

Newton Divided Differences and Nested Multiplication

The following algorithm overwrites given values $a_0, \dots, a_n$ with their Newton divided differences at distinct $x_0, \dots, x_n$.

NEWTON DIVIDED DIFFERENCES:

Given $a_0, \dots, a_n$ and distinct $x_0, \dots, x_n$,

For $k = 1, \dots, n$

For $j = n, \dots, k$

Update $a_j \leftarrow (a_j - a_{j-1}) / (x_j - x_{j-k})$.

The following algorithm evaluates the interpolating polynomial in Newton form at a point x , given the Newton divided differences $a_0, \dots, a_n$ at distinct $x_0, \dots, x_n$.

NESTED MULTIPLICATION:

Given $a_0, \dots, a_n$ and distinct $x_0, \dots, x_n$,

Set $pval = a_n$.

For $k = n - 1, \dots, 0$

Update $pval \leftarrow a_k + (x - x_k) \cdot pval$.