satisfaction in subways, (Zhang et al., 2020). How headways effect the time it takes for trains to reach their destination is shown in figure 2, (Ko et al., 2024). The model to find optimal headways will work by sending trains whenever stations reach maximum capacity or when passenger satisfaction drops to a certain point. The time it takes for these conditions to be met will be averaged and used as the optimal headways. This method takes inspiration from other leading models to compare the differences between using the fluid dynamics boarding model and not (Ko et al., 2024).

Figure 2: How headways affect express and local trains (Ko et al, 2024)



The central hypothesis of this proposal is that using fluid dynamics to model passenger boarding times will provide a more accurate and dynamic simulation of subway operations, leading to the identification of optimal headway times. The long-term goal of this project is to develop a fluid dynamics-based model to determine the time it takes to board a subway car and compare it with leading models which use flat rates with the goal of seeing if there is a substantial difference in proposed optimized headways between both models. If there is a significant difference, then we would directly compare the two headways and see which one leads to a more optimized system.

Section II: Specific Aims

This proposal's objective is to develop a mathematical model that simulates the time it takes for passengers to board a subway and test the impact it has on the predicted most optimal headway time.

Our long-term goal is to develop an accurate and robust model for simulating boarding times for passengers on subway systems, which will contribute to optimization of headway and to a larger extent improve subway efficiency (Gentile, 2015).

The central hypothesis of this proposal is that using fluid dynamics to model passenger boarding times will provide a more accurate and dynamic simulation of subway operations, leading to the identification of optimal headway times. The rationale is that current models that use flat boarding times do not accurately capture the time it takes for subway cars to board, which directly impacts the time subways are at the station. The boarding rate directly impacts how long it takes trains to leave, which is part of what decides headway times.

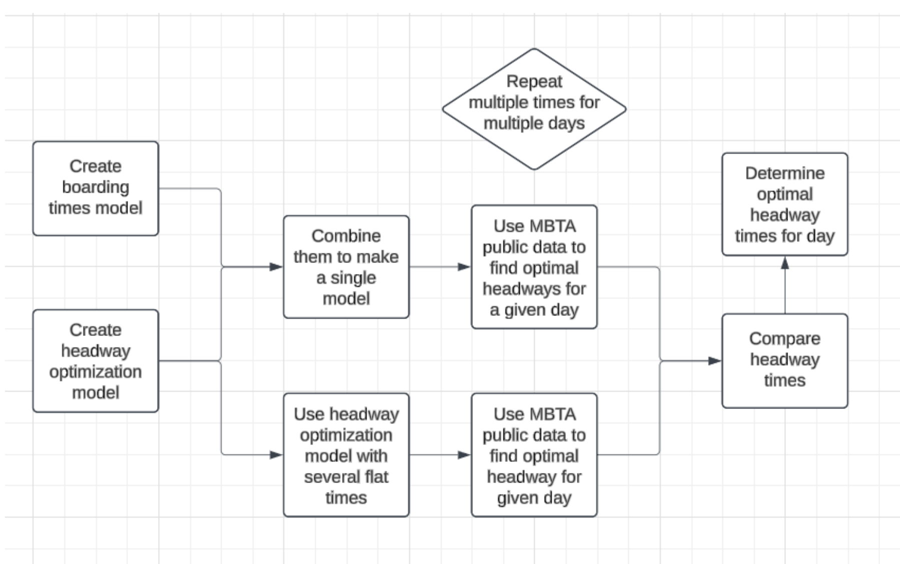


Figure 3: Graphical display of process

This process is seen graphically in figure 3.

The work we propose here will involve the development of a mathematical model using fluid dynamics. We will use a separate model based of leading headway optimization models and use publicly available MBTA data to test the headway times with the fluid dynamics model, and without by using a variety of flat times. This process will allow us to see the differences in accuracy between the two models and identify the most optimal headway times. The differences can be used to study the impact of how crowded subway cars and platforms have on the time it takes for trains to leave. It can also be determined if the assumptions made by our peers in assuming a standard 1-3 minutes is a justified decision and if it does not impact the predicted most optimal headway, (Assis & Milani, 2004; Gentile, 2015; Peng et al., 2025). We can determine this by applying predicted most optimal headways and seeing which one leads to the most passenger satisfaction. The optimal headway can be determined by having each model simulate a full 24-hour cycle, and in that time sum the combined number of minutes every person had waited in that day; while dividing that by the number of trips the trains had made that day (Özgür Yalçınkaya & Bayhan, 2009). Whichever proposed headway has the smallest number would have the most optimized headway due to keeping the most amount of people happy while using the least amount of city resources (Zhang et al., 2020).

Specific Aim 1: To create a model that is an accurate representation of people boarding trains using fluid dynamics.

Specific Aim 2: To create a model that determines the optimal headway time of a given subway system.

Specific Aim 3: To combine the two models and determine the most optimal headway time and compare it to if flat times were used.

The expected outcome of this work is to develop an accurate and robust model for simulating passenger boarding times which would allow for the identification of optimal headway times much more accurately than before. This knowledge could be used in improving the efficiency of subway operations and increasing passenger satisfaction.

Section III: Project Goals and Methodology

Relevance/Significance

Subway systems are a key part of recreational and economic life in many cities. These cities rely on subway systems to be fast and efficient when transporting people. Accurate modeling is what allows for new ideas to be proposed and tested. Simulations are part of the backbone of improving these systems. We can allow for the improvement of thousands of people's lives by creating more accurate simulations (Singhania & Marinov, 2017). By testing a long-standing assumption in subway simulations, we can potentially open up an entire subsector of subway simulations and allow for accurate models to help so many people.

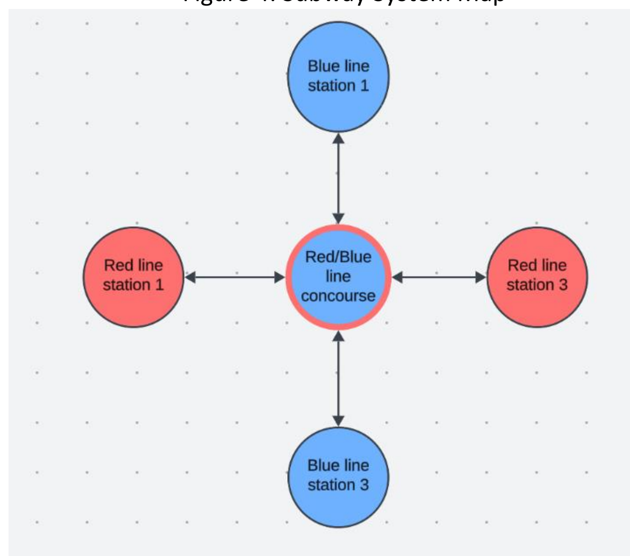
Innovation

The application of fluid dynamics to model passenger boarding times is innovative in multiple ways. The idea to use fluid dynamics has been used rarely in this field and, it has the opportunity to lead to more accurate simulations because it encourages others to use other systems we understand to try and model hard to simulate behaviors. It encourages more novel approaches to simulating things that are traditionally hard to simulate (Gentile, 2015). It also is innovative because it critically examines an assumption that very few in the field have looked at. Dynamic adjustment to boarding times allows for behaviors to be studied that haven't been well documented before due to the lack of attention it gets.

Methodology

Using the software SIMULINK, we first develop a series of equations using fluid dynamics to determine the time it would take for a group of people to board the subway, given 2 variables: the amount of people on the train and the amount of people on the platform. It is worth noting that in this proposal, we are going to assume each subway car has 2 doors and each subway train has 3 subway cars. We can then move onto creating the actual subway system itself. The system will be developed as 2 lines, each with 3 stations, crossing at the center platform of each line, which can be seen in figure 4. Each station will have a flow rate that will be determined by analyzing the public MBTA datasets, which includes every piece of data the MBTA has collected since 2021, and headway data since 2016. The passenger flow rate will be constant at each station but differs between stations. The time it takes to board the train will also be constant. Testing will involve trains that will be sent out with an initial headway of 10 minutes. As this goes on the algorithm will either increase or decrease the headway of the train as the day progresses. It will change the headway depending on a multitude of factors, namely, the amount of people on the train, the amount of people waiting at the station, and how long they have been waiting. Factors may include adding more trains to each line. It is also noteworthy to mention that the distance between stations, top speed of the train, and acceleration will all be calculated beforehand so in the simulation, trains will wait a set amount of time before appearing at the next station. This calculation will also result in the time between stations is the same. Eventually this algorithm will settle into a consistent headway, that being the one the algorithm finds most optimal.

Figure 4: Subway System Map



The previously developed model which, determines the time it takes to load the trains, will be implemented into the system. The trains will start with the headway already found to be the most optimal. The algorithm will run, and if the algorithm determines a different headway to be the most optimal then, we have a number that we believe to be more accurate for the true optimal headway (Gentile, 2015). This entire process will be repeated multiple times across different passenger flow rates and, each optimal headway with and without the fluid dynamics model will be recorded.

To test if the headway the fluid dynamics model found was truly the most optimal, we can compare the times the fluid dynamics model predicted for people to board trains to the times trains spend in stations as reported by the MBTA. We can calculate how much of this time was spent accelerating and decelerating and have an indicator of how precise our prediction was, and thus we can prove our optimal headway is the most optimal.

Specific Aim #1:

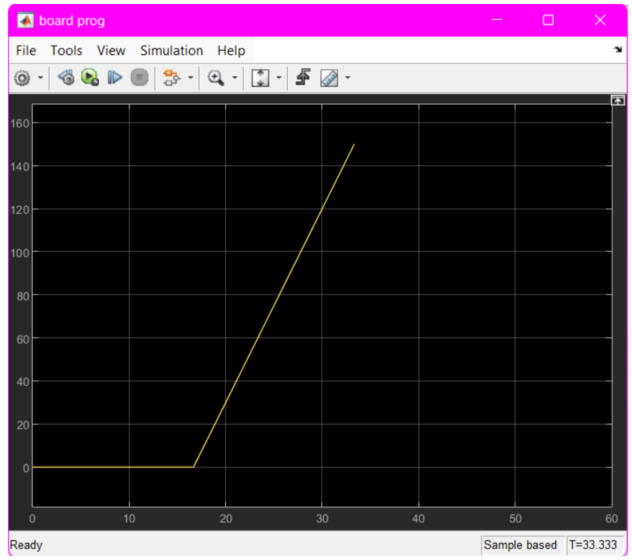
The objective is to create a model that accurately represents passenger boarding times using fluid dynamics, given the amount of people leaving and boarding the train. The way this will be done is by using Bernoulli's principle and treating the train as an area of high pressure and platform having low pressure; we can find the average flow rate of people (Gentile, 2017). The average flow rate can be used to find the boarding time. All of the computations will be done inside Simulink.

Justification and Feasibility

These methods are incredibly useful towards the main aim. Doing this allows us to test our hypothesis on a headway simulation by using the boarding times found in this method. We are able to create a simulation that calculates the time it takes to get on a train, which is our specific aim.

Summary of Preliminary Data.

Figure 2: Graph of boarding



The data we collected in figure 2 shows the amount of people on the train who were initially on the platform. In this scenario, there were 150 people on the train and 150 people on the station. It can be seen that it takes a bit over 16 seconds for the people who started on the train to leave and another 16 for the people who started on the platform to board for a total time of 33.3 seconds.

Expected Outcomes.

The expected outcome of this aim is to develop an accurate and robust model for telling the time it takes to board a subway that can be used in future models.

Potential Pitfalls and Alternative Strategies.

There were no pitfalls we came across in developing this. There were potential issues with calculating fluid dynamics, but they were avoided due to Simulink's built-in fluids modeler.

Specific Aim #2:

The objective is to create a model that determines optimal headway time of a given system. The model would be based on other leading models, which would mean that it would dynamically change depending on the amount of people per station and in the train. It would keep adjusting until an equilibrium is met and an optimal headway is reached (Gentile, 2015). The rationale behind this is that it allows us a model to test our new boarding model on and test it.

Justification/Feasibility

Building the model allows us to integrate the boarding model to test the central hypothesis. Integrating the boarding model allows us to test the headways with and without it and learn how the headway changes.

Summary of preliminary data

The data we expect to see from this will give us the optimal headway times of different subway lines. We can expect to see data that matches with other leading models and plan on testing with numbers of other papers have used to try and get a similar result, such as other dynamic adjustment models (Assis & Milani, 2004).

Expected outcomes

The outcome of this aim is to develop a model for optimizing headways which, will help us test our central hypothesis of how much the boarding model impacts the optimal headway.

Potential Pitfalls and Alternative Strategies.

Potential challenges include computational resource limitations and time constraints when building the secondary model. The alternative strategies are making a simpler model or using preexisting data.

Specific Aim #3:

The objective is to combine the two models and determine if the boarding model had a substantial difference in the optimal headway time and if it created a more accurate model. This aim will be done by running multiple scenarios of differing variables such as number of stations, influx of passengers, and train speed. Then, the models will be combined and, the scenarios retested to see if the optimal headways found differ.

Justification and Feasibility.

It is important because it allows us to deduce how big of an impact proper subway boarding time shave on headway. It also allows us to get more accurate headway times which can be used to help guide new subway projects.

Summary of Preliminary Data

We expect to see the preliminary datasets show a significant difference between the optimized headway with and without the boarding model. Additionally, we expect the predicted optimal headways with the boarding model to be more accurate when compared to preexisting MBTA data.

Expected outcomes

The overall outcome is to determine the most optimal headway time using the combined model and compare it to already existing MBTA data to determine its accuracy. We expect it improve upon flat time models and ensure a more optimized travel time across many instances.

Potential pitfalls and alternative strategies

Potential challenges include model integration issues, which can be circumvented by pre-calculating the boarding time and changing it manually in the optimization model. Another potential issue is MBTA data not being too wildly fluctuating which could be circumvented by only looking at times and places that represent our test cases.

Section III: Resources/Equipment

The computational tools used will be Simulink and Desmos and data will be given by MBTA blue book.

Section V: Ethical Considerations

Everything is done virtually, and all data is open source so there are no ethical considerations.

Section VI: Timeline

11/30 – Develop boarding model

12/7 – Finish testing boarding model and begin work on headway model

1/4 – Finish building headway model and begin integrating the two

1/25 – Finish integrating the 2 models and begin testing hypothesis

2/15 – Make a conclusion about the impact the boarding model had on optimal headways

Section VIII: References

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