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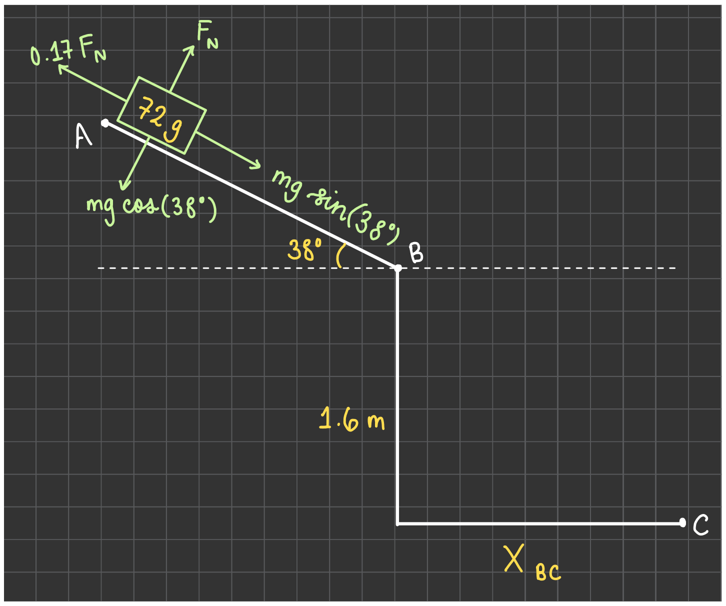
Problem Statement

A 72 g object starts at rest on a ramp 2.9 m up from the base. The object has a coefficient of friction of 0.17 with the ramp. The ramp itself makes a 38° angle with the horizontal, and the bottom of the ramp is 1.6 m up from the floor. How far away will the object land from the bottom of the ramp with the given information, and what angle of the ramp would maximize this distance?

Process

We started by drawing a free-body diagram of the problem.



We found that the forces acting on the block were two components of gravity (perpendicular and parallel to the ramp), normal force, and friction. Next, we realized that we would have to find the velocity of the block by the time it reached the end of the ramp. This would be used to calculate the motion of the block as a projectile once it left the ramp. In total, we had four main steps for this first part of the problem:

1. Calculate acceleration of the block while on the ramp by using the forces parallel to the ramp
2. Calculate final velocity at the end of the ramp by using the no-t kinematic equation
3. Find time projectile spent in the air using vertical component of velocity and no-V kinematic equation

4. Find horizontal distance using air time and horizontal component of velocity (no-*a* kinematic equation)

After that, we had to find the angle that would maximize the puck's horizontal distance. We used Desmos to graph the steps above and find the maximum. We also coded the equations in python and looped through the angle values from 0 to 90 to find the maximum x value. Both of these methods gave the same maximum angle and corresponding distance. We also checked these methods with the angle 38° from the last part to make sure it gave us the same angle

Solution

STEP 1

$$(\cancel{m}g * \sin(38)) - (0.17 * \cancel{m}g * \cos(38)) = \cancel{m}a$$

$$(9.8 * \sin(38)) - (0.17 * 9.8 * \cos(38)) = a$$

$$a = 4.7\ m/s^2$$

STEP 2

$$v_f^{\ 2} = v_0^{\ 2} + 2a\Delta x$$

$$v_f^{\ 2} = 0 + (2 * 4.7 * 2.9)$$

$$v_f^{\ 2} = 27.26$$

$$v_f = 5.22\ m/s$$

(We do not know why random variables turned red – save for the solutions, the colors do not mean anything.)

STEP 3

$$\Delta y = v_0 + \frac{1}{2}at^2$$

Rearrange kinematic equation into a quadratic in order to solve for the positive value of (t)

$$t = \frac{(v * \sin(38) - \sqrt{(v * \sin(38))^2 - (4 * a * c)})}{2a}$$

$$t = \frac{(5.22 * \sin(38) - \sqrt{(5.22 * \sin(38))^2 - (4 * 4.9 * -1.6)})}{-9.8}$$

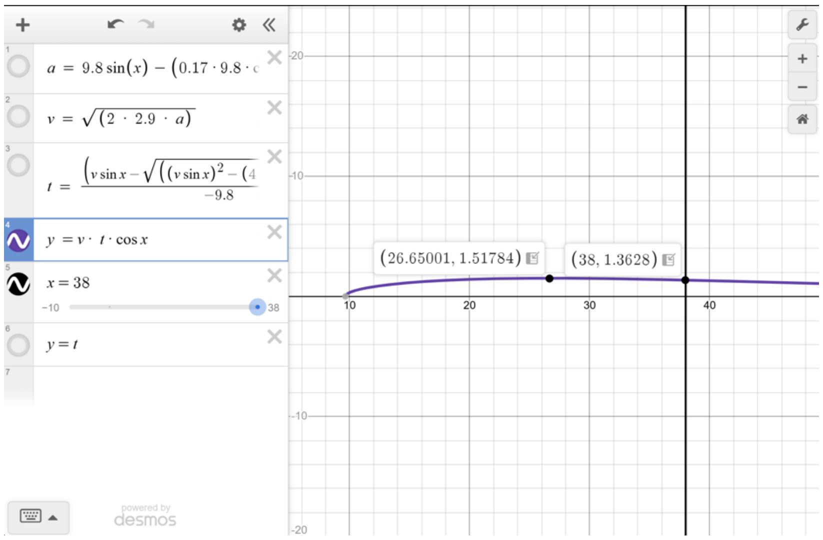
$$t = 0.331\ seconds$$

STEP 4

$$\Delta x = v * t * \cos(x)$$

$$\Delta x = 5.22 * 0.33 * \cos(38)$$

$$\Delta x = 1.36\ meters$$


1 $a = 9.8\sin(x) - (0.17 \cdot 9.8 \cdot$ c

2 $v = \sqrt{(2 \cdot 2.9 \cdot a)}$

3 $t = \frac{(v\sin x - \sqrt{((v\sin x)^2 - (4}}{-9.8}$

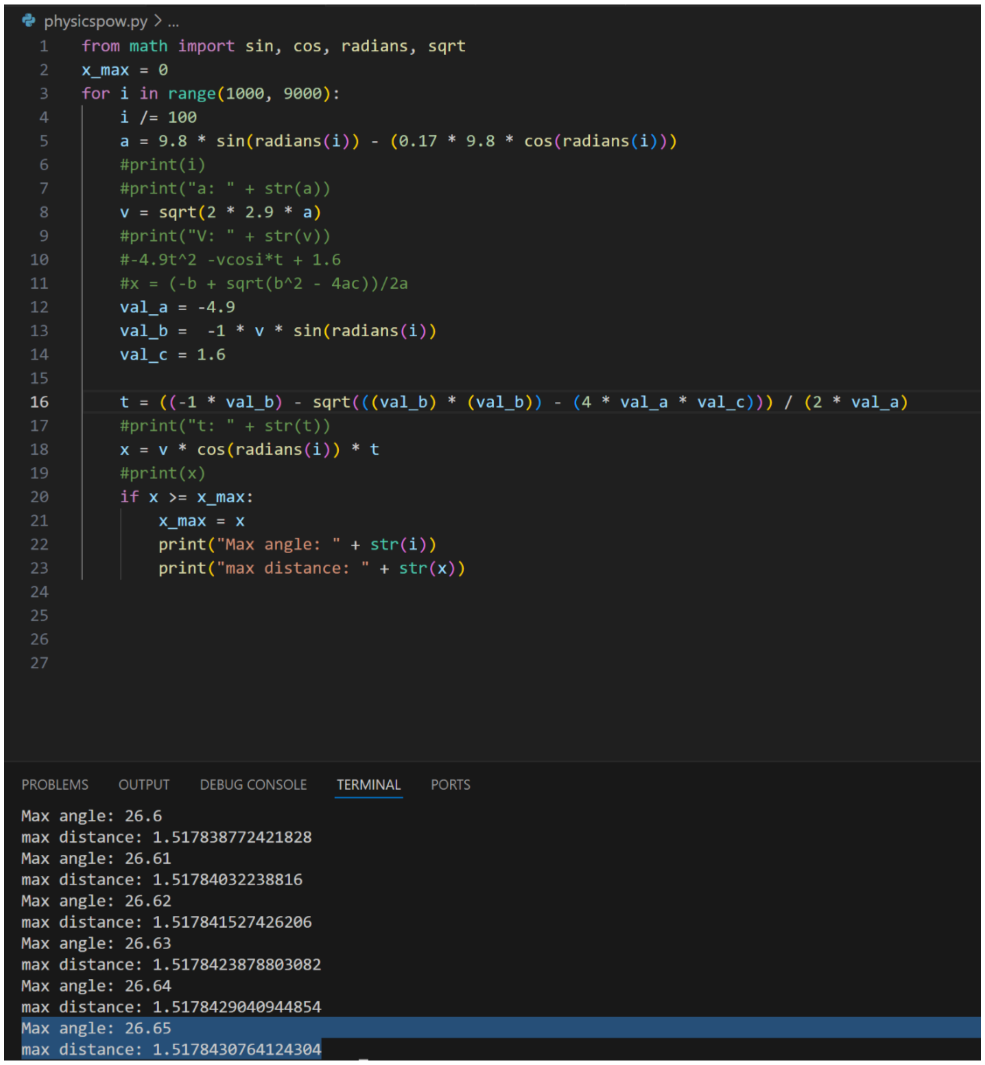
4 $y = v \cdot t \cdot \cos x$

5 $x = 38$ −10 38

6 $y = t$

7

We used parametric functions to model the process we used in the first part of the problem, inputting the resultant variable of each step into the next step. The maximum angle turned out to be 26.65° according to Desmos, with a distance of 1.52m. We also made sure that 38° still gave us 1.36m, which it did.



```
physicspow.py > ...
from math import sin, cos, radians, sqrt
x_max = 0
for i in range(1000, 9000):
    i /= 100
    a = 9.8 * sin(radians(i)) - (0.17 * 9.8 * cos(radians(i)))
    #print(i)
    #print("a: " + str(a))
    v = sqrt(2 * 2.9 * a)
    #print("V: " + str(v))
    #-4.9t^2 -vcosi*t + 1.6
    #x = (-b + sqrt(b^2 - 4ac))/2a
    val_a = -4.9
    val_b =  -1 * v * sin(radians(i))
    val_c = 1.6

    t = ((-1 * val_b) - sqrt(((val_b) * (val_b)) - (4 * val_a * val_c))) / (2 * val_a)
    #print("t: " + str(t))
    x = v * cos(radians(i)) * t
    #print(x)
    if x >= x_max:
        x_max = x
        print("Max angle: " + str(i))
        print("max distance: " + str(x))
```

```
Max angle: 26.6
max distance: 1.517838772421828
Max angle: 26.61
max distance: 1.51784032238816
Max angle: 26.62
max distance: 1.517841527426206
Max angle: 26.63
max distance: 1.5178423878803082
Max angle: 26.64
max distance: 1.5178429040944854
Max angle: 26.65
max distance: 1.5178430764124304
```

The Python program was the exact same, but manually coding each equation. Also, it only showed us the result if it was higher than the previous distance, to avoid clutter. This gave us the same answer as Desmos, with an angle of 26.65°, and a maximum corresponding distance of 1.52m. The precision could be increased by adding zeroes to the value of i, but it still gave 26.65 when this was done, while slowing down the program.

Extensions

If the coefficient friction changes to 0.5, what is the maximum distance? Is it more or less than before? How far can the block go if there is no coefficient of friction? *One way to model this is by using the slider function on Desmos.*