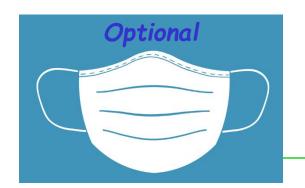
WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS ME-3320, B'2025

Lecture 15

December 2025





Notches and stress concentrations

- ☐ Master Examples 6-1 and 6-2: estimating S-N diagrams
- Master Example 6-3: determining fatigue stress-concentration factors





Notches and stress concentrations

□ Notches introduce stress-concentrations. See lectures 07-08, 13, and 14

Shaft with keyway



- \square Correcting for stress-concentrations. Stress concentration factors in fatigue: $K_f,K_{f\!s}$
- ☐ Use of stress concentration factors in fatigue:

$$\sigma = K_f \sigma_{\text{nominal}}$$

$$\tau = K_{fs} \ \tau_{\text{nominal}}$$





Notches and stress concentrations

☐ Stress concentration factors in fatigue:

$$K_f = 1 + q(K_t - 1)$$

- \square Theoretical (static) stress-concentration factor: K_t
- \square Notch sensitivity factor: $q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$
- □ Neuber's constant (depends on the value of the ultimate tensile strength of the material used).
 See, for example, Tables 6-6, 6-7, and 6-8

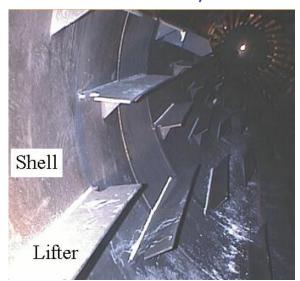




Residual stresses: must be taken into account

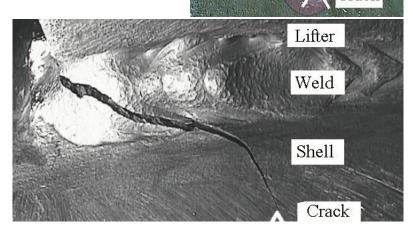
- Residual stress are built-in or introduced (typically during manufacturing) to an unloaded part.
- ☐ Residual stresses can be the cause of crack initiation and, therefore, fatigue failure

Example: rotary dryer. Welding lifters to a rotary shell



Source: ASM International

Residual stresses introduced during welding caused crack initiation

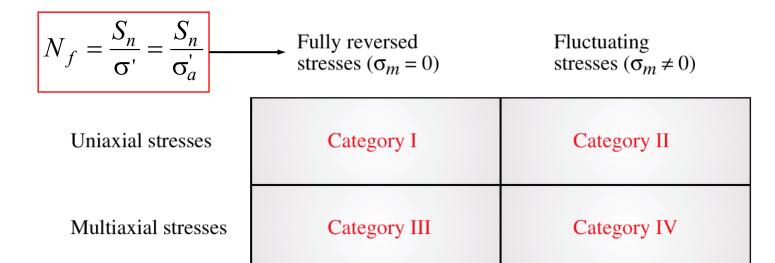






Designing for HCF

☐ Fatigue design situations



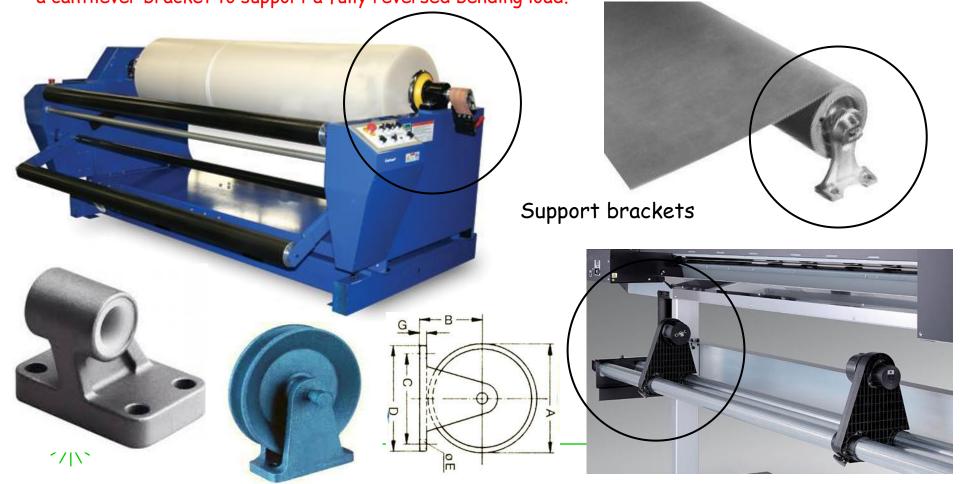




Designing for HCF. $N_f=2.5$

□ Review Example 6-4: under fully-reversed bending: parametric approach

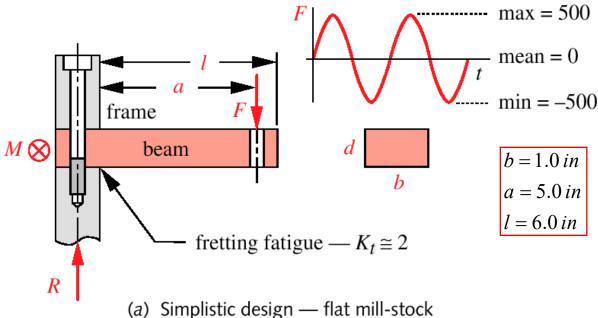
A feed-roll assembly is to be mounted at each end on support brackets cantilevered from a machine frame. Examples of cantilevered bracket configurations are shown in the figures. Task is to design a cantilever bracket to support a fully reversed bending load.



Designing for HCF. $N_f=2.5$

☐ Review Example 6-4: under fully-reversed bending: parametric approach

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from the machine frame as shown in the Figure. The feed rolls experience a fully reversed load of 1000 lb amplitude, split equally between the two support brackets. Design a cantilever bracket to support a fully reversed bending load of 500 lb amplitude for 109 cycles with no failure. Its dynamic deflection cannot exceed 0.01 in.



Other initial assumptions:

$$r/d = 0.5;$$

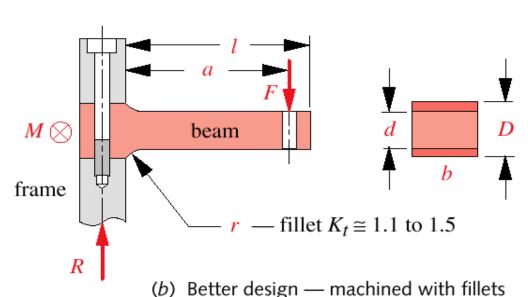
 $D/d = 1.125;$
 $b/d = 2$
 Low -carbon steel:
 $S_{ut} = 80ksi$





Designing for HCF. $N_f=2.5$

☐ Review Example 6-4: under fully-reversed bending: iterative approach



- ☐ Fully reversed load of 1000 lb (amplitude is, therefore, 500 lbs)
- ☐ Life of about 10 ⁹ cycles
- □ Material: steel/machined
- □ Operating conditions: room temp.

Initial assumptions: r/d=0.5; D/d=1.125

b = 1.0 in d = 0.75 in D = 0.94 in r = 0.25 in a = 5.0 inl = 6.0 in

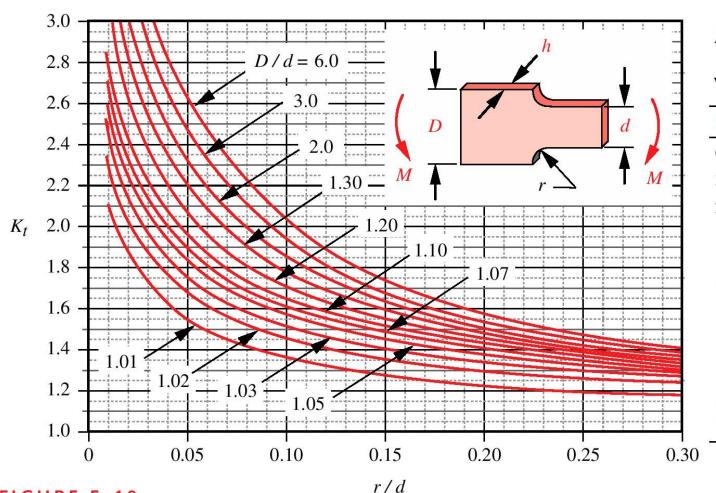
☐ Comment: use MathCad example to perform design iterations (file is included in the CD-ROM that came with your textbook)





Designing for HCF

□ Review Example 6-4: under fully-reversed bending



$$K_t \cong A \left(\frac{r}{d}\right)^b$$

where:

D/d	\boldsymbol{A}	b
6.00	0.895 79	-0.358 47
3.00	0.907 20	-0.333 33
2.00	0.932 32	-0.303 04
1.30	0.958 80	-0.272 69
1.20	0.995 90	-0.238 29
1.10	1.016 50	-0.215 48
1.07	1.019 90	-0.203 33
1.05	1.022 60	-0.191 56
1.03	1.016 60	-0.178 02
1.02	0.995 28	-0.170 13
1.01	0.966 89	-0.154 17
(6)		

FIGURE E-10

Designing for HCF

□ Review Example 6-4: under fully-reversed bending

Table 6-6

Neuber's Constant for Steels

S _{ut} (ksi)	\sqrt{a} (in ^{0.5})	
50	0.130	
55	0.118	
60	0.108	
70	0.093	
80	0.080	
90	0.070	
100	0.062	
110	0.055	
120	0.049	
130	0.044	
140	0.039	
160	0.031	
180	0.024	
200	0.018	
220	0.013	
240	0.009	

Table 6-7

Neuber's Constant for Annealed Aluminum

S _{ut} (kpsi)	\sqrt{a} (in ^{0.5})
10	0.500
15	0.341
20	0.264
25	0.217
30	0.180
35	0.152
40	0.126
45	0.111
	•

Table 6-8

Neuber's Constant for Hardened Aluminum

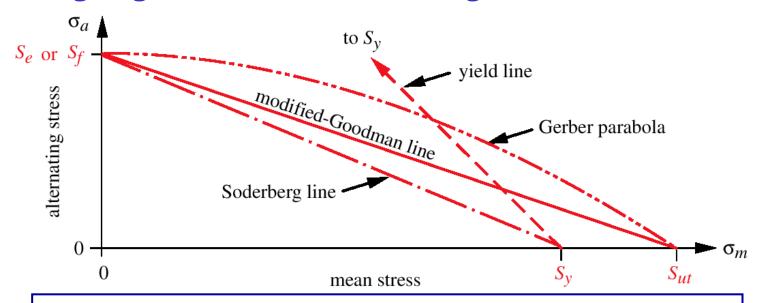
S _{ut} (kpsi)	\sqrt{a} (in ^{0.5})
15	0.475
20	0.380
30	0.278
40	0.219
50	0.186
60	0.162
70	0.144
80	0.131
90	0.122

May need to do curve fitting in order to determine Neuber's constant functions:

$$y = f(x)$$
 $y = \text{Neuber' s constant} = \sqrt{a}$
 $x = S_{\text{ut}}$



Designing for HCF: fluctuating uniaxial stresses



$$S_a = S_e \left(1 - \frac{\sigma_m^2}{\sigma_{ut}^2} \right)$$

Gerber parabola: $S_a = S_e \left(1 - \frac{\sigma_m^2}{\sigma_m^2} \right)$ (Fits experimental data: useful to study failed parts)

Modified-Goodman line:
$$S_a = S_e \left(1 - \frac{\sigma_m}{\sigma_{ut}} \right)$$
 (Conservative theory)

Soderberg line:
$$S_a = S_e \left(1 - \frac{\sigma_m}{\sigma_y} \right)$$
 (Overly conservative theory)



Modified Goodman-diagram

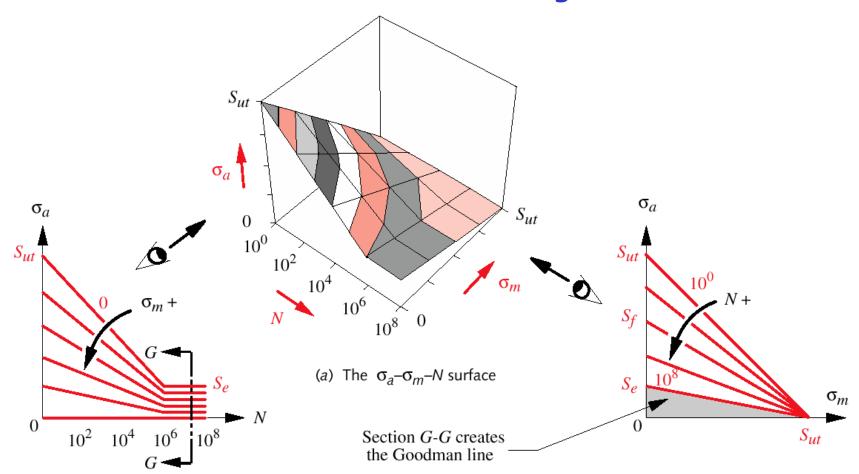


FIGURE 6-43

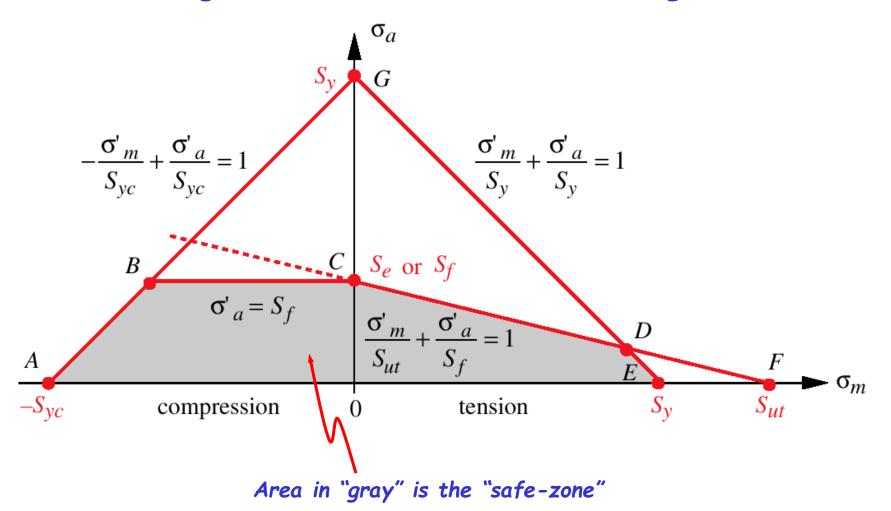
(b) The σ_a -N projection (S-N diagrams)



(c) The σ_a - σ_m projection (constant life diagrams)



Augmented modified Goodman-diagram

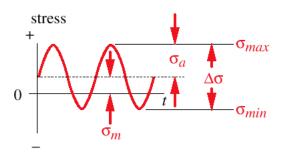




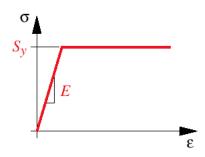


Stress-concentration factors in fluctuating stresses

Note that component may "yield" locally



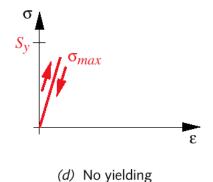
 $F \longrightarrow G > S_y$ $G > S_y$

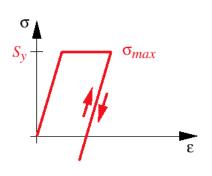


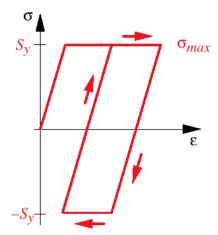
(a) Fluctuating stress

(b) Possible plastic zones

(c) Elastic-perfectly plastic material







(e) Yielding on first cycle

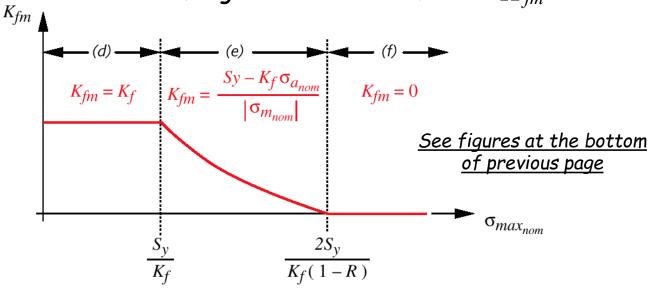
(f) Reversed yielding





Stress-concentration factors in fluctuating stresses

Mean stress fatigue-concentration factor: K_{fm}



(g) K_{fm} as a function of the maximum nominal stress $\sigma_{max_{nom}}$

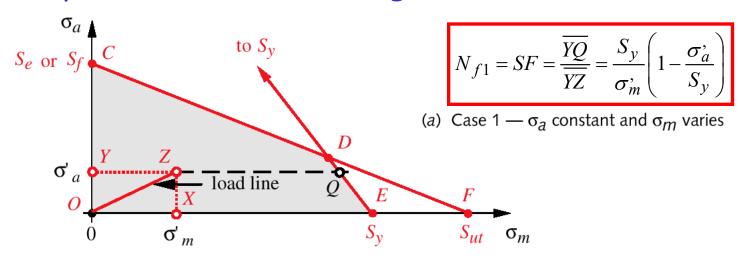
$$\begin{cases} & \text{if } K_f | \sigma_{\text{max}_{\text{nominal}}} | < S_y & \longrightarrow & K_{fm} = K_f \\ & \text{if } K_f | \sigma_{\text{max}_{\text{nominal}}} | > S_y & \longrightarrow & K_{fm} = \frac{S_y - K_f \sigma_{a_{\text{nominal}}}}{\sigma_{m_{\text{nominal}}}} \\ & \text{if } K_f | \sigma_{\text{max}_{\text{nominal}}} - \sigma_{\text{min}_{\text{nominal}}} | > 2S_y & \longrightarrow & K_{fm} = 0 \end{cases}$$

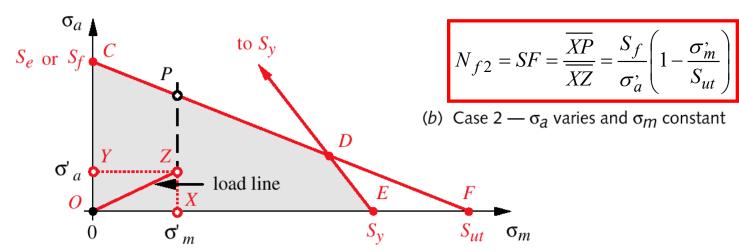




Fatigue failure - Modified Goodman's diagram

Safety factors in fluctuating stresses: Cases 1 and 2

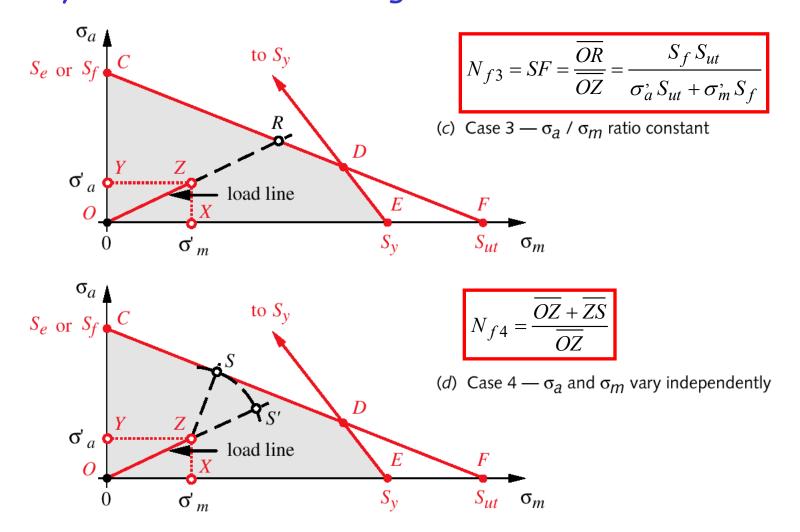








Fatigue failure - Modified Goodman's diagram Safety factors in fluctuating stresses: Cases 3 and 4



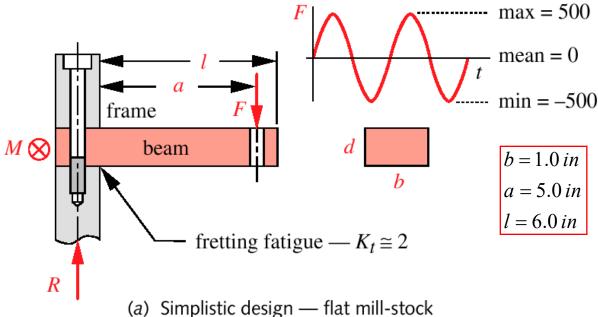




Designing for HCF. $N_f=2.5$

☐ Review Example 6-4: under fully-reversed bending: parametric approach

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from the machine frame as shown in the Figure. The feed rolls experience a fully reversed load of 1000 lb amplitude, split equally between the two support brackets. Design a cantilever bracket to support a fully reversed bending load of 500 lb amplitude for 109 cycles with no failure. Its dynamic deflection cannot exceed 0.01 in.



Other initial assumptions:

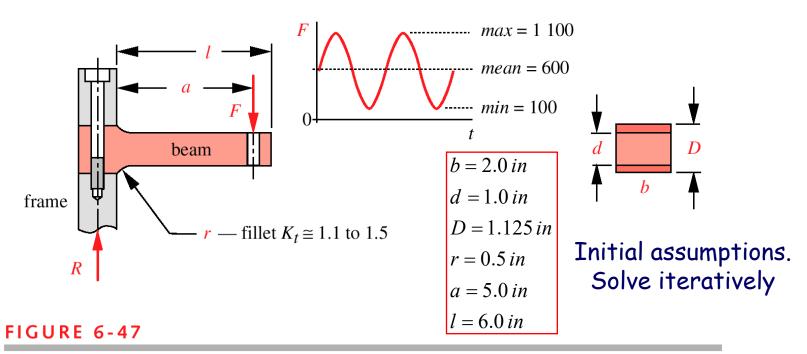
$$r/d = 0.5;$$
 $D/d = 1.125;$
 $b/d = 2$
 Low -carbon steel:
 $S_{ut} = 80ksi$





Review examples

☐ Example 6-5: fatigue under fluctuating bending. Design bracket to support the load. <u>Verify for maximum deflections</u>



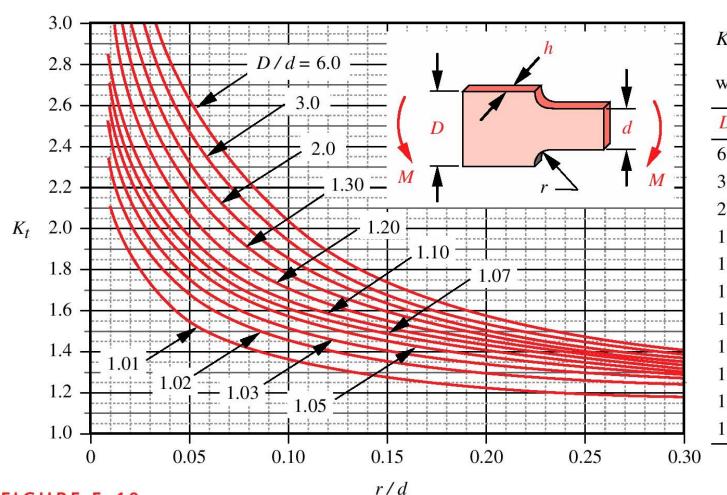
Design of a Cantilever Bracket for Fluctuating-Bending Loading





Designing for HCF

□ Review Example 6-4: under fully-reversed bending



$$K_t \cong A \left(\frac{r}{d}\right)^b$$

where:

D/d	\boldsymbol{A}	b
6.00	0.895 79	-0.358 47
3.00	0.907 20	-0.333 33
2.00	0.932 32	-0.303 04
1.30	0.958 80	-0.272 69
1.20	0.995 90	-0.238 29
1.10	1.016 50	-0.215 48
1.07	1.019 90	-0.203 33
1.05	1.022 60	-0.191 56
1.03	1.016 60	-0.178 02
1.02	0.995 28	-0.170 13
1.01	0.966 89	-0.154 17
(6)		

FIGURE E-10

Review examples

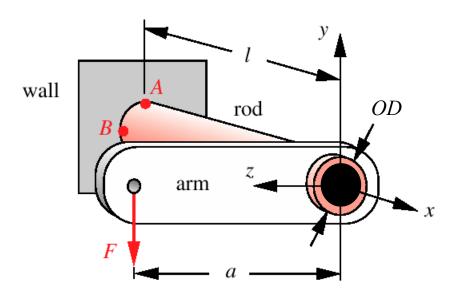
For next lecture: Master Examples 6-5 and and 6-6; use and understand corresponding MathCad solutions (in the CD that came with your book and/or on Norton's Machine Design website)





Review examples

☐ Example 6-6: multiaxial fluctuating stresses. Verify the design against failure (e.g., by determining safety factors, other?)



Notch radius (wall) is 0.25", $K_t=1.70$, $K_{ts}=1.35$

- ☐ Applied load: sinusoidal [-200,340] lb
- ☐ Finite life of about 6x10 7 cycles
- Material: Al 2024-T4
- Operating conditions: room temp.

Initial dimensions:

$$ID = 1.5 in$$

 $OD = 2 in$
 $a = 8.0 in$
 $l = 6.0 in$

☐ Comment: use MathCad example to perform design iterations (file is included in the CD-ROM that came with your textbook)



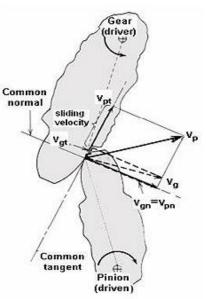


Review examples

☐ Example 6-6: multiaxial fluctuating stresses.

Example of mechanical configurations that can be idealized with arrangement shown in Example 6-6.









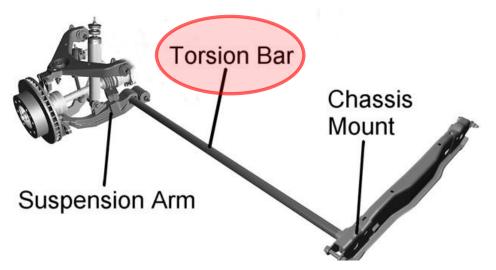


Review examples

☐ Example 6-6: multiaxial fluctuating stresses.

Example of mechanical configurations that can be idealized with arrangement shown in Example 6-6.









Fatigue failure: Review examples

☐ Example 6-6: Multiaxial Fluctuating Stresses

Problem Determine the safety factors for the bracket tube shown in Figure 5-7.

Units $ksi := 10^3 \cdot psi$

Given The material is 2024-T4 aluminum

Yield strength $S_y := 47 \cdot ksi$

Tensile strength $S_{ut} := 68 \cdot ksi$

Tube length $l := 6 \cdot in$ Arm length $a := 8 \cdot in$ Tube OD $od := 2.0 \cdot in$ Tube ID $id := 1.5 \cdot in$

Load $F_{min} := -200 \cdot lbf$ $F_{max} := 340 \cdot lbf$

Assumptions The load is dynamic and the assembly is at room temperature.

Consider shear due to transverse loading as well as other stresses.

A finite life design will be sought with a life of $N := 6.10^7$ cycles.

The notch radius at the wall is r = 0.25 in and

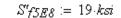
stress-concentration factors are for bending $K_t = 1.7$, and for

shear, $K_{ts} := 1.35$.

Solution See Figure 5-7 and Mathcad file EX06-06. Also see Example 4-9 for

a more complete explanation of the stress analysis for this problem.

Aluminum does not have an endurance limit. Its endurance strength at 5E8 cycles can be estimated from equation 6.5c. Since the S_{uf} is larger than 48 ksi, the uncorrected S'_{fOSES} is







2 The correction factors are calculated from equations 6.7 and used to find a corrected idurance strength at the standard 5E8 cycles.

fatigue - $C_{load} := 1.0$ for bending $Ag_5 := 0.0105 \cdot od^2$ $Ag_5 = 0.042 in^2$ Make sure to know how to evaluate A_{05} $C_{size} := 0.869 \cdot \left(\frac{d_{eq}}{in}\right)^{-0.097}$ $C_{size} = 0.895$ Note "negative" A := 2.7 b := -0.265Table 6-3 constants exponent $C_{surf} := A \left(\frac{S_{ut}}{t_{res}} \right)^b$ (a)

$$S_{ut}$$
 is used in $kpsi$

$$C_{temp} := 1$$

$$C_{reliab} := 0.753$$

for 99.9%

Note that this is only 16.6% of the S_{ut} (and 24 % of the S_v)

$$S_{f5E8} := C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S_{f5E8}$$
 (b)

$$\rightarrow$$
 $S_{f5E8} = 11.30 \, ksi$

Note that the bending value of C_{load} is used despite the fact that there is both bending and torsion present. The torsional shear stress will be converted to an equivalent tensile stress with the von Mises calculation. C_{surf} is calculated from equation 6.7e using data from Table 6-3. This corrected fatigue strength is still at the tested number of cycles, N = 5E8.



3 This problem calls for a life of 6E7 cycles, so a strength value at that life must be estimated from the S-N line of Figure 6-33b using the corrected fatigue strength at that life. Equation 6.10a for this line can be solved for the desired strength after we compute the values of its coefficients a and b from equation 6.10c.

 $S_m = 61.2 \, ksi$

 $S_m := 0.90 \cdot S_{t/t}$

This is calculated differently for materials with an
$$S_e$$

From Table 6-5 for 5E8
$$z := 5.699$$

$$b := -\frac{1}{z} \cdot log \left(\frac{S_m}{S_{f5E8}} \right) \qquad b = -0.1288 \qquad (c)$$

$$a := \frac{S_m}{10^{3 \cdot b}} \qquad a = 148.9 \, ksi$$

$$S_n := a \cdot N^b \qquad S_n = 14.84 \, ksi$$

Note that S_m is calculated as 90% of S_{ut} because loading is bending rather than axial (see Eq. 6.9). The value of z is taken from Table 6-5 for N=5E8 cycles. This is a corrected fatigue strength for the shorter life required in this case and so is larger than the corrected test value, which was calculated at a longer life.

4 The notch sensitivity of the material must be found to calculate the fatigue stress-concentration factors. Table 6-8 shows the Neuber factors for hardened aluminum. Interpolation gives a value of a := 0.147² in at the material's S_{ut}. Equation 6.13 gives the resulting notch sensitivity for the assumed notch radius.

$$q := \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \qquad \qquad q = 0.773 \tag{a}$$

K_t and K_{ts} are given (we are lucky!)

5 The fatigue stress-concentration factors are found from equation 6.11b using the given geometric stress-concentration factors for bending and torsion, respectively.

$$K_f := 1 + q \cdot (K_l - 1)$$
 $K_f = 1.541$ (e)

$$K_{f\bar{s}} := 1 + q \cdot (K_{t\bar{s}} - 1)$$
 $K_{f\bar{s}} = 1.270$ (f)





6 The bracket tube is loaded in both bending (as a cantilever beam) and in torsion. The shapes of the shear, moment and torque distributions are shown in Figure 4-30. All are maximum at the wall. The alternating and mean components of the applied force, moment, and torque at the walls are

Forces: evaluated for amplitude and mean components

Loads
$$F_{\alpha} := \frac{F_{max} - F_{min}}{2} \qquad F_{\alpha} = 270 \, lbf$$

$$F_{m} := \frac{F_{max} + F_{min}}{2} \qquad F_{m} = 70 \, lbf$$

Moments: evaluated for amplitude, mean, and maximum components

Moments
$$M_{\alpha}:=F_{\alpha}\cdot l$$
 $M_{\alpha}=1620\,lbf\cdot in$ (h) $M_{m}:=F_{m}\cdot l$ $M_{m}=420\,lbf\cdot in$ $M_{max}:=M_{\alpha}+M_{m}$ $M_{max}=2040\,lbf\cdot in$

$$a:=8.0 \cdot in$$

$$T_{a}:=F_{a} \cdot a \qquad T_{a}=2160 \, lbf \cdot in$$

$$T_{m}:=F_{m} \cdot a \qquad T_{m}=560 \, lbf \cdot in$$

Evaluated for amplitude and mean components

7 The fatigue stress-concentration factor for the mean stresses depends on the relationship between the maximum local stress in the notch and the yield strength as defined in equation 6.17, a portion of which is shown here.

Outer fiber
$$c:=0.5 \cdot od$$
 $c=1.000 in$

Moment of $I:=\frac{\pi}{64} \cdot \left(od^4 - id^4\right)$ $I=0.5369 in^4$
 $J:=2 \cdot I$ $J=1.0738 in^4$

If
$$K_f \cdot |\sigma_{max}| < S_y$$
 then $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$

Torques

$$K_f \cdot \left| \frac{M_{max} \cdot c}{I} \right| = 5.86 \, ksi$$

which is less than
$$S_y = 47 \text{ ksi so}$$
, $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$

Compensate for local "yield," if any

(j)



In this case, there is no reduction in stress-concentration factors for the mean stress because there is no yielding at the notch to relieve the stress concentration.



8 The largest tensile bending stress will be in the top or bottom outer fiber at points A or A'. The largest torsional shear stress will be all around the outer circumference of the tube. (See Example 4-9 for more details.) First take a differential element at point A or A' where both of these sresses combine. (See Figure 4-32.) Find the alternating and mean components of the normal bending stress and of the torsional shear stress on point A using equations 4.11b and 4.24b, respectively.

9 Find the alternating and mean von Mises effective stresses at point A from equation 6.22b.

Evaluate equivalent Mises, amplitude and mean, stresses
$$\sigma_{xa} := \sigma_{a} \qquad \sigma_{ya} := 0 \cdot psi \qquad \tau_{xya} := \tau_{a}$$

$$\sigma_{a}' := \sqrt{\sigma_{xa}^{2} + \sigma_{ya}^{2} - \sigma_{xa} \cdot \sigma_{ya} + 3 \cdot \tau_{xya}^{2}} \qquad \sigma_{a}' = 6.42 ksi$$

$$\sigma_{xm} := \sigma_{m} \qquad \sigma_{ym} := 0 \cdot psi \qquad \tau_{xym} := \tau_{m}$$

$$\sigma_{m}' := \sqrt{\sigma_{xm}^{2} + \sigma_{ym}^{2} - \sigma_{xm} \cdot \sigma_{ym} + 3 \cdot \tau_{xym}^{2}} \qquad \sigma_{m}' = 1.66 ksi$$

10 Because the moment and torque are both caused by the same applied force, they are synchronous and in-phase and any change in them will be in a constant ratio.

This is a Case 3 situation and the safety factor is found using equation 6.18e.

Evaluate for all cases, if unsure about which case

$$N_f := \frac{S_n \cdot S_{ut}}{\sigma'_{a} \cdot S_{ut} + \sigma'_{m} \cdot S_{n}}$$

$$N_f = 2.2$$
 At point A



Note the use of

(finite life)

 S_n in this equation

11 Since the tube is a short beam, we need to check the shear due to transverse loading at point B on the neutral axis where the torsional shear is also maximal. The maximum transverse shear stress at the neutral axis of a hollow, thin-walled, round tube was given as equation 4.15d.

Cross-section area
$$A := \frac{\pi}{4} \cdot (od^2 - id^2)$$
 $A = 1.374 in^2$

$$au_{abend} := K_{fs} \cdot \frac{2 \cdot F_a}{A}$$
 $au_{abend} = 499 \, psi$

$$au_{mbend} := K_{fsm} \cdot \frac{2 \cdot F_m}{A}$$
 $au_{mbend} = 129 \, psi$
 $au_{mbend} = 129 \, psi$

Account for transversal shear point B is subjected to pure shear

Point B is in pure shear. The total shear stress at point B is the sum of the transverse shear stress and the torsional shear stress which act on the same planes of the element.

$$au_{atotal} := au_{abend} + au_{a}$$
 $au_{atotal} = 3055 \, psi$

$$au_{mtotal} := au_{mbend} + au_{m}$$
 $au_{mtotal} = 792 \, psi$

12 Find the alternating and mean von Mises effective stresses at point B from equation 6.22b.

Mises, amplitude and mean, stresses

Evaluate equivalent Mises, amplitude and mean, stresses
$$\sigma_{xa} := 0 \cdot psi \qquad \sigma_{ya} := 0 \cdot psi \qquad \tau_{xya} := \tau_{atotal}$$

$$\sigma_{a} := \sqrt{\sigma_{xa}^{2} + \sigma_{ya}^{2} - \sigma_{xa} \cdot \sigma_{ya} + 3 \cdot \tau_{xya}^{2}} \qquad \sigma_{a} = 5.29 \, ksi$$

$$\sigma_{xm} := 0 \cdot psi \qquad \sigma_{ym} := 0 \cdot psi \qquad \tau_{xym} := \tau_{mtotal}$$

$$\sigma_{m} := \sqrt{\sigma_{xm}^{2} + \sigma_{ym}^{2} - \sigma_{xm} \cdot \sigma_{ym} + 3 \cdot \tau_{xym}^{2}} \qquad \sigma_{m} = 1.37 \, ksi$$

13 The safety factor for point B is found using equation 6.18e.

Evaluate for all cases, if unsure about which case

$$Nf := \frac{S_n \cdot S_{ut}}{\sigma'_{a} \cdot S_{ut} + \sigma'_{m} \cdot S_{ut}}$$

$$N_f = 2.7$$
 At point B

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Note the use of

(finite life)

 S_n in this equation

Reading

- Chapters 6 of textbook: Sections 6.5 to 6.8
- Review notes and text: ES2501, ES2502

Homework assignment

- * Author's: as indicated in Website of our course
- Solve: as indicated in Website of our course



