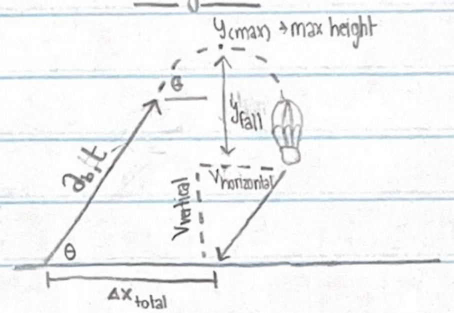
Multi-Step Rocket Problem

Given variables

- Launch angle (θ) = 56°
- Engine burn time (t) = 6.9 sec
- Net acceleration while rocket burns (a_b) = 4.2 m/s^2
- Vertical distance from max height before parachute opens (y_{fall}) = 84 m
- Rocket with parachute constant vertical speed ($V_{vertical}$) = 6.0 m/s
- Wind and rocket with parachute constant horizontal speed ($V_{horizontal}$) = 12 m/s

Diagram



Strategy

- First part → Solve for Δx with rocket burning
    - Solve for vertical and horizontal acceleration using sin and cos
    - Solve for Δx and y using the no-v equation, 0 for initial velocity, and respective acceleration
    - Solve for ending horizontal and vertical velocities using the no Δx equation
- Second part → Solve for Δx of parabola
    - Solve for time to max height by using the no-v equation and dividing by 2
    - Solve for max height using no-v equation
    - Solve for parachute fall distance by substracting y_{fall} from max height
    - Solve for fall time till parachute opens by using no-v equation where Δy is $-y_{fall}$
    - Solve for total time by adding time to max height and fall height
    - Solve for Δx using $\Delta x = vt$ where v is ending horizontal velocity of rocket burning and t is total time

- Third part → Solve for Δx of parachute and rocket
  - Find time they fall using $\Delta y = vt$ where Δy is parachute fall distance
  - Find Δx using $\Delta x = vt$
- Final
  - Add Δx from first and second parts, then substract Δx from third part

First part

$a_x = a_0 \cos\theta$

$a_x = 4.2\cos(56)$

$a_x = 2.3486\ m/s^2$

$a_y = a_0 \sin\theta$

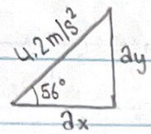
$a_y = 4.2\sin(56)$

$a_y = 3.482\ m/s^2$

$\Delta x_1 = V_{0x}t + \frac{1}{2}a_x t^2$

$\Delta x_1 = 0(6.9) + \frac{1}{2}(2.3486)(6.9)^2$

$\Delta x_1 = 55.908\ m$



$y_1 = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

$y_1 = 0 + 0(6.9) + \frac{1}{2}(3.482)(6.9)^2$

$y_1 = 82.888\ m$

$V_x = V_{0x} + a_x t$

$V_x = 0 + (2.3486)(6.9)$

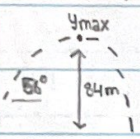
$V_x = 16.2054\ m/s$

$V_y = V_{0y} + a_y t$

$V_y = 0 + (3.482)(6.9)$

$V_y = 24.0255\ m/s$

Second part



$y = y_0 + V_y t_p + \frac{1}{2}a t_p^2$

$0 = 0 + 24.0255t_p + \frac{1}{2}(-9.8)t_p^2$

$t_p = 4.903\ sec$

$t_{max} = \frac{t_p}{2}$

$t_{max} = \frac{4.903}{2}$

$t_{max} = 2.452\ sec$

$y_{max} = y_1 + V_y t_{max} + \frac{1}{2}a t_{max}^2$

$y_{max} = 82.888 + 24.0255(2.452) + \frac{1}{2}(-9.8)(2.452)^2$

$y_{max} = 112.3382\ m$

$y_{parachute} = y_{max} - y_{fall}$

$y_{parachute} = 112.3382 - 84$

$y_{parachute} = 28.3382\ m$

$-y_{fall} = V_{0y}t_{fall} + \frac{1}{2}a t_{fall}^2$

$-84 = 0t_{fall} + \frac{1}{2}(-9.8)t_{fall}^2$

$t_{fall} = 4.14\ sec$

$t_{whole} = t_{max} + t_{fall}$

$t_{whole} = 2.452 + 4.14$

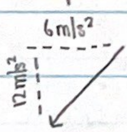
$t_{whole} = 6.592\ sec$

$\Delta x_2 = V_x t_{whole}$

$\Delta x_2 = (16.2054)(6.592)$

$\Delta x_2 = 106.8261\ m$

Third part



$y_{parachute} = V_{vertical} \cdot t_{parachute}$

$28.3382 = 6t_{parachute}$

$t_{parachute} = 4.723\ sec$

$\Delta x_3 = V_{horizontal} \cdot t_{parachute}$

$\Delta x_3 = 12(4.723)$

$\Delta x_3 = 56.676\ m$

Total Δx

$\Delta x_{total} = \Delta x_1 + \Delta x_2 - \Delta x_3$

$\Delta x_{total} = 55.908 + 106.8261 - 56.676$

$\Delta x_{total} = 106.0581$

106.1 m east