Ramp-Projectile POW

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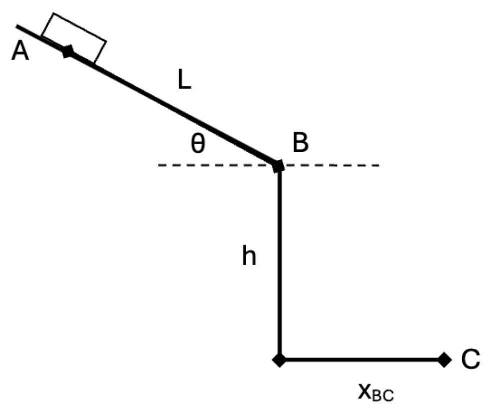
Problem Statement:

We were given a physics problem that depicted a small puck sliding down an angled ramp, continuing on to fall down further after the it passes over the decline. (see diagram below)



We were given the following information...

- The puck starts at rest
- $\theta = 38°$
- Mass the puck is 72g (0.072kg)
- $\mu = 0.17$ (for friction between the ramp and the puck)
- Ramp length (L) = 2.9m
- Drop height (h) = 1.6

The question asked us to calculate the horizontal distance that the puck would travel after sliding off the ramp. This distance is represented in the diagram by X_{BC}. The next part of the question asked us to make a spreadsheet or program to calculate the angle that would maximize this horizontal distance.

Process:

To tackle this problem, we realized that we would have to apply knowledge from different topics in class: Dynamics and kinematics. The first part would be pertinent to the inclined plane that the puck is initially on, requiring us to use dynamics to calculate the forces acting upon it before it slides off the ramp and becomes a projectile. Then, using calculation methods for projectiles with kinematics, we could ultimately calculate the distance X_{BC} traveled.

After we calculated it on our own, we tried to make an excel spreadsheet to maximize the distance the puck would travel based on θ of the ramp. When trying to construct a spreadsheet, we started trying to replicate our formulas in Excel but encountered some challenges. At first, we put in some of the constant values such as the first angle with the ramp, the mass of the puck, the initial velocity, acceleration due to gravity, the coefficient of friction, and the height of the ramp. We then started trying to solve for the acceleration on the ramp in the Excel, but were not getting the right value, because we did not realize that SIN and COS functions rely on angle values in radians and not degrees. After fixing that, we found the acceleration and final velocity at point B.

We then attempted to find the time that the puck took to fall to the ground by utilizing the equation with the y-component of velocity. However, this required the quadratic formula, which was challenging to do in excel because there were two values for the solution. But after putting the coefficients a, b, and c which are found in the general formula $ax^2 + bx + c = 0$. After doing this, we calculated the time by using the goal seek function in excel. First, we wrote the quadratic equation using the values of a, b, and c. Then, using the goal seek function, we set the value of the cell with the quadratic equation equal to 0 in relation to another cell, which solved the equation in that cell.

After finding the time, we were able to put that into the $\Delta x = Vt$ equation and find the final change in distance. When we found that change, we were able to maximize it and find what angle would optimize the puck landing the farthest away.

Solution:

1. Solve for acceleration:

We first used dynamics to calculate the forces acting upon the puck on the incline, in order to eventually calculate its acceleration. These included normal force, friction, gravity along the ramp, and gravity angled vertically.

To calculate acceleration, we applied the F=ma equation to the horizontal forces acting on the puck, which were friction and a component of gravity.

Friction was calculated by multiplying the given mu by the normal force ($F_N = mg\cos\theta$), which gave us 0.095 newtons. The component of gravity along the decline was calculated with $mg\sin\theta$

After applying these forces to the F=ma equation, we get:

$F=ma \rightarrow$

$mg\sin\theta - F_f = ma \rightarrow$

$(0.072)(9.8)(\sin 38) - 0.095 = (0.072)a$

And after solving, we get $a = 4.72\ m/s^2$

2. Solve for final velocity

We then used this acceleration to find the puck's velocity at the end of the declining ramp, right before it starts falling. This was done with the kinematic equation $V^2 = V_O^2 + 2a\Delta x$.

After plugging in the known values, we can solve:

$V^2 = 0^2 + 2(4.72)(2.9)$

$V = 5.23\ m/s^2$

3. Projectile Motion

The final step was to apply our knowledge of projectile motion to find the horizontal distance the puck would travel. By splitting its motion into x and y components, we can first solve for the time from the vertical component and use it to find the distance in the horizontal component.

Vertical component calculation:

$\Delta y = V_0 t + 1/2at^2$

$-1.6 = (5.23\sin38)t + ½(9.8)t^2$

t = 0.33s

Horizontal component calculation:

$\Delta x = Vt$

$\Delta x = (5.23\cos38)(0.33)$

$\Delta x = 1.36m$

4. Angle for maximum distance

The second part of the question required us to create a spreadsheet or code to run every angle and determine which one maximized the distance. Using the equation we created in the spreadsheet (which was based on the methodology we used to solve part A), we determined the farthest distance.

Steps 1 and 2:

theta	theta radians	Acceleration	Final Velocity at B
27	0.4712385	2.964686251	4.146707159

Step 3:

a	b	c	Quadratic Formula	Time	Distance (XBC)
-4.9	-1.882564185	1.6	9.65264E-07	0.410755	1.517634523

s

The highest distance was 1.518 meters, which was when there was an angle of ~27 degrees.

Extensions:

1. One extension includes the addition of a ramp to the problem. After the puck falls and hits the ground at the calculated horizontal distance X_{BC}, it immediately continues down another ramp. This would add a bit more complexity to the problem, asking solvers to calculate X_{BC} as well as the final velocity after the second ramp.
2. Similarly, we could also possibly place two ramps side by side, at different angles, affecting the acceleration and velocity of the puck before it falls. After the ramp shown in the problem, we could place another ramp that is connected, but has a shallower angle, slowing down the puck before it becomes a projectile. Students would be asked the same question as in the initial problem, to calculate X_{BC}.