WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

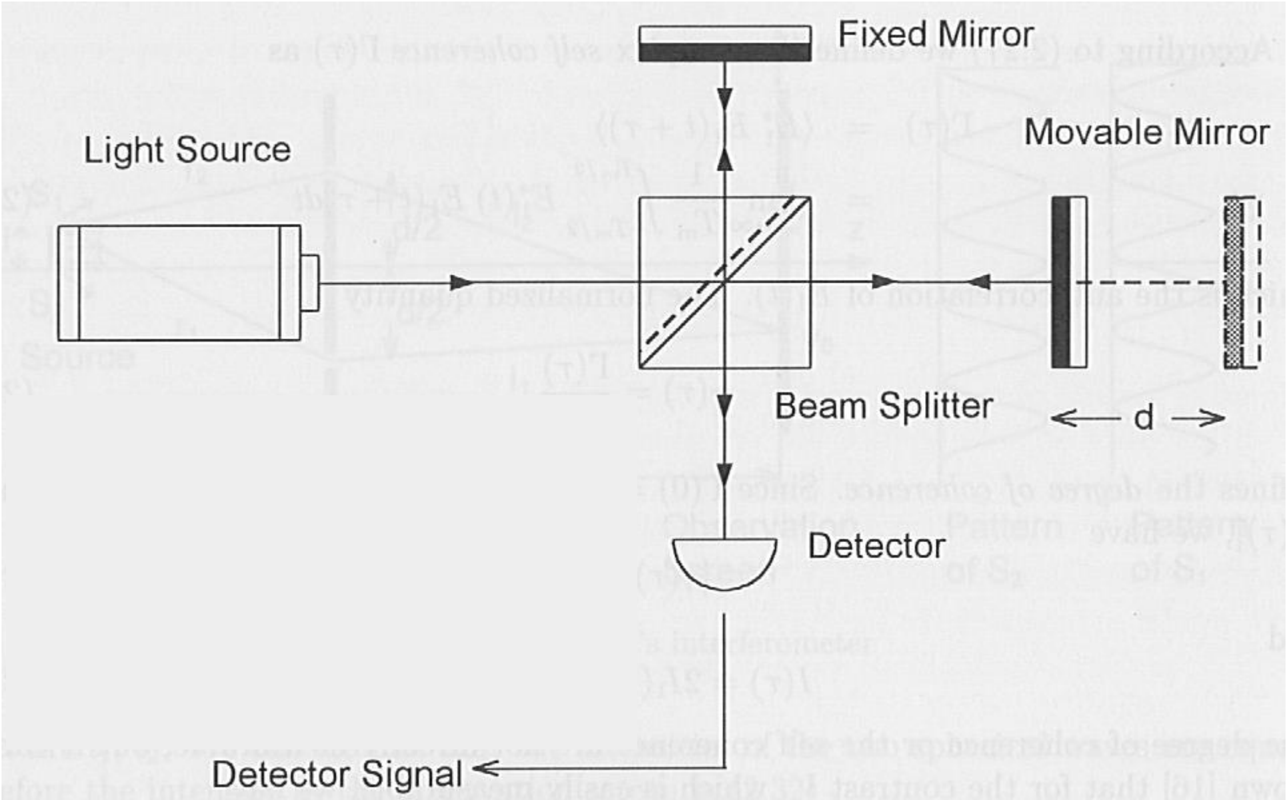
Optical Metrology and NDT
ME-593L/ME-5304, C'2025

Introduction: Wave Optics
February 2025

Wave optics: coherence

Temporal coherence

- Review interference equation: Lecture 04
- Michelson interferometer to test for coherence length (temporal)

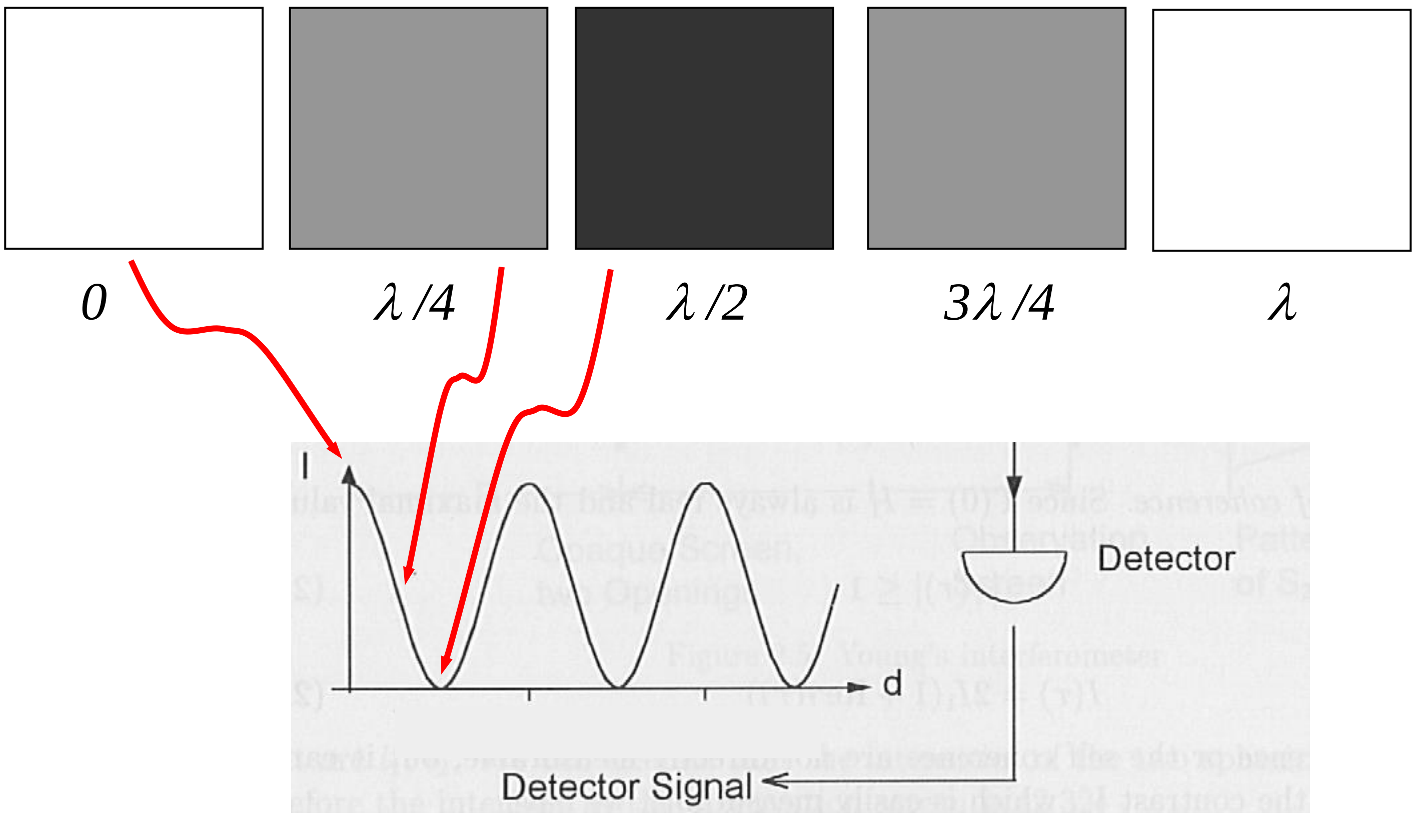


Wave optics: coherence

Temporal coherence

- Assume 100% flat and orthogonal mirrors

Detected intensities as a function of position of movable mirror



Wave optics: coherence

Temporal coherence

- Temporal coherence describes the self-correlation of a wave. Consider the normalized autocorrelation function

$$g(\tau) = \frac{G(\tau)}{G(0)} = \frac{\left\langle U^*(t)\,U(t+\tau)\right\rangle}{\left\langle U^*(t)\,U(t)\right\rangle}, \qquad (1)$$

where τ is the time delay, related optical path length changes introduced by the movable mirror.

Equation 1 is called the **degree of temporal coherence** with

$$0 \le |g(\tau)| \le 1. \qquad (2)$$

Usually, $|g(\tau)|$ drops from its largest values $|g(\tau)| = 1$ as τ increases and the fluctuations become uncorrelated for sufficiently large time delay τ.

Wave optics: coherence

Temporal coherence

- Coherence length is defined as

$$l_c = c\,\tau_c \ , \qquad (3)$$

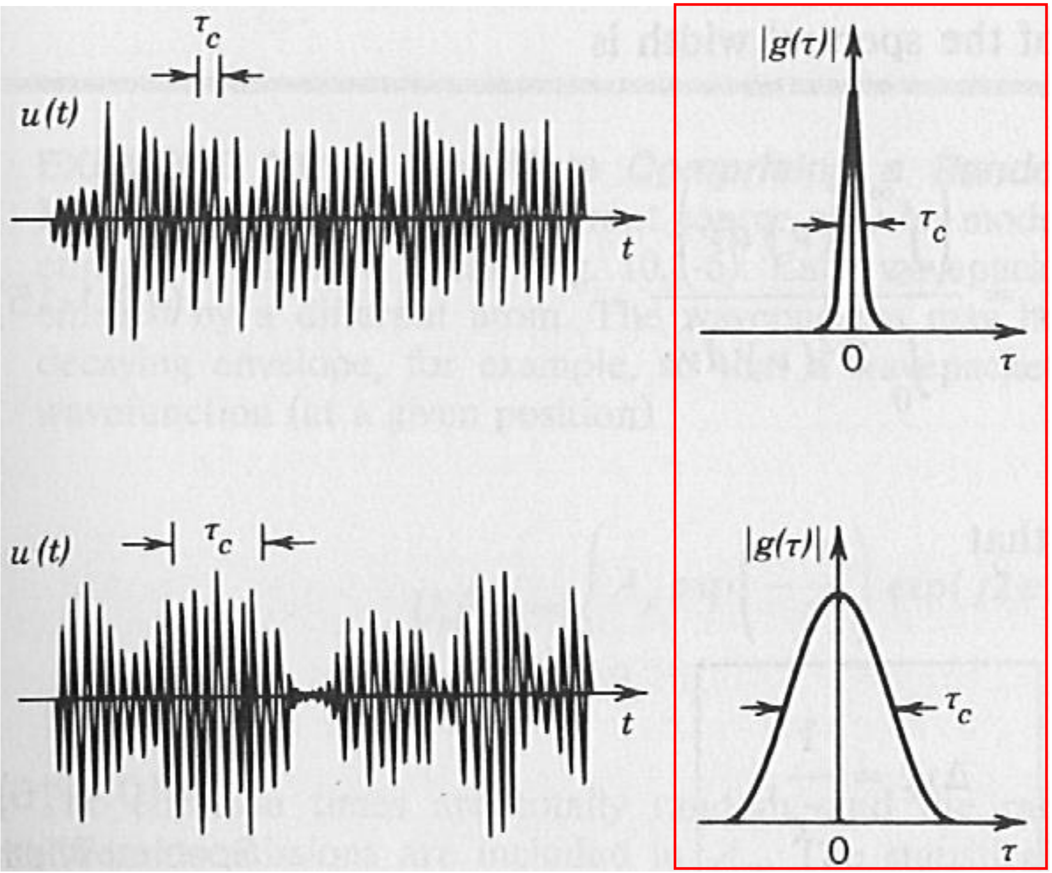
where c is the speed of light and τ_c is the power equivalent width of the function $|g(\tau)|$ given as

$$\tau_c = \int_{-\infty}^{\infty} |g(\tau)|^2 d\tau \ . \qquad (4)$$

Wave optics: coherence

Temporal coherence

Magnitude of the **degree of temporal coherence** $|g(\tau)|$



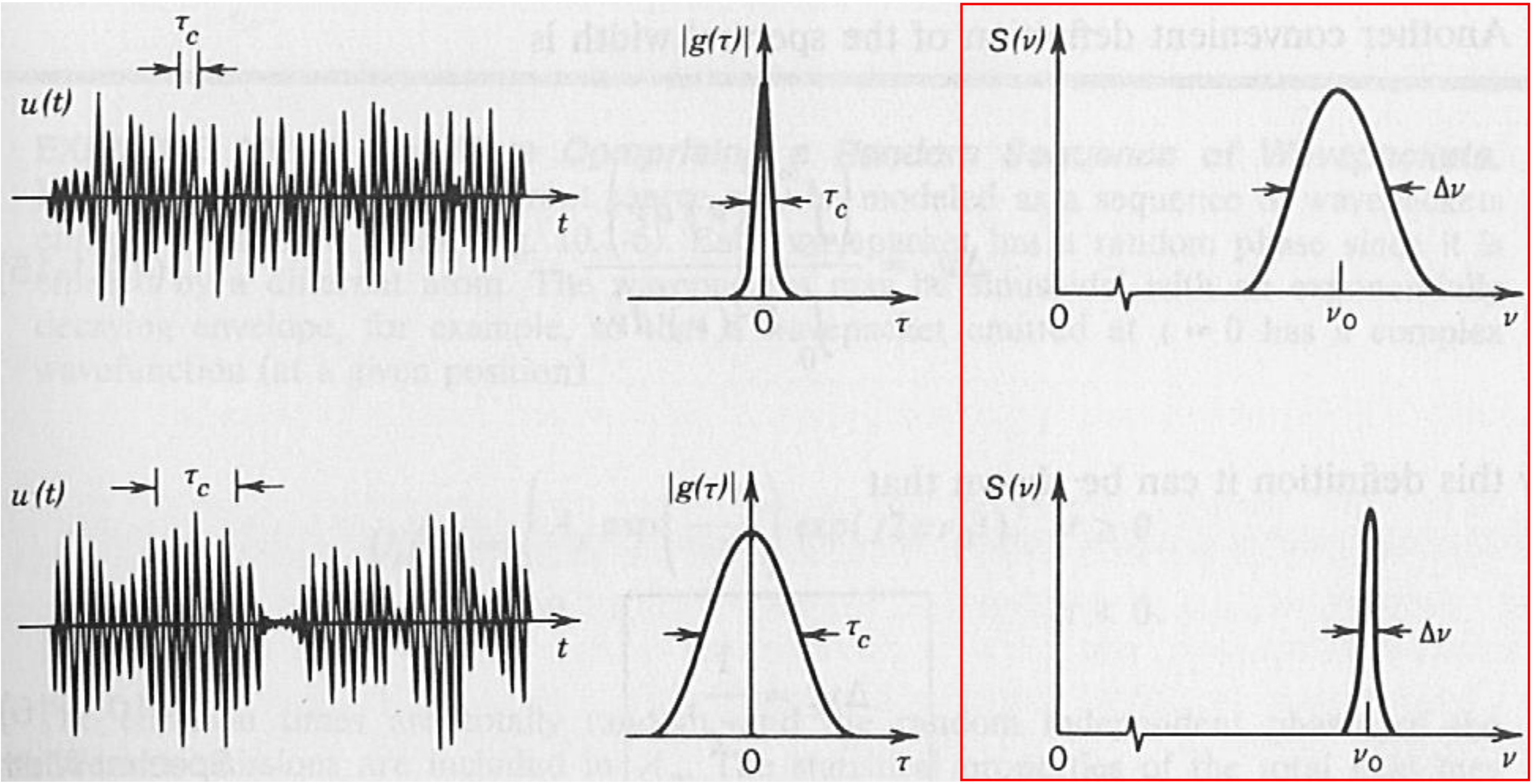
Short coherence time

Long coherence time

Wave optics: coherence

Temporal coherence

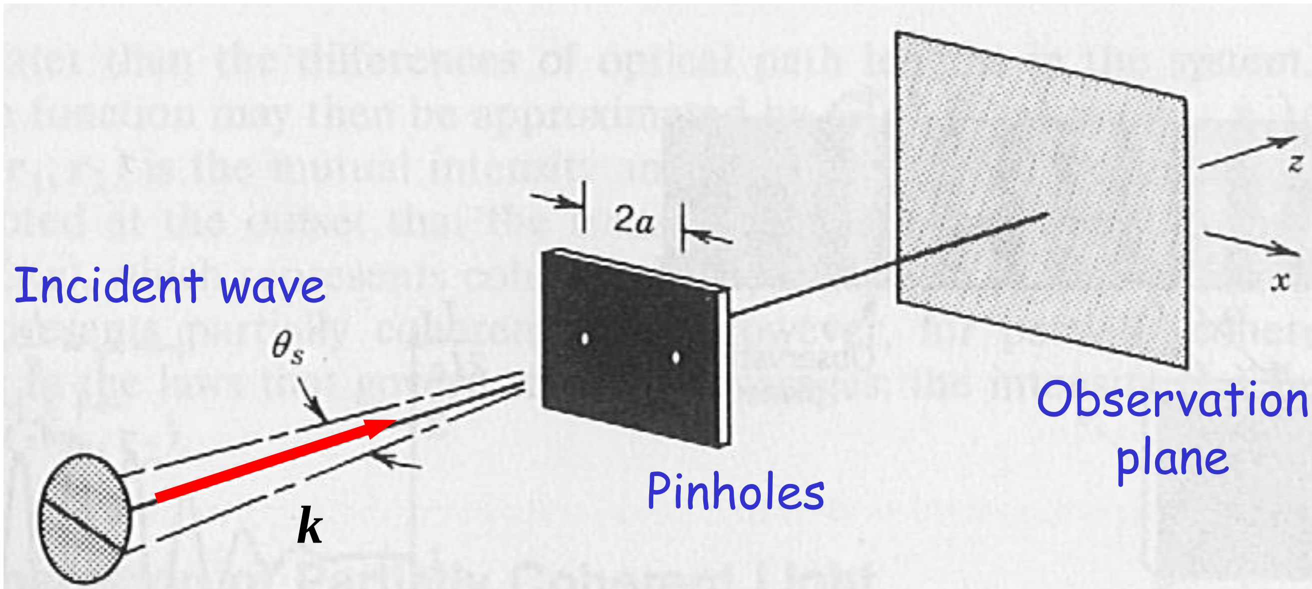
Power spectral density: $S(\nu) = \int_{-\infty}^{\infty} g(\tau) \exp(-j2\pi\nu\tau) d\tau$



Wave optics: coherence

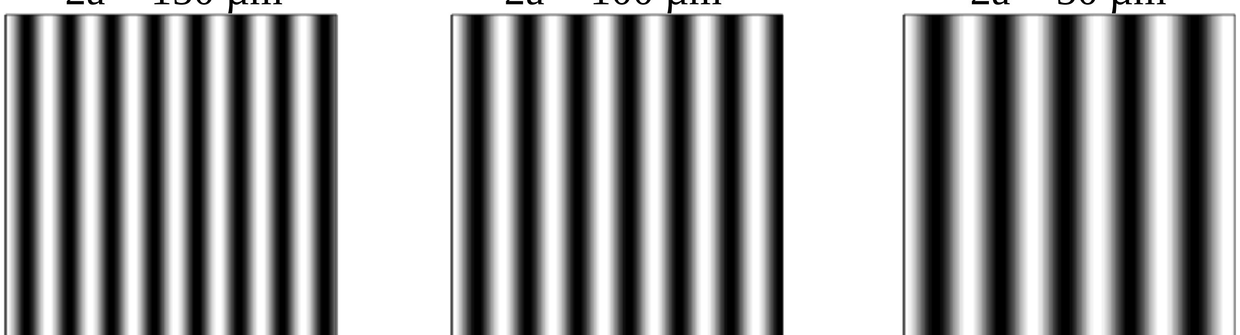
Spatial coherence

Young's double pinhole interferometer



Predicted fringe patterns corresponding to difference pinholes distances

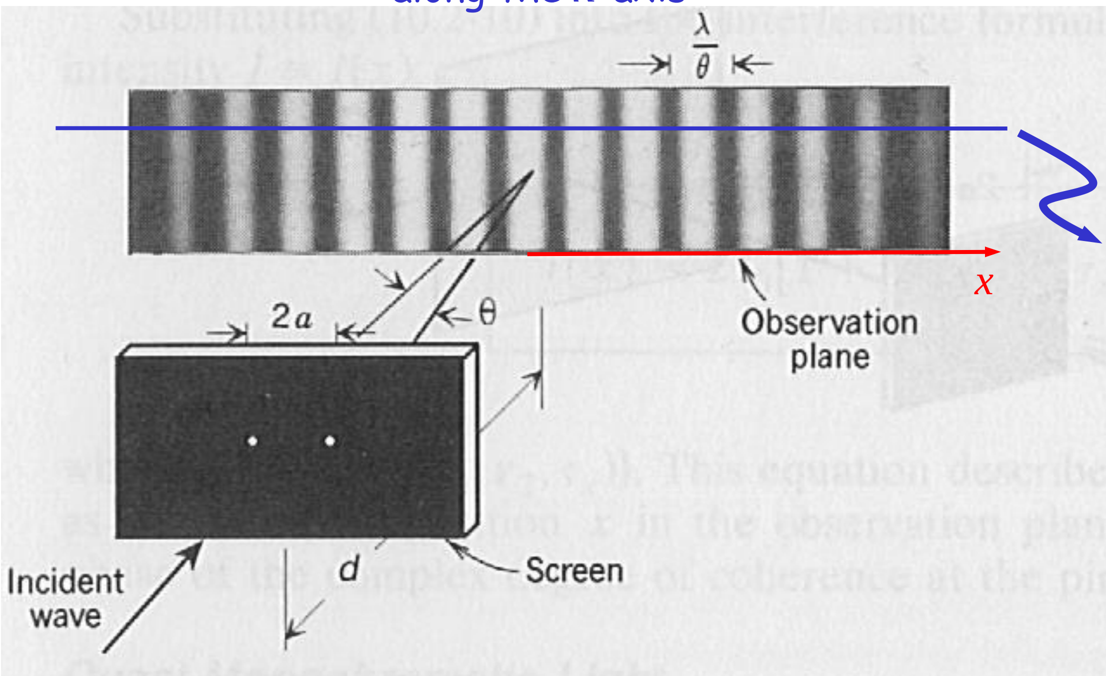
2a = 150 μm | 2a = 100 μm | 2a = 50 μm



Wave optics: coherence

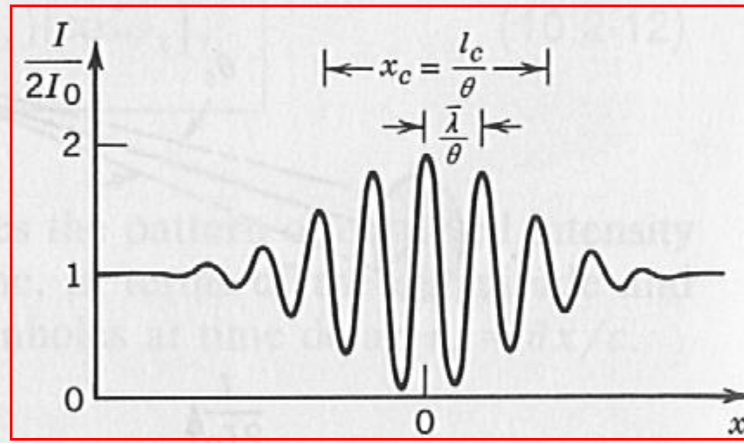
Spatial coherence

Actual fringe pattern: note variation of intensity along the x-axis



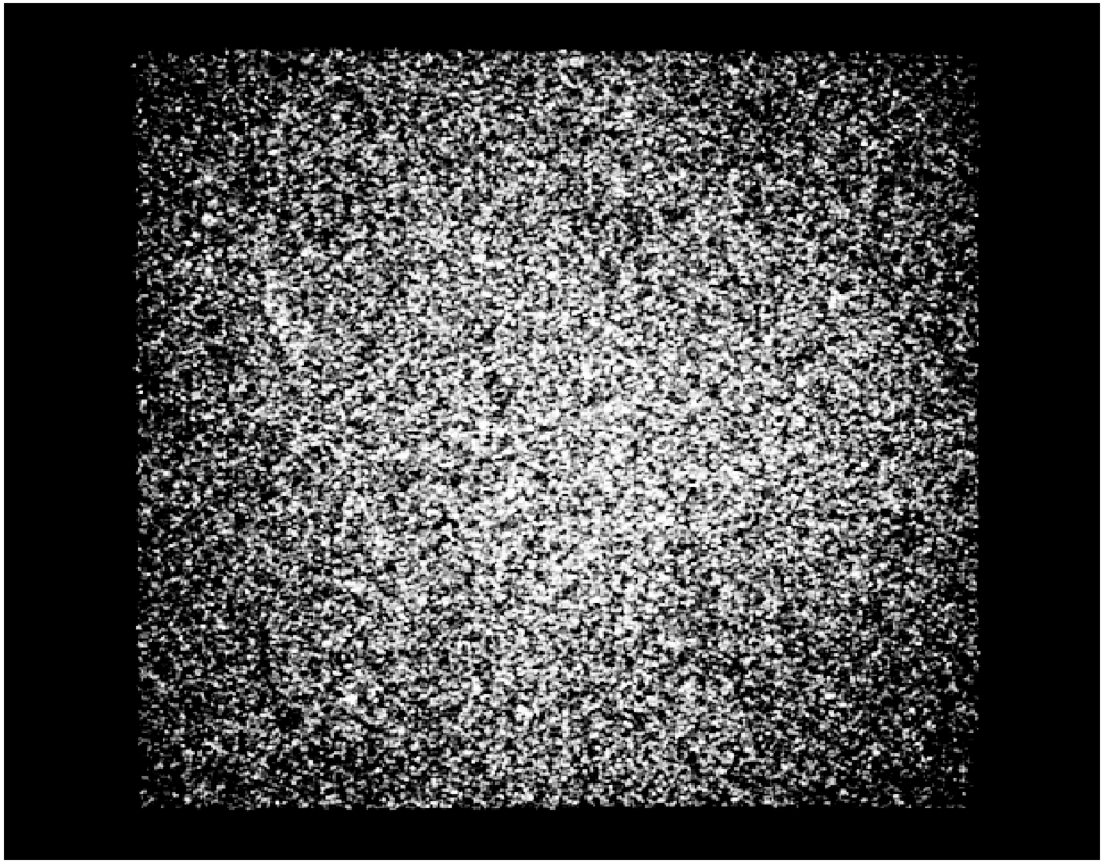
Spatial coherence length is:

$$x_c = \frac{l_c}{\theta}$$



Wave optics: light scattering

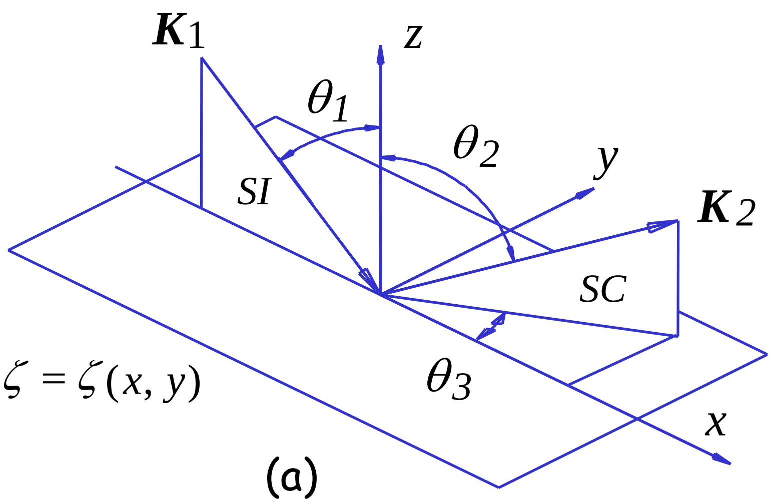
Speckle pattern observed on the flat surface of an object illuminated with a coherent light source with characteristic wavelength λ = 994.1 nm and horizontally polarized. Average surface roughness is 5 μm, object distance is 30 mm, and f/2.8 aperture is used. The surface is illuminated and observed normal to the plane where the surface lies.



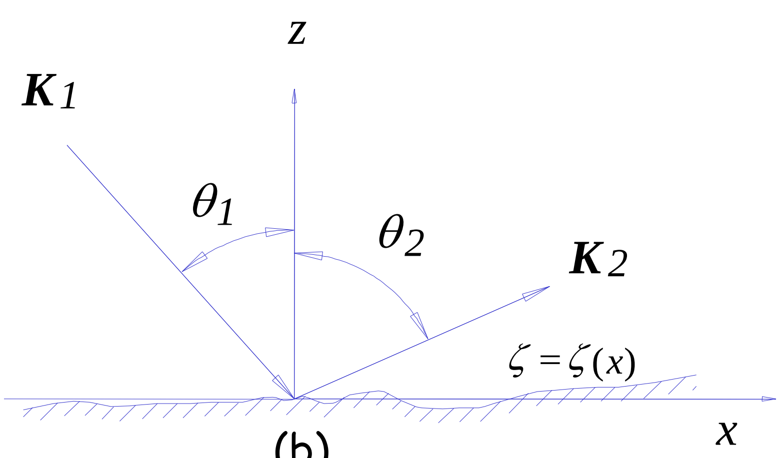
Wave optics: light scattering

Illumination and scattering geometry from surfaces defined as $\zeta = \zeta(x, y)$ and $\zeta = \zeta(x)$.

Vectors of illumination and scattering: $\boldsymbol{K}_1 = k\,\hat{K}_1$, $\boldsymbol{K}_2 = k\,\hat{K}_2$



(a)



(b)

Wave optics: light scattering

Scattering from a smooth surface

- The complex amplitude of the scattered light, F_o, at point p, can be predicted using the Kirchoff integral theorem

$$F_o = \frac{1}{4\pi}\left\{\iint_s \frac{1}{r}\exp(-jkr)\nabla U \cdot d\mathbf{S} - \iint_s U\ \nabla\left[\frac{1}{r}\exp(-jkr)\right]\cdot d\mathbf{S}\right\} \quad (5)$$

which is derived from the Helmholtz equation.

- For a smooth surface, $\zeta(x) \approx 0$, extending from $[-L, L]$:

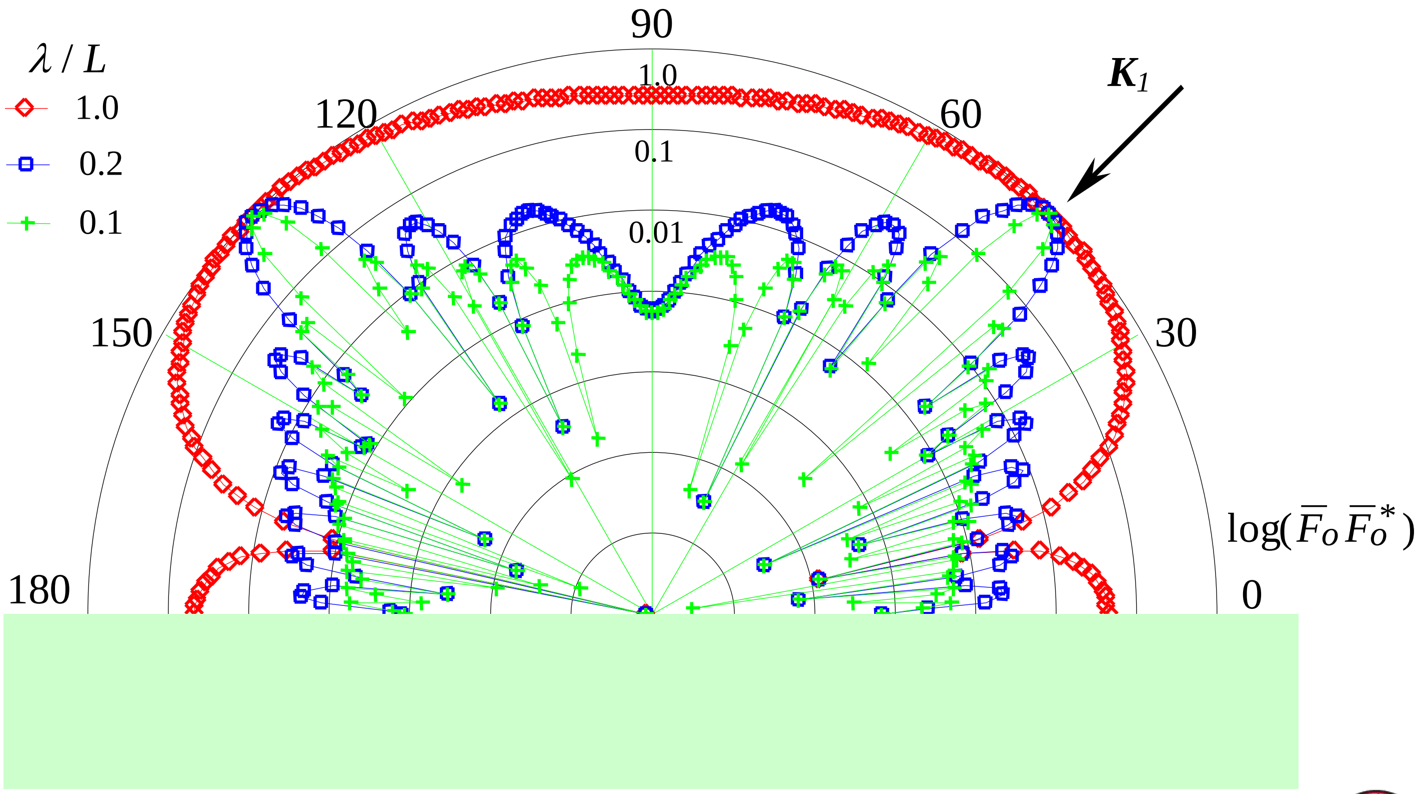
$$F_o = F_o(\theta_1, \theta_2) = \frac{\sin[kL(\sin\theta_1 - \sin\theta_2)]}{kL(\sin\theta_1 - \sin\theta_2)} \quad (6)$$

$$= \mathrm{sinc}\left[\frac{2\pi}{\lambda} L(\sin\theta_1 - \sin\theta_2)\right]$$

Wave optics: light scattering

Scattering from a smooth surface

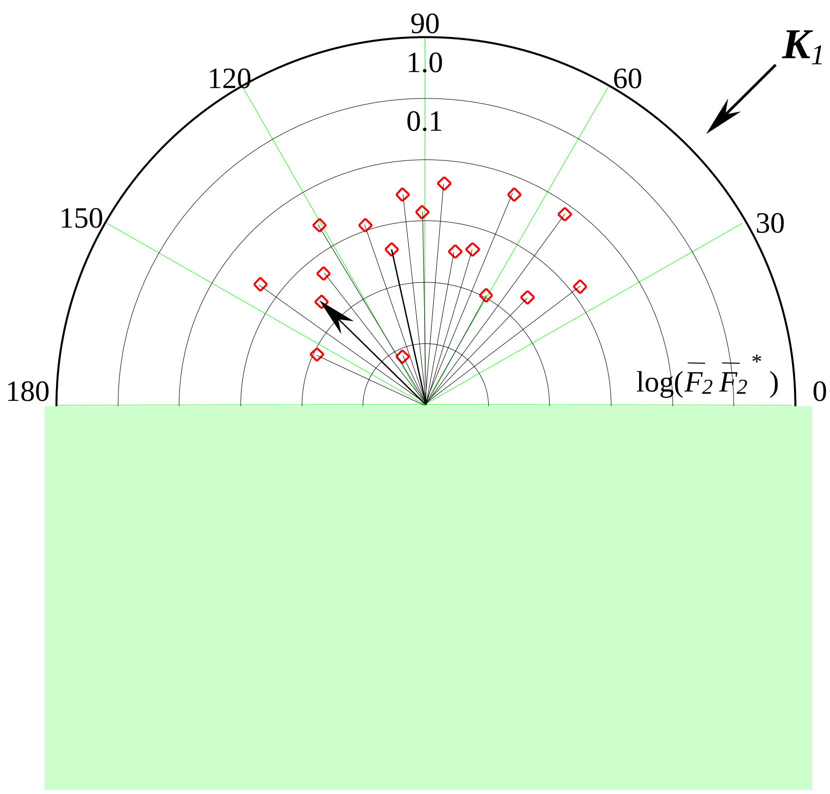
- Scattering diagram for a smooth surface characterized by the ratio λ / L.
- Note that when $\lambda / L \to 0$ scattering of light concentrates in the direction of specular reflection



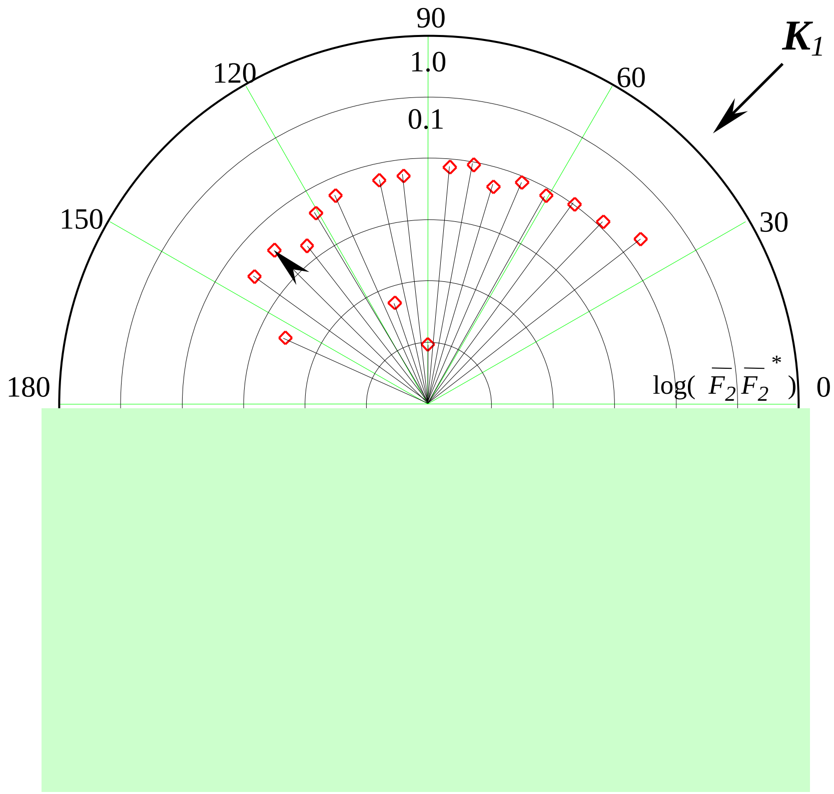
Wave optics: light scattering

Scattering from a surface characterized by a periodic function:

$\Lambda = 10\lambda\,, \quad \theta_1 = \pi/4\,, \quad \lambda / h = 0.033$



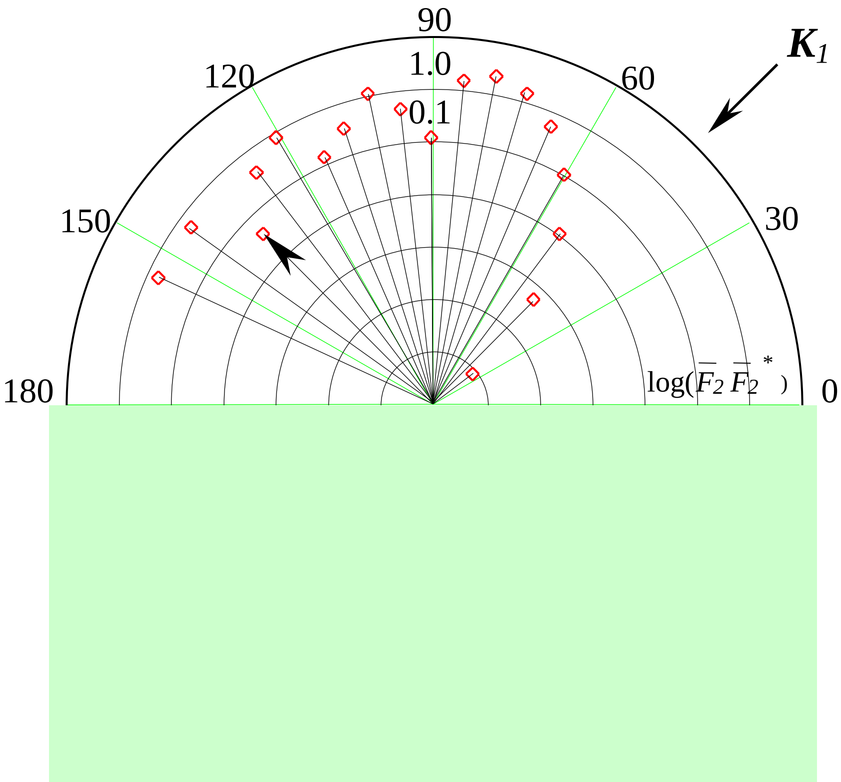
$\Lambda = 10\lambda\,, \quad \theta_1 = \pi/4\,, \quad \lambda / h = 0.066$



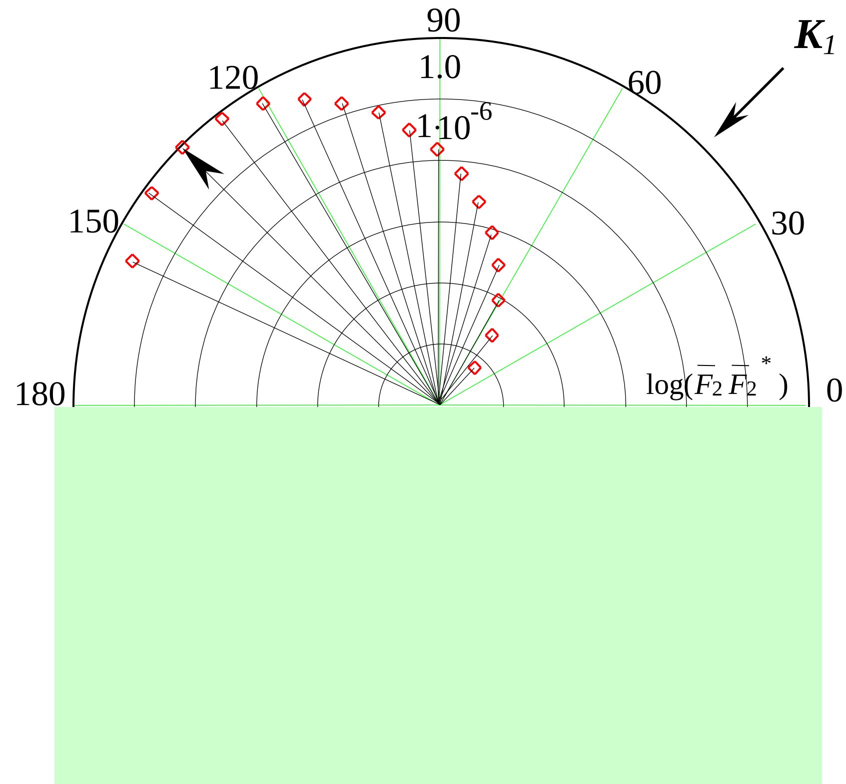
Wave optics: light scattering

Scattering from a surface characterized by a periodic function

$\Lambda = 10\lambda\,,\quad \theta_1 = \pi/4\,,\quad \lambda/h = 1.0$



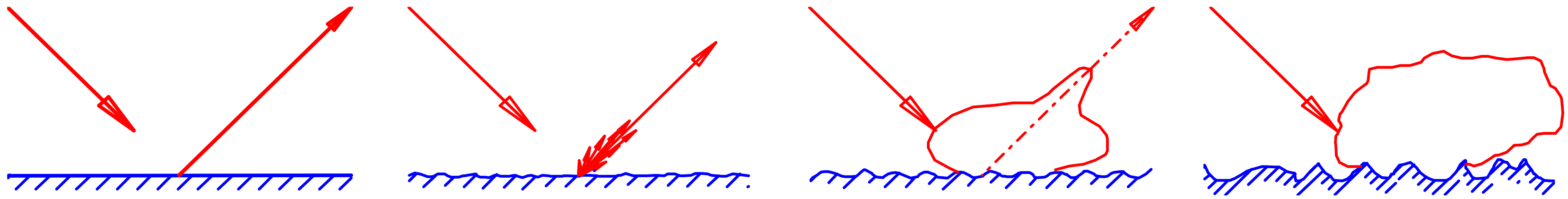
$\Lambda = 10\lambda\,,\quad \theta_1 = \pi/4\,,\quad \lambda/h = 10.0$



Wave optics: light scattering

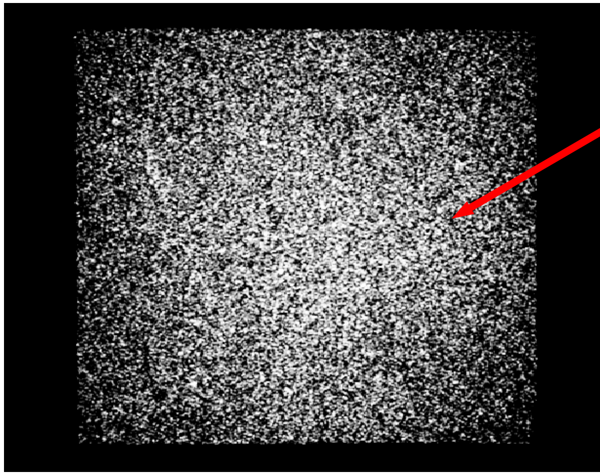
Scattering from a surface characterized by a random function

Transition from specular to diffuse scattering reflection as a function of surface roughness:



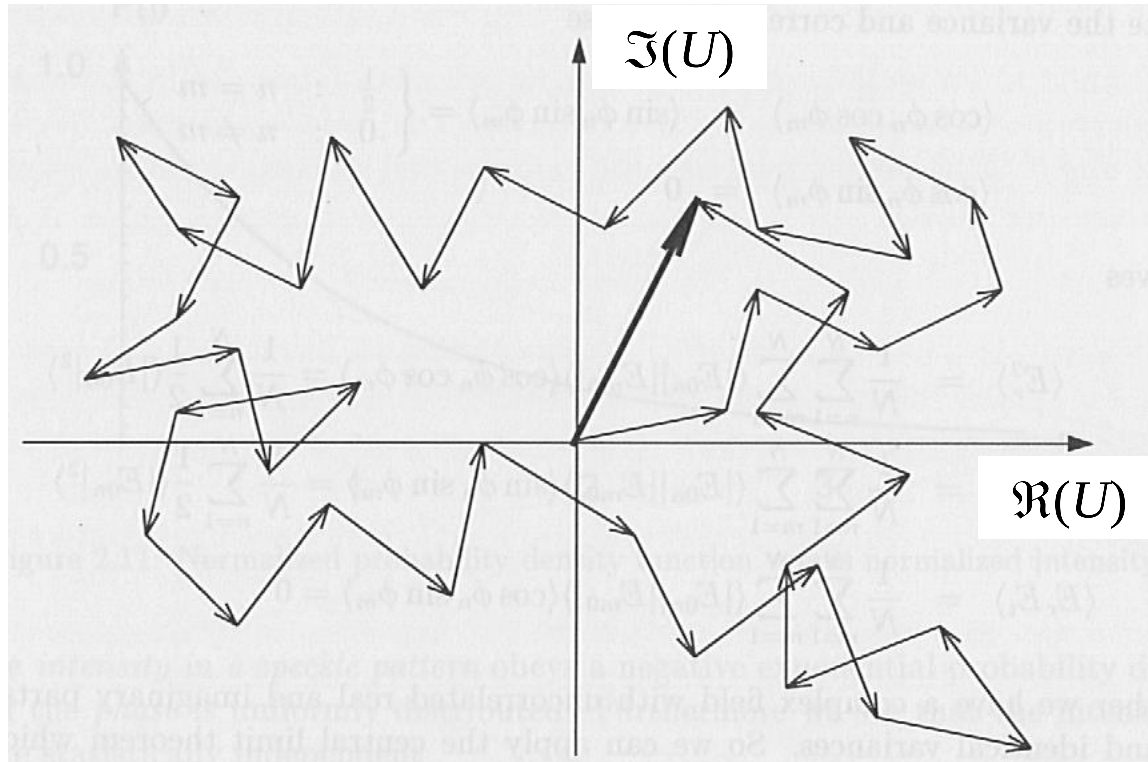
Speckles

Speckle pattern



(x,y,z)

Adding contribution of scattered components at point (x,y,z)



Speckles

Speckle properties: first order statistics

- Sum of N complex amplitude components:

$$U(x,y,z) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |A_k| \exp[j\phi_k(x,y,z)] \qquad (7)$$

- Real an imaginary components:

$$\Re\{U\} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |a_k| \cos(\phi_k)$$

$$\Im\{U\} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |a_k| \sin(\phi_k) \qquad (8)$$

- Amplitude and phase are statistically independent.

Speckles

Speckle properties: first order statistics

- Probability distribution for the intensity is (mean = $2\sigma^2$, variance = $<I^2>$)

$$P_I(I) = \int_{-\pi}^{\pi} P_{I,\phi}(I,\phi)\,d\phi = \begin{cases} \dfrac{1}{2\sigma^2}\exp\left(-\dfrac{I}{2\sigma^2}\right) & ; \quad I \geq 0 \\ 0 & ; \text{ otherwise} \end{cases} \qquad (9)$$

- Probability distribution for the phase is:

$$P_\phi(\phi) = \int_{0}^{\infty} P_{I,\phi}(I,\phi)\,dI = \begin{cases} \dfrac{1}{2\pi} & ; -\pi \leq \phi \leq \pi \\ 0 & ; \text{ otherwise} \end{cases} \qquad (10)$$

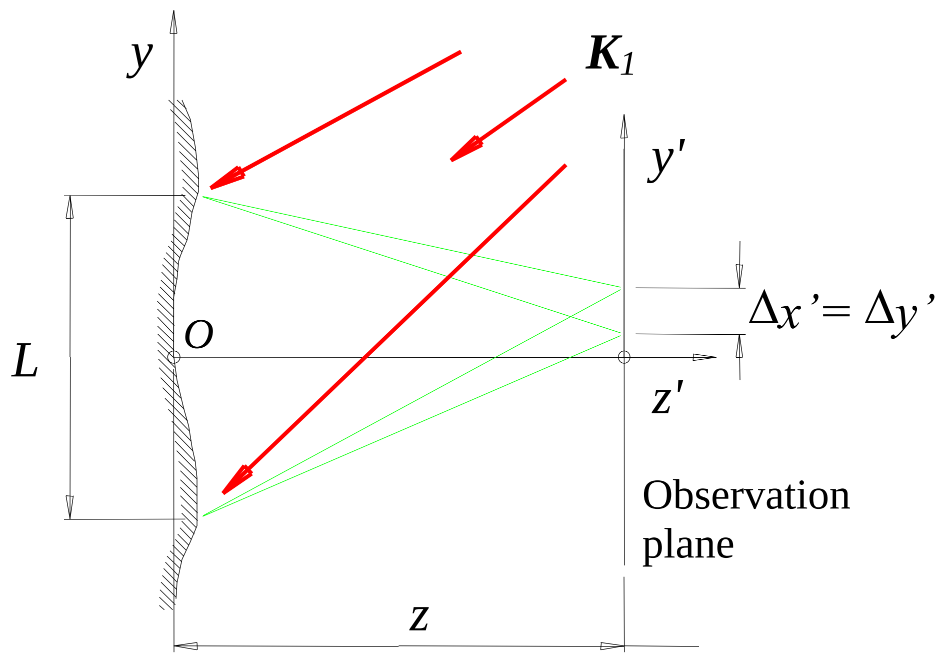
Speckles

Speckle properties: second order statistics

- Observed speckle size is (without imaging system):

$$\delta x_S = 2\frac{\lambda z \pi}{L} \qquad (11)$$

Speckle field formation without imaging system



Speckles

Speckle properties: second order statistics

- Observed speckle size is (with imaging system):

$$\delta r_s = 2.44\frac{\lambda z}{D} \quad (12)$$

Speckle field formation with imaging system