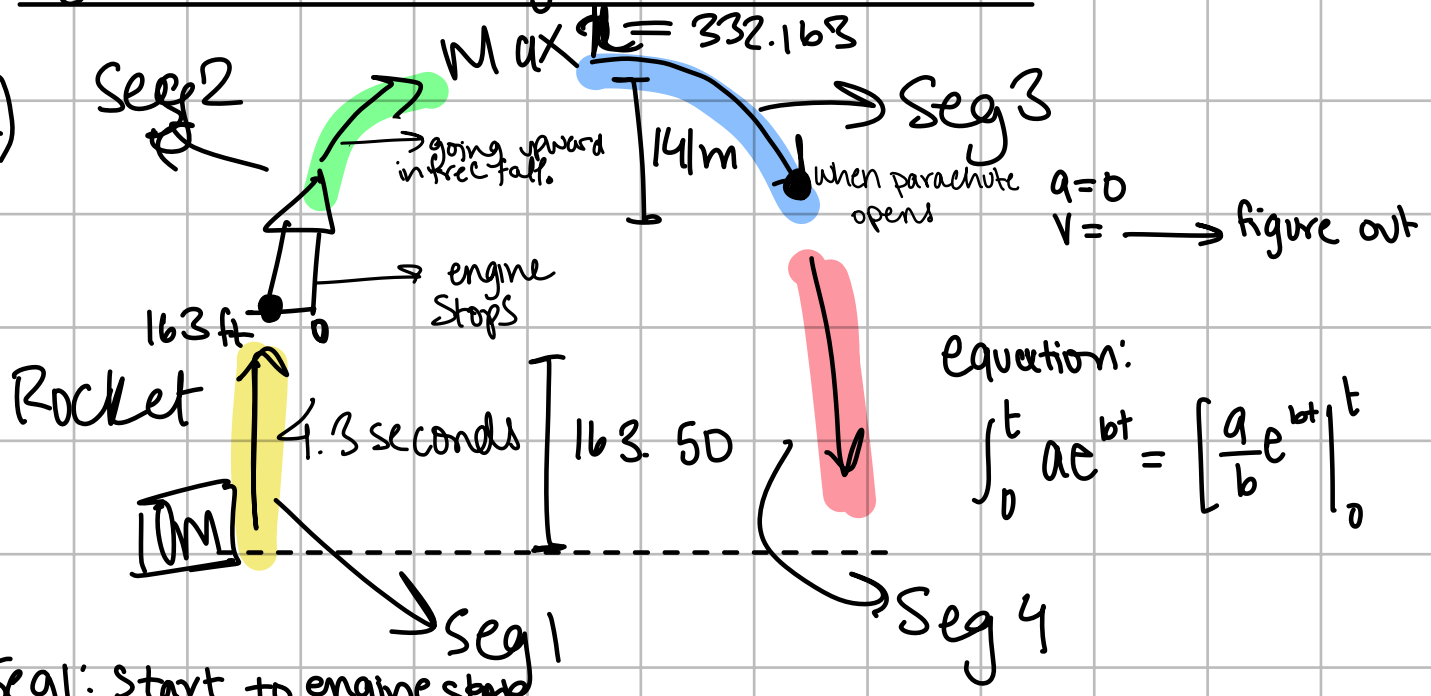
Physics: Multi-Stage Rocket Problem

a)



Seg1: start to engine stop

seg2:

Given:

Net Energy: $a = -1.4t^2 + 22\ m/s^2$

Engine Burn Time: $t_e = 4.3$ sec

Distance between max & parachute: $h = 141\ m$

Speed of rocket with parachute: $V_p = -17(1-e^{-\frac{4}{17}t})\ \frac{m}{s}$

$\hookrightarrow V_p = -17 + 17e^{-\frac{4}{17}t}\ \frac{m}{s}$

Calculate entire time in air.

1) Engine Distance Covered

Segment Between launch to engine stop is 4.3 sec.

Figure out distance

$$= \int_0^{4.3} -1.4t^2 + 22 = \left[\frac{-1.4}{3}t^3 + 22t\right|_0^{4.3}$$

$$= -\frac{1.4}{3}(4.3)^3 + 22(4.3) = 57.496 \text{ m/s}$$

$$- \left(-\frac{1.4}{3}(0)^3 + 22(0)\right) = \boxed{57.496 \text{ m/s}}$$

$\frac{14}{10} \div 3 = \frac{14}{30} = \frac{7}{15}$

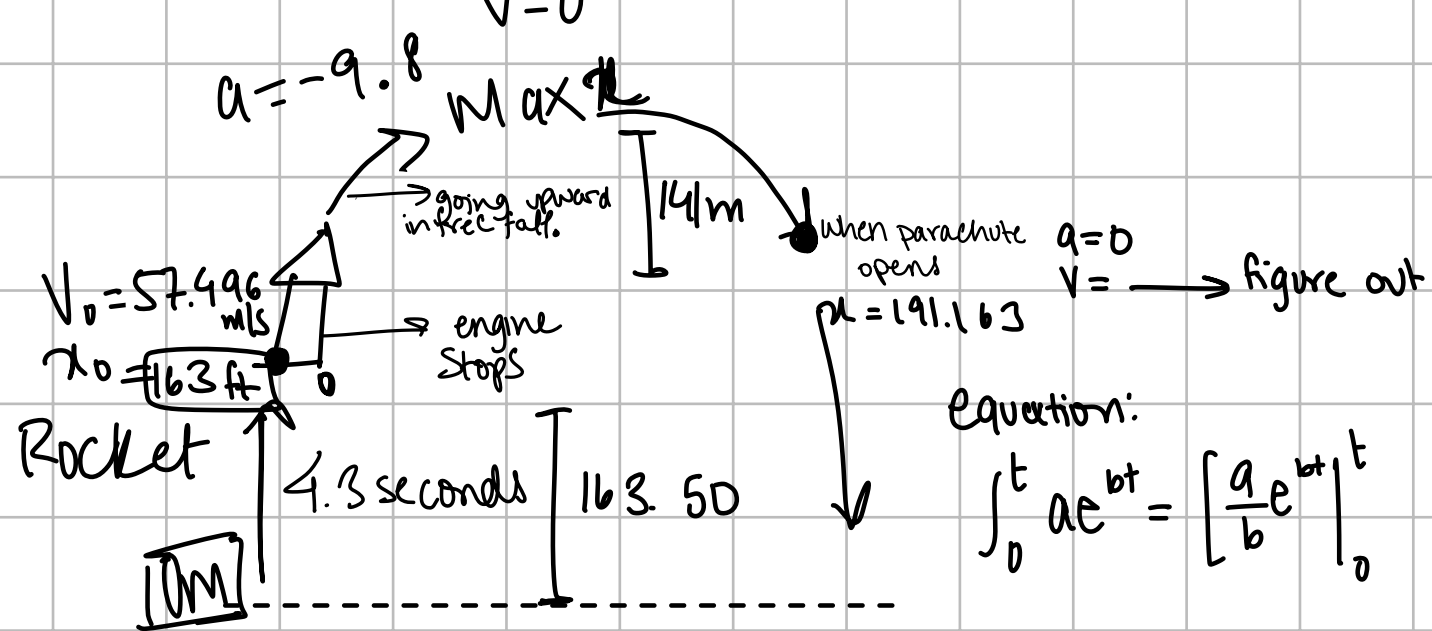
↳ rocket velocity reaching till end of segment 1

$$= \int_0^{4.3} -\frac{7}{15}t^3 + 22t = \left[-\frac{7}{60}t^4 + 11t^2\right]_0^{4.3}$$

$$= -\frac{7}{60}(4.3)^4 + 11(4.3)^2 - \left(-\frac{7}{60}(0)^2 + 11(0)^2\right)$$

$= \boxed{163.50 \text{ m}}$ distance covered by rocket b/w launch to engine stop

2) Figuring out Free Fall Phase (Segment 2)



equation:

$$\int_0^t ae^{bt} = \left[\frac{a}{b}e^{bt}\right]_0^t$$

$x = ?$

$x_0 = 163.50\text{m}$

$$0^2 = (57.496)^2 + 2(-9.8)\Delta x$$

$V_0 = 57.496$ m/s

$V = 0$ m/s

$a = -9.8$ m/s^2

$0^2 = 3305.790 - 19.6\Delta x$

$19.6\Delta x = 3305.790$

$x = 168.663$ distance between engine stop & max

3) Figuring Time in Segment 2 Based of distance

$x = 163.50 + 168.663$ m

$x_0 = 163.50$ m

$V_0 = 57.496$ m/s

$V = 0$ m/s

$a = -9.8$ m/s^2

$t = ?$

$V = V_0 + at$

$0 = 57.496 + (-9.8)(t)$

$-57.496 = -9.8t$

$t = 5.866$ sec

4) Calculate time from max to parachute.

Max $= 168.663 + 163.50$

Max $= 332.163$ m

Parachute $= 332.163\text{m} - 141\text{m} =$ 191.163 m

$x = 191.163$m

$x_0 = 332.163$m

$V = ? = 52.570$ m/s

$V_0 = 0$

$a = -9.8$

$t = ?$

$V^2 = V_0^2 + 2a\Delta x$

$V^2 = 0^2 + 2(-9.8)(-141)$

$V^2 = 2763.6$

$V = 52.570$ m/s

Figuring out t:

$$52.570 \text{ m/s} = 0 + 9.8t$$

$$\boxed{t = 5.364 \text{ sec}}$$

5) **Calculate parachute phase.**

Integrating the equation for V_p

$$V_p = -17\left(1 - e^{-t/9}\right)$$

$$V_p = -17 + 17e^{-t/9}$$

$$\int_0^t 17e^{-\frac{1}{9}t} - 17 = \left[-153e^{-1/9t} - 17t\right]_0$$

↳ put in t & 0

↳ graph

$$= -153e^{-1/9t} - 17t - \left(-153e^{-1/9(0)} - 17(0)\right)$$

↙ Position of parachute following

$$-191.163 = -153e^{-1/9t} - 17t - 153$$

$$0 = -153e^{-1/9t} - 17t + 153 + 191.163$$

344.163

$x = 19.176 \text{ sec}$

6) Total time:

$= 19.176 + 5.364 + 5.866 + 4.3$

$= 34.71 \text{ sec}$

Final Diagram