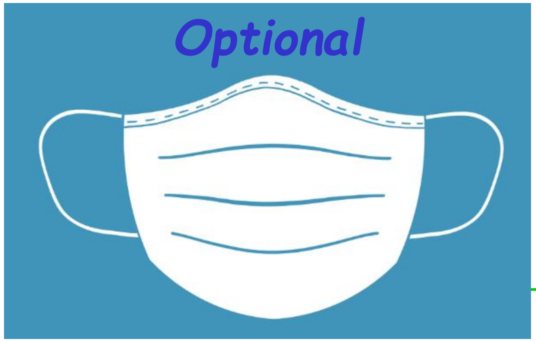
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS
ME-3320, B'2025

Lecture 10-11

November 2025



Static failure theories

Accepted failure theories that apply to **ductile** materials:

- *Total strain energy theory*
- *Distortion energy theory*
- *Pure shear-stress theory*
- *Maximum shear-stress theory*
- *Maximum normal stress theory (limited application)*

Accepted failure theories that apply to **brittle** materials:

- *Maximum normal stress theory (even material)*
- *Maximum normal stress theory (uneven material)*
- *Coulomb-Mohr theory*
- *Modified Mohr theory*

Static failure theories

Ductile materials

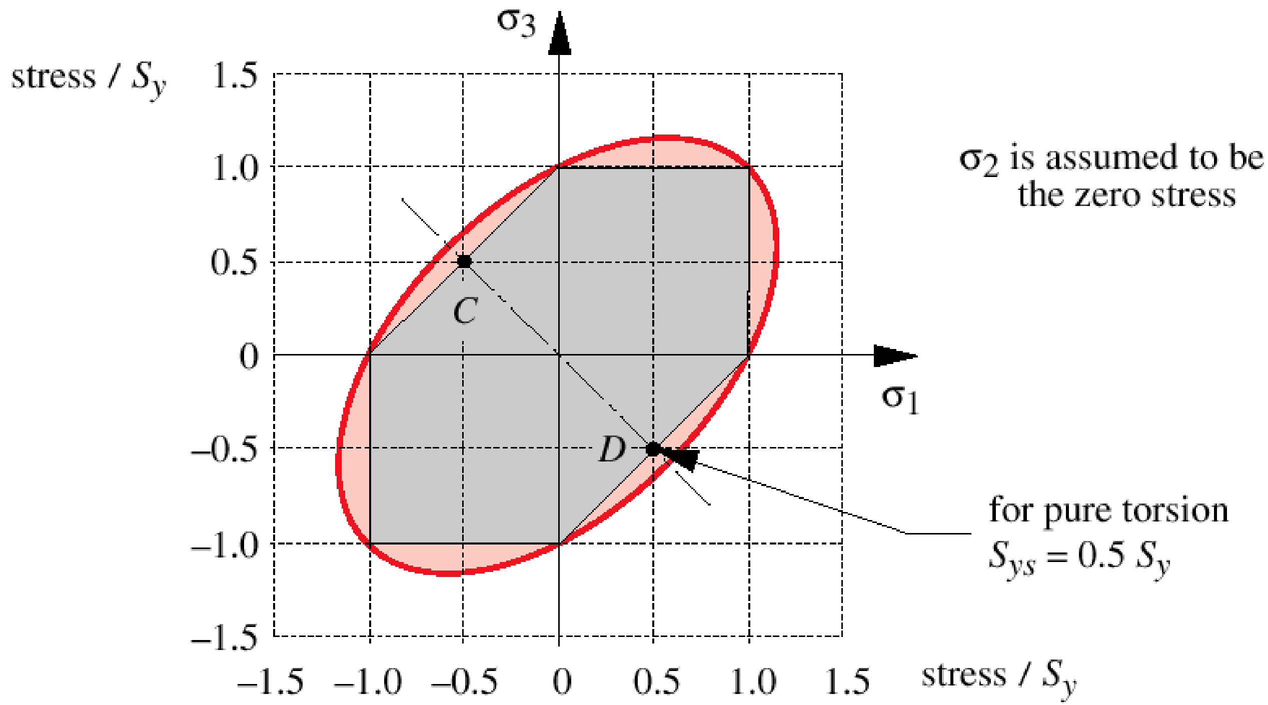


FIGURE 5-5

The 2-D Shear-Stress Theory Hexagon Inscribed Within the Distortion-Energy Ellipse

Static failure theories: experimental verifications
Ductile & brittle materials

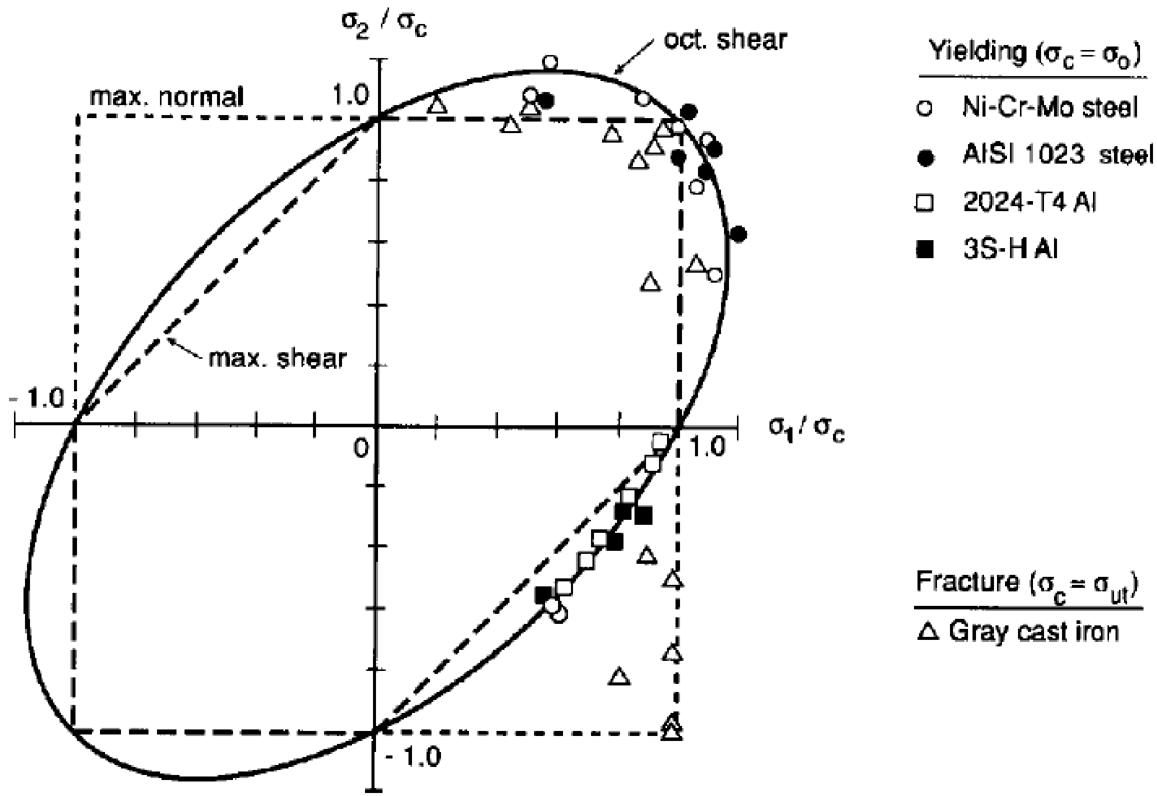


FIGURE 5-8

Experimental Data from Tensile Tests Superposed on Three Failure Theories *(Reproduced from Fig. 7.11, p. 252, in Mechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993)*

Static failure theories
Brittle materials

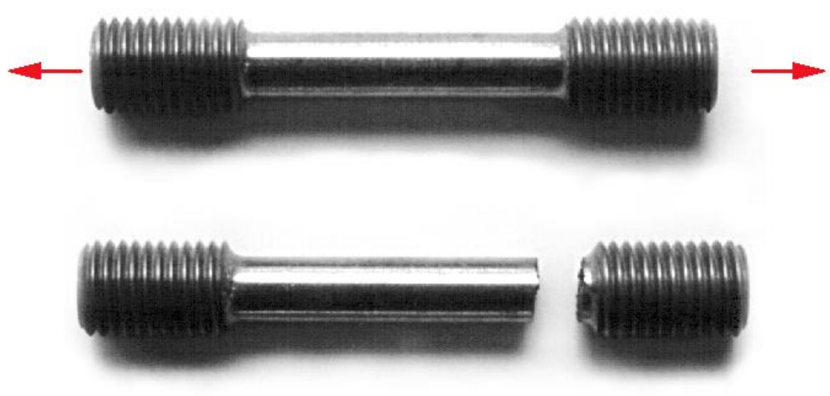


FIGURE 2-5

A Tensile Test Specimen of Brittle Cast Iron After Fracture

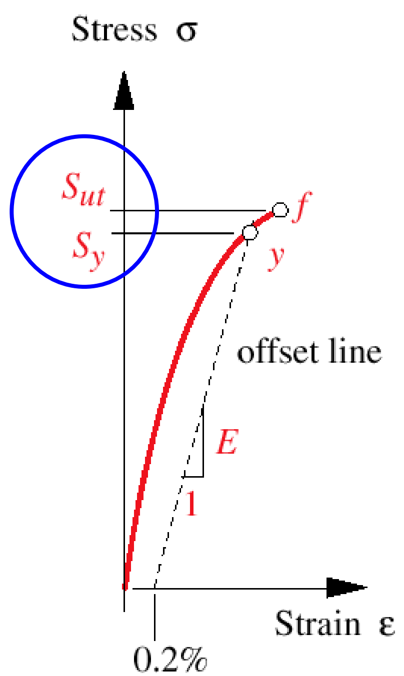
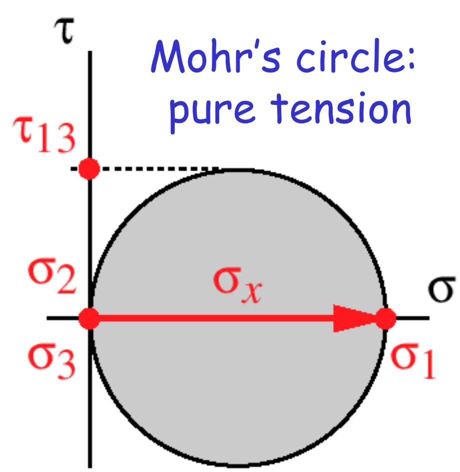


FIGURE 2-4

Stress-Strain Curve of a Brittle Material



Static failure theories

Brittle materials

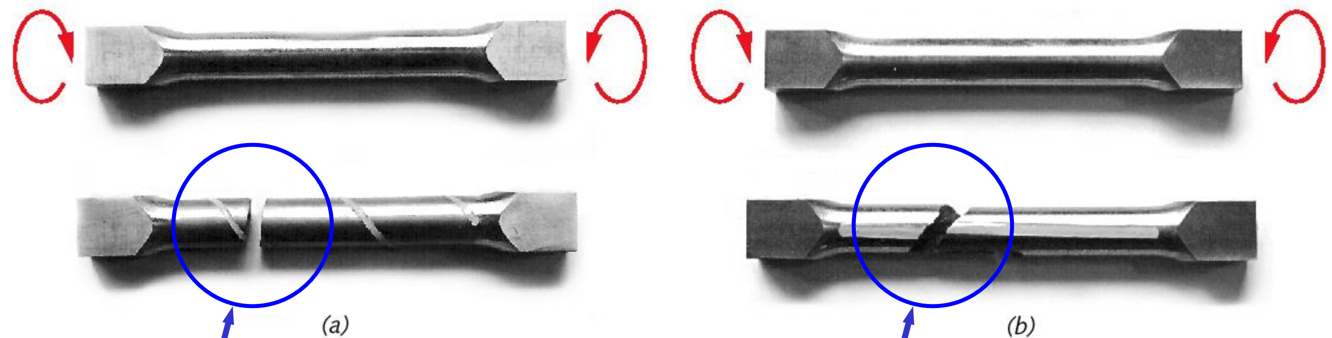
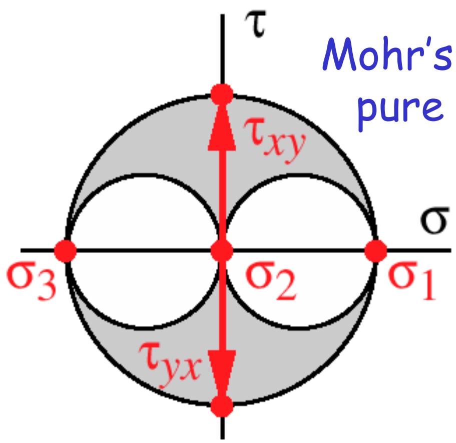


FIGURE 2-8

Torsion Test Specimens After Failure (a) Ductile Steel (b) Brittle Cast Iron

Pure shear condition

Pure shear condition



Mohr's circle: pure shear

Static failure theories

Brittle materials: even and uneven materials

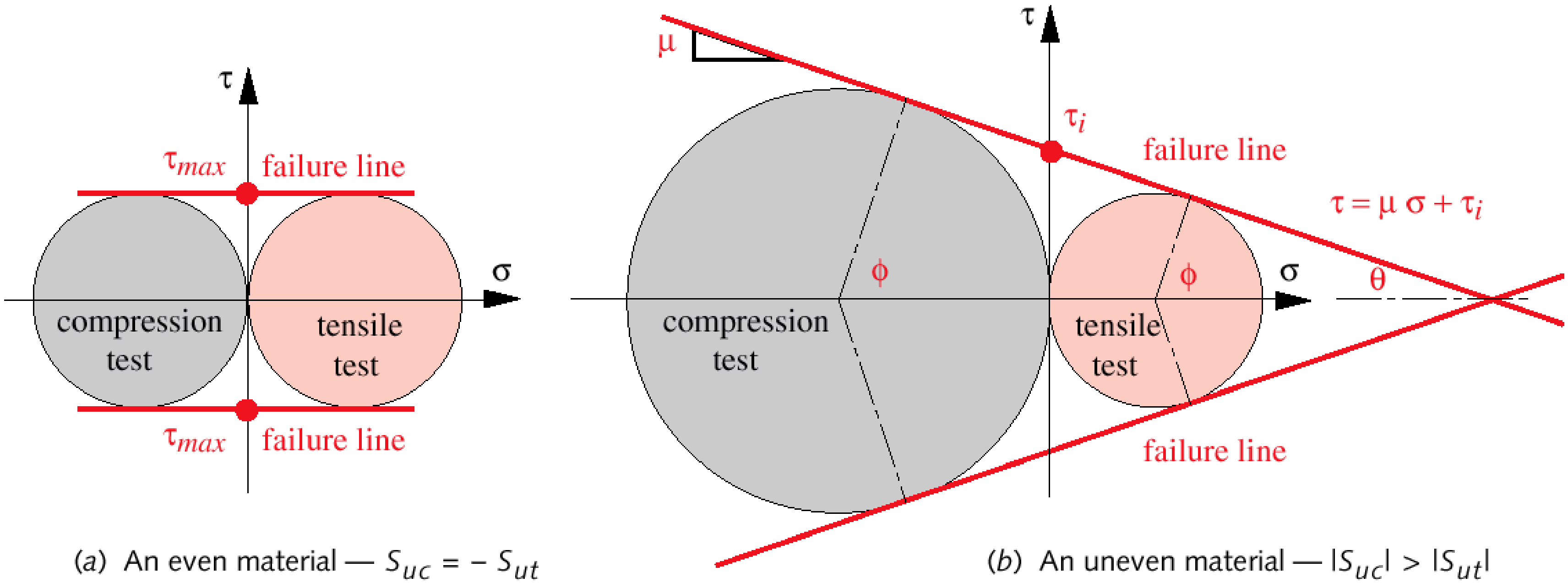


FIGURE 5-10

Mohr's Circles for Both Compression and Tensile Tests Showing the Failure Envelopes for (*a*) *Even* and (*b*) *Uneven* Materials

Static failure theories

Brittle materials: *Coulomb-Mohr, modified-Mohr, and normal stress theories*

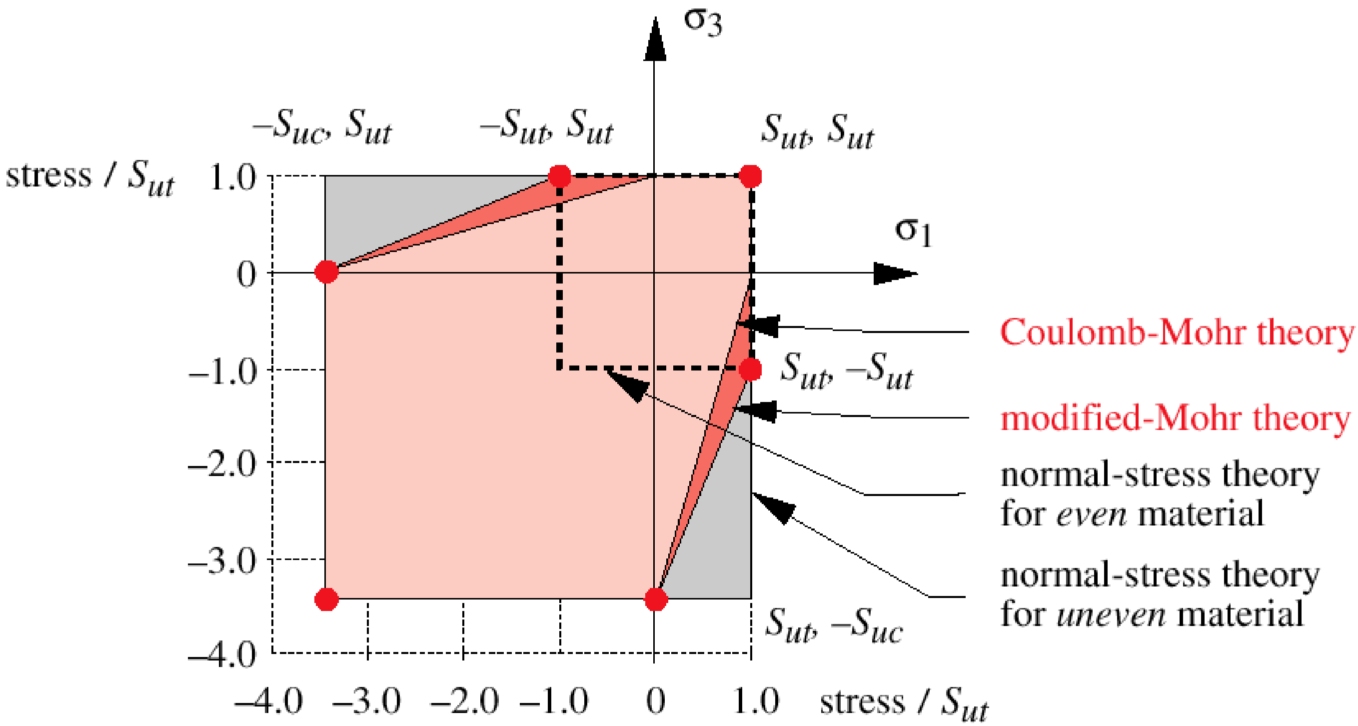


FIGURE 5-11

Coulomb-Mohr, Modified-Mohr, and Maximum Normal-Stress Theories for Uneven Brittle Materials

Static failure theories: brittle materials

Coulomb-Mohr, modified-Mohr, and normal stress theories

Experimental observations

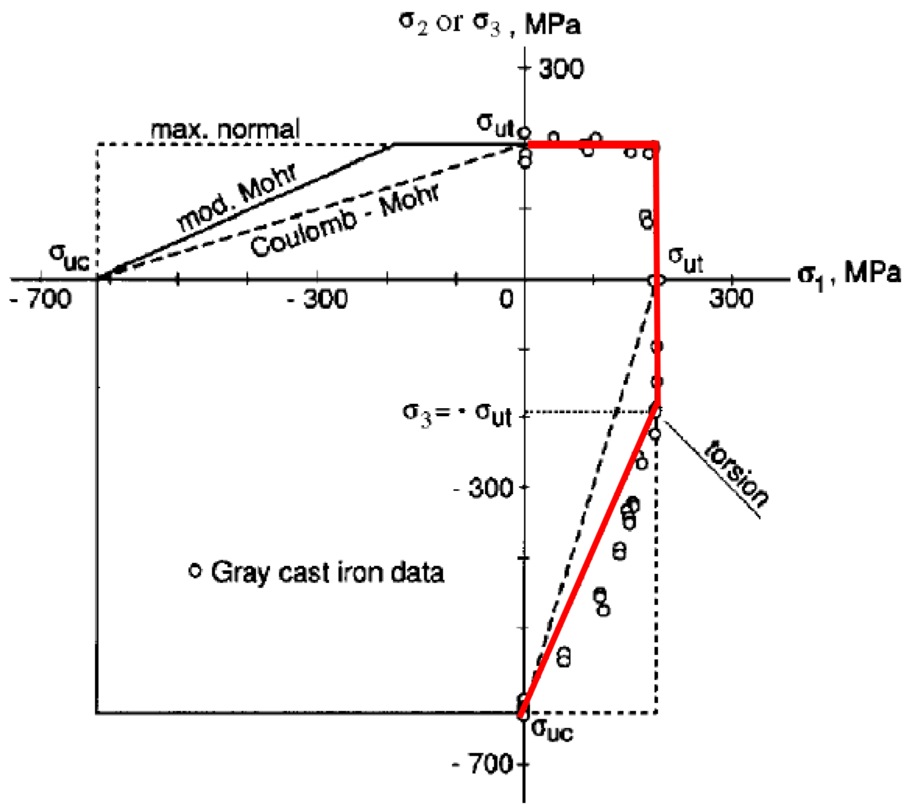


FIGURE 5-12

Biaxial Fracture Data of Gray Cast Iron Compared to Various Failure Criteria *(From Fig 7.13, p. 255, in Mechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993. Data from R. C. Grassi and I. Cornet, "Fracture of Gray Cast Iron Tubes under Biaxial Stresses," J. App. Mech, v. 16, p.178, 1949)*

Static failure theories: brittle materials

Modified-Mohr theory: quadrants of interest

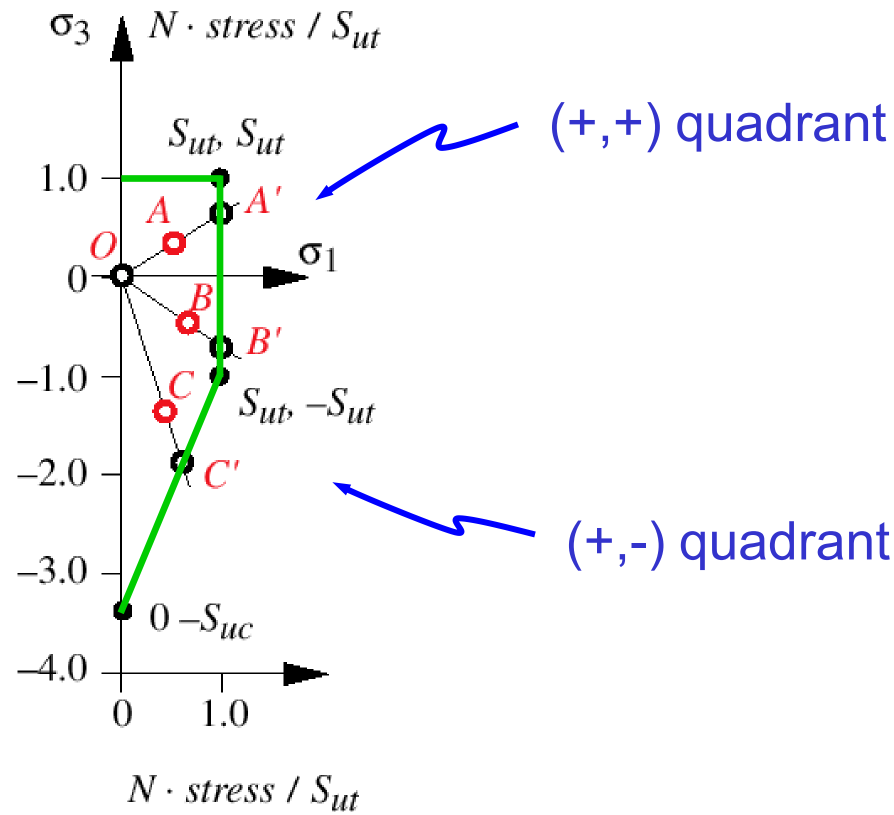


FIGURE 5-13

Modified-Mohr Failure Theory for Brittle Material

Static failure theories: brittle materials

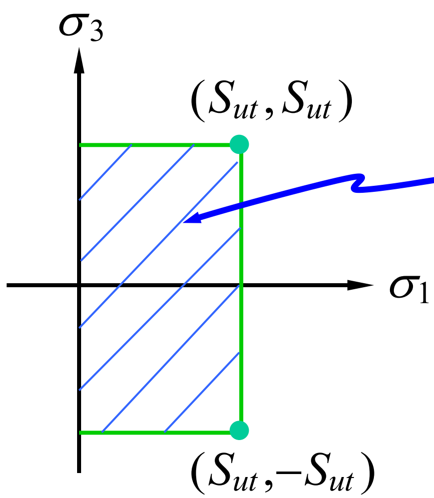
Modified-Mohr theory

Safety factor: zone I:

Modified-Mohr theory: $$\boxed{SF = N = \frac{S_{ut}}{\sigma_1}}$$

Ultimate strength of the material in tension

Max. principal normal stress



Modified-Mohr theory: applicable inside this area

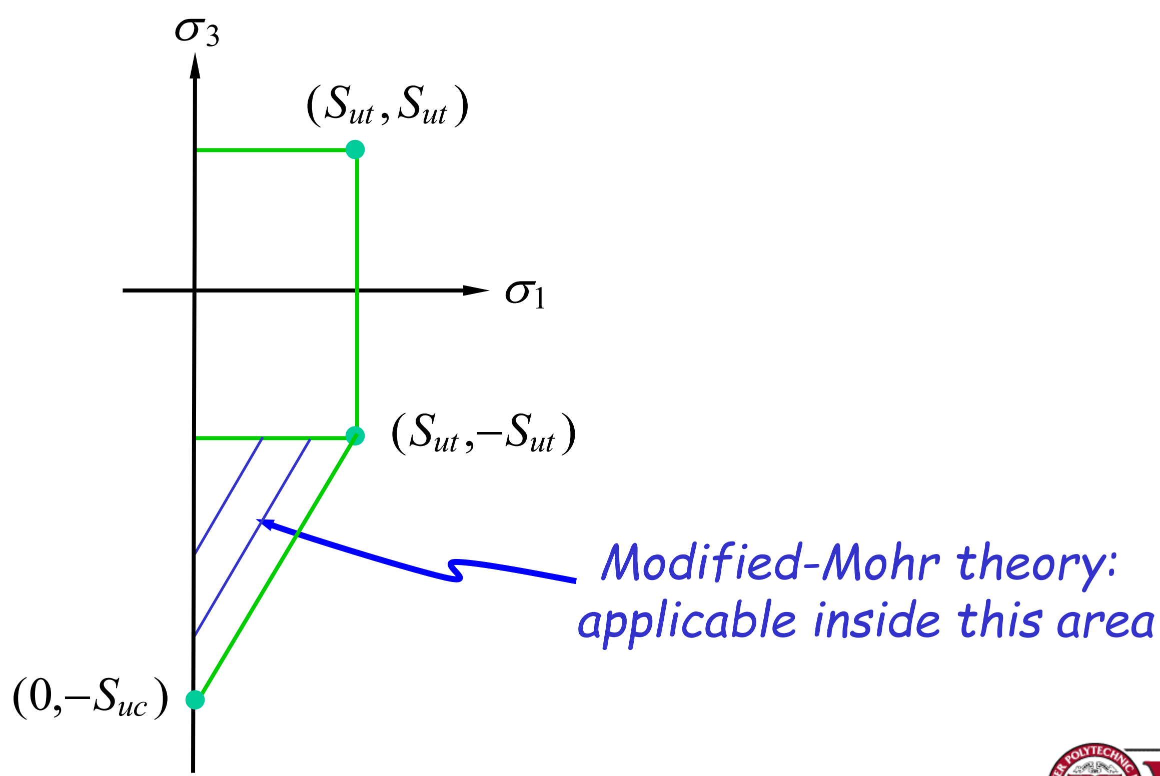
Static failure theories: brittle materials

Modified-Mohr theory

Safety factor: zone II

Modified-Mohr theory:

$$\boxed{SF = N = \frac{S_{ut}\left|S_{uc}\right|}{\left|S_{uc}\right|\sigma_1 - S_{ut}(\sigma_1 + \sigma_3)}}$$



Static failure theories: brittle materials

Modified-Mohr theory

Safety factor: zone II

Modified-Mohr theory:

$$SF = N = \frac{S_{ut}\left|S_{uc}\right|}{\left|S_{uc}\right|\sigma_1 - S_{ut}(\sigma_1 + \sigma_3)}$$

EC: derive expression for the SF ***in Zone II***

Understand: state of stresses at points A, B, and C.

What do points A', B', and C' represent?

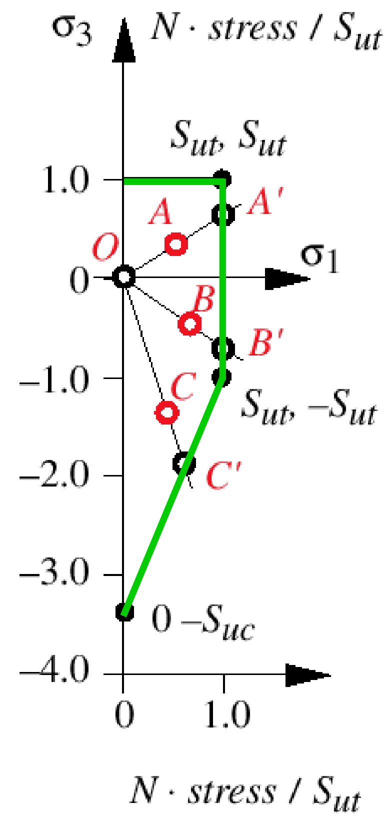


FIGURE 5-13

Modified-Mohr Failure Theory for Brittle Material

Static failure theories: brittle materials

*Effective stress: **Dowling indexes***

(Similar concept as the equivalent von Mises stress in ductile materials)

$$C_1 = \frac{1}{2}\left[\left|\sigma_1 - \sigma_2\right| + \frac{2S_{ut} - \left|S_{uc}\right|}{-\left|S_{uc}\right|}\left(\sigma_1 + \sigma_2\right)\right]$$

$$C_2 = \frac{1}{2}\left[\left|\sigma_2 - \sigma_3\right| + \frac{2S_{ut} - \left|S_{uc}\right|}{-\left|S_{uc}\right|}\left(\sigma_2 + \sigma_3\right)\right]$$

$$C_3 = \frac{1}{2}\left[\left|\sigma_1 - \sigma_3\right| + \frac{2S_{ut} - \left|S_{uc}\right|}{-\left|S_{uc}\right|}\left(\sigma_1 + \sigma_3\right)\right]$$

Static failure theories

Ductile materials

Safety factors:

Distortion energy theory:

$$SF = N = \frac{S_y}{\sigma'}$$

Yield strength of the material

von Mises effective stress

(Obtained from)

Distortion energy theory (pure shear):

$$SF = N = \frac{S_{ys}}{\tau_{\max}}$$

$$S_{ys} = 0.577 S_y$$

Max. shear-stress

Max. shear-stress theory:

$$SF = N = \frac{S_{ys}}{\tau_{\max}}$$

$$S_{ys} = 0.5 S_y$$

Maximum shear-stress

Static failure theories: brittle materials

Modified-Mohr theory: ***effective stress***

Safety factor:

Modified-Mohr theory.
Effective stress:

$$\boxed{SF = N = \frac{S_{ut}}{\tilde{\sigma}}}$$

Ultimate strength of the material in tension

Effective stress. Obtained as:

$$\tilde{\sigma} = MAX(\sigma_1, \sigma_2, \sigma_3, C_1, C_2, C_3)$$

and

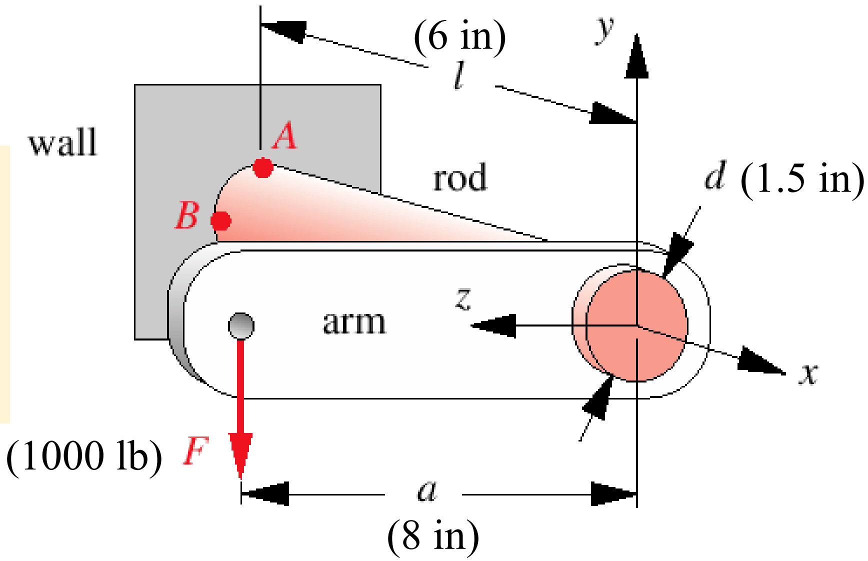
$\tilde{\sigma} = 0$ *if MAX < 0, use a different approach*

Static failure theories: ductile & brittle materials

Review and Master: Examples 5-1 and 5-2

Determine the safety factors for the bracket rod shown considering: (a) ductile; and (b) brittle materials.

Do analyses with & without stress concentrations at the wall/rod interface



Ductile case:

Al 2024-T4 (consult Appendix C)

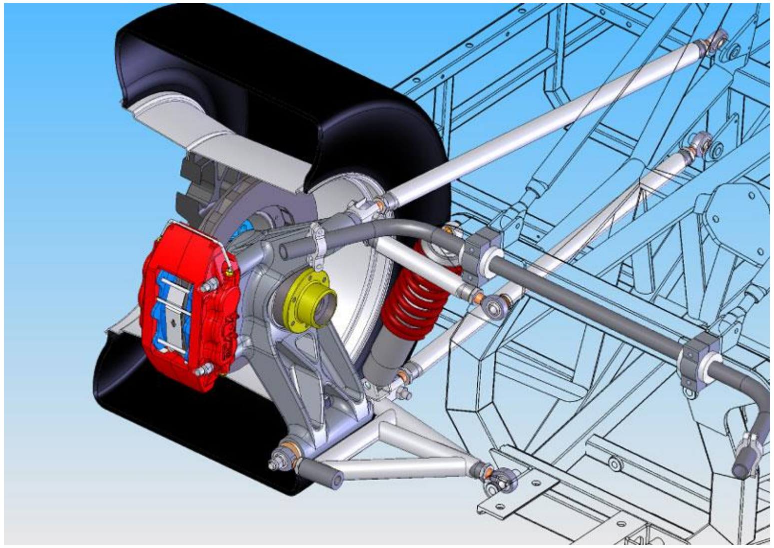
S_y = 47 kpsi

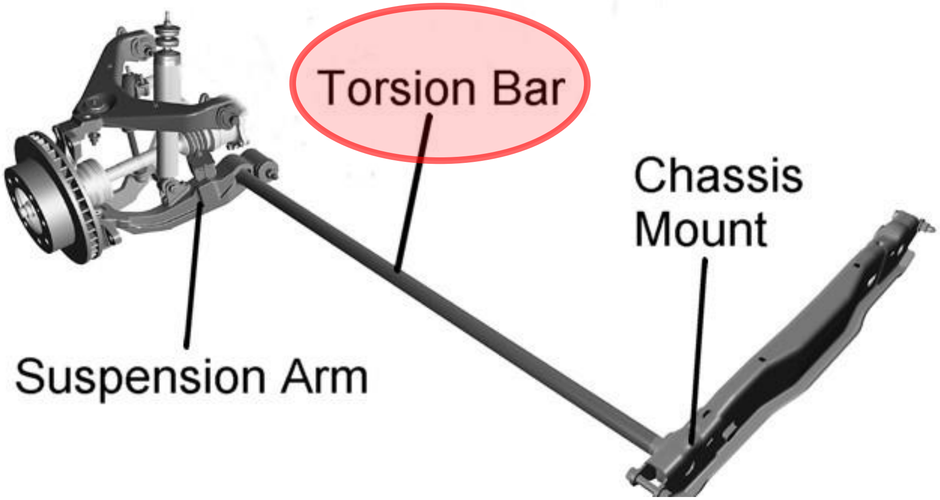
Brittle case:

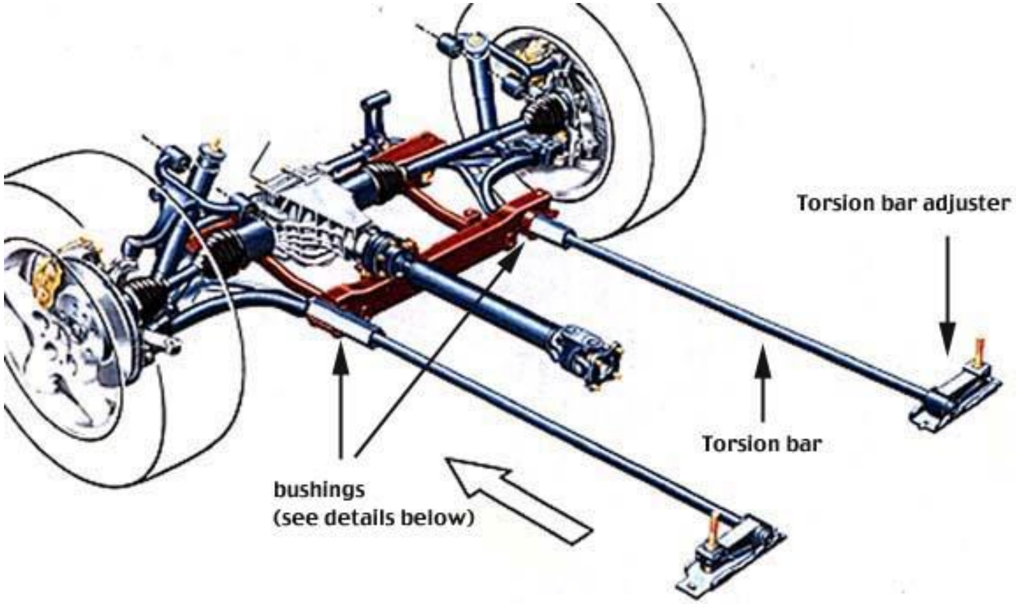
Class 50 gray cast iron (consult Appendix C)

S_{ut} = 52.5 kpsi. S_{uc} = 164 kpsi

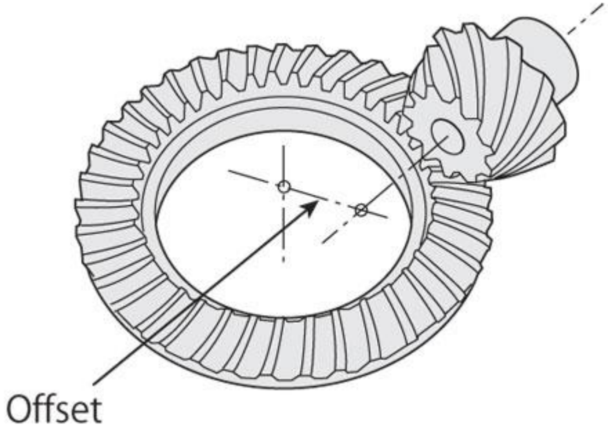
Uses of the <u>bracket model</u> configuration: suspension system



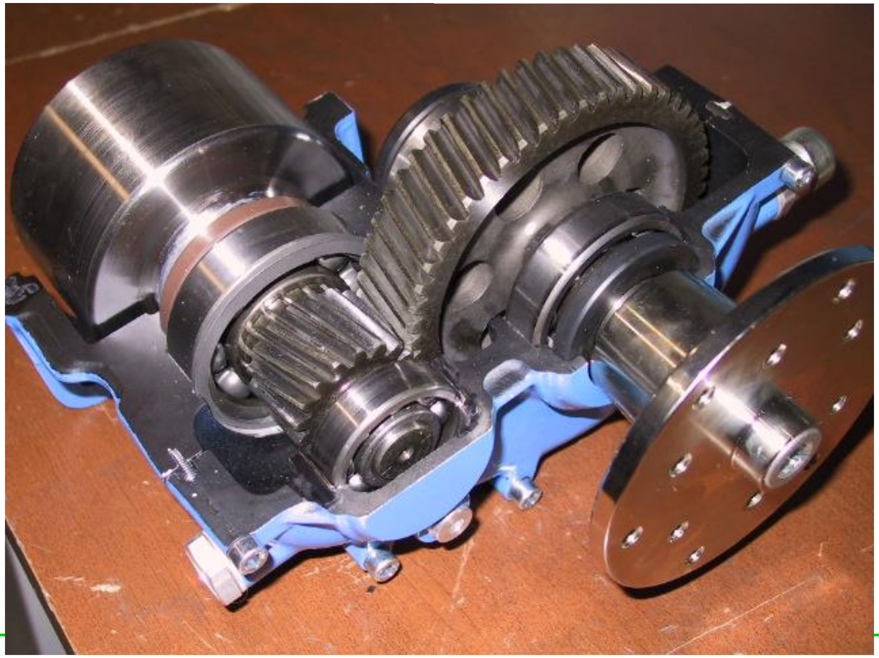


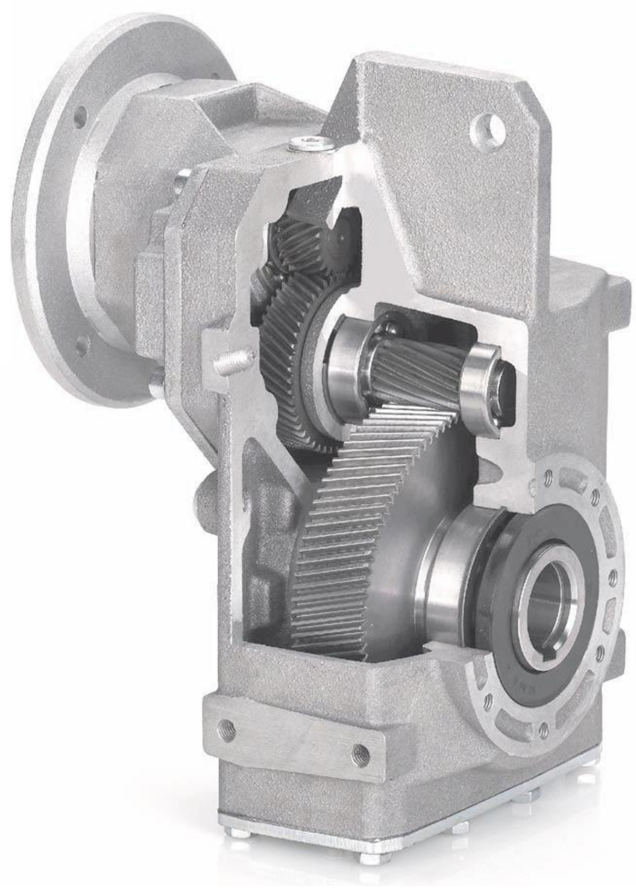


Uses of the bracket model configuration: transmissions



Hypoid Gear





Static failure theories

Ductile materials

Safety factors:

Distortion energy theory:

$$SF = N = \frac{S_y}{\sigma'}$$

Yield strength of the material

von Mises effective stress

(Obtained from)

Distortion energy theory (pure shear):

$$SF = N = \frac{S_{ys}}{\tau_{max}}$$

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Max. shear-stress

Max. shear-stress theory:

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Maximum shear-stress

Static failure theories: brittle materials

Modified-Mohr theory: ***effective stress***

Safety factor:

Modified-Mohr theory.
Effective stress:

$$\boxed{SF = N = \frac{S_{ut}}{\tilde{\sigma}}}$$

Ultimate strength of the material in tension

Effective stress. Obtained as:

$$\tilde{\sigma} = MAX(\sigma_1, \sigma_2, \sigma_3, C_1, C_2, C_3)$$

and

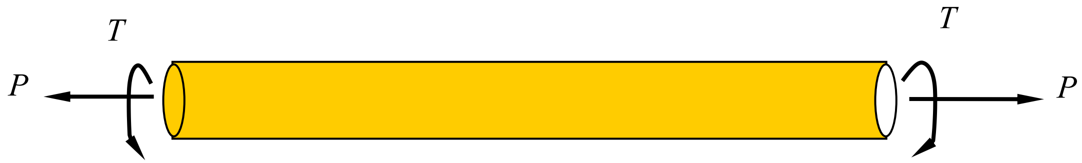
$$\tilde{\sigma} = 0$$ *if MAX < 0, use a different approach*

Review
Example

A circular rod is subjected to combined loading consisting of a tensile load P = 10 kN and a torque T = 5 kN·m. Rod is 50 mm in diameter.

1) Draw stress element (cube) at the most highly stressed location on the rod, and

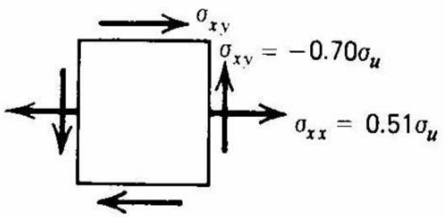
2) draw corresponding Mohr's circle(s).



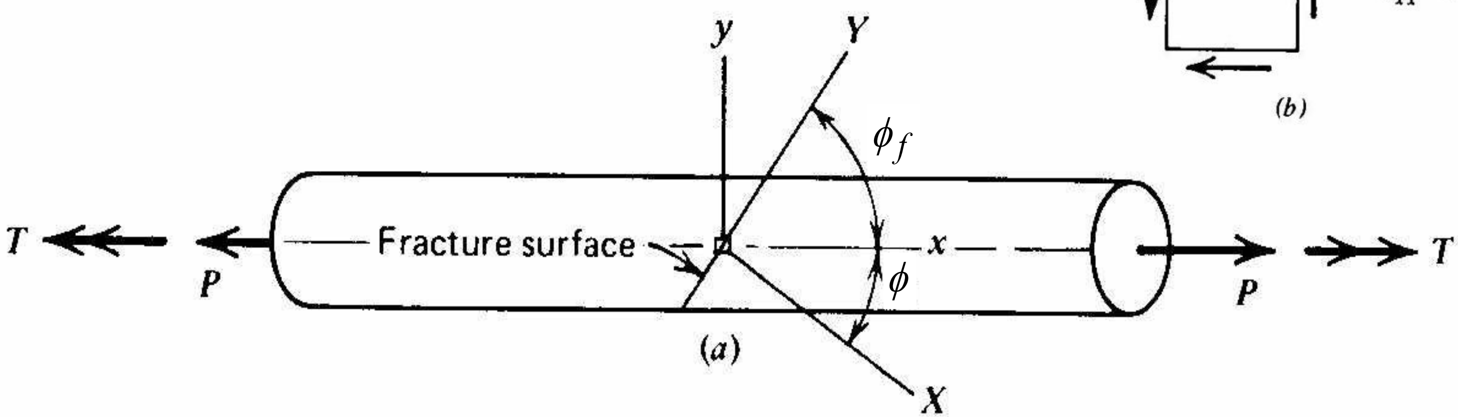
Review
Example

A piece of chalk is subjected to combined loading consisting of a tensile load $\boldsymbol{P}$ and a torque $\boldsymbol{T}$, see figure. The chalk has an ultimate strength σ_u as determined by a tensile test. The load $\boldsymbol{P}$ remains constant at such a value that it produces a tensile stress of $0.51 \cdot \sigma_u$ on any cross-section. The torque $\boldsymbol{T}$ is increased gradually until fracture occurs on some inclined surface.

Assuming that fracture takes place when the maximum principal stress σ_1 reaches the ultimate strength σ_u, determine the magnitude of the torsional shearing stress produced by the torque $\boldsymbol{T}$ at fracture and determine the orientation of the fractured surface.



(b)



(a)

Reading assignment

- Chapters 5 of textbook: Sections 5.2 to 5.5
- Review notes and text: ES2501, ES2502

Homework assignment

- **Author's:** As indicated in website of our course
- **Solve:** As indicated in website of our course