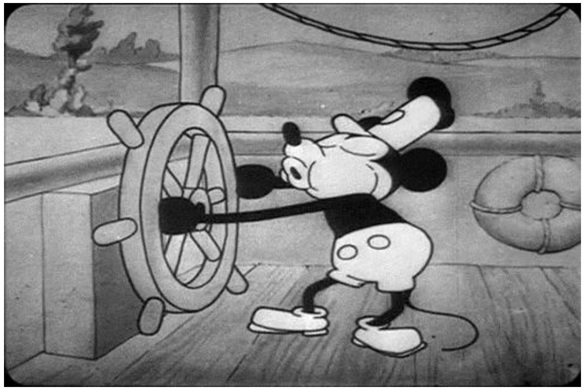
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



11 December 2025

WORCESTER POLYTECHNIC INSTITUTE
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STRESS ANALYSIS
ES-2502, B'2025

Lecture 27:

Course Summary

11 December 2025

General information

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Summary

Average normal stress in an axially loaded bar

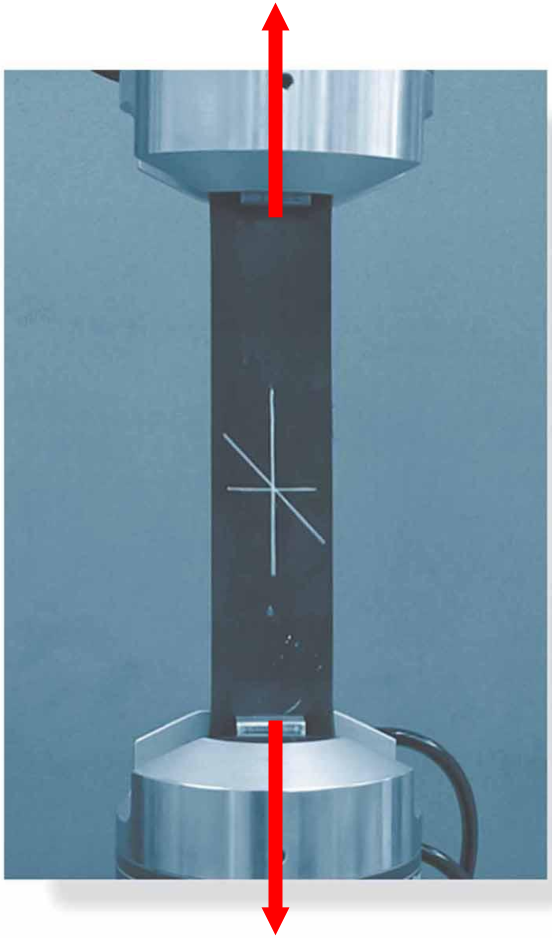


Figure: 02-01-A-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

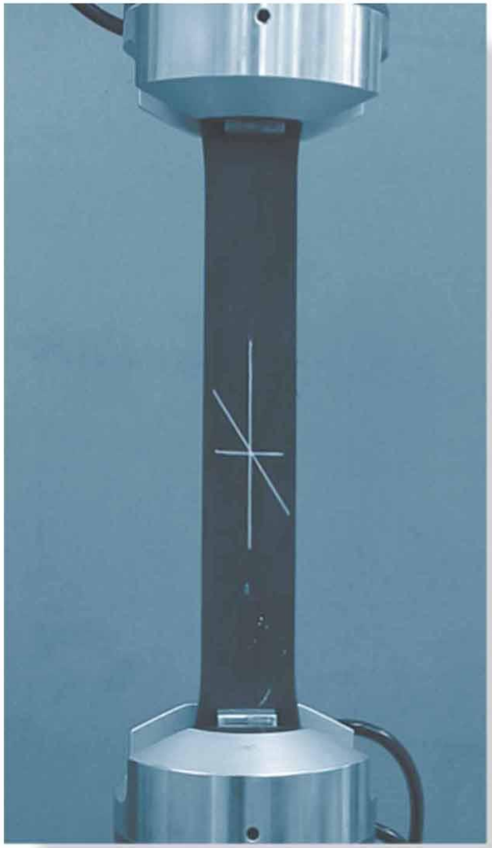
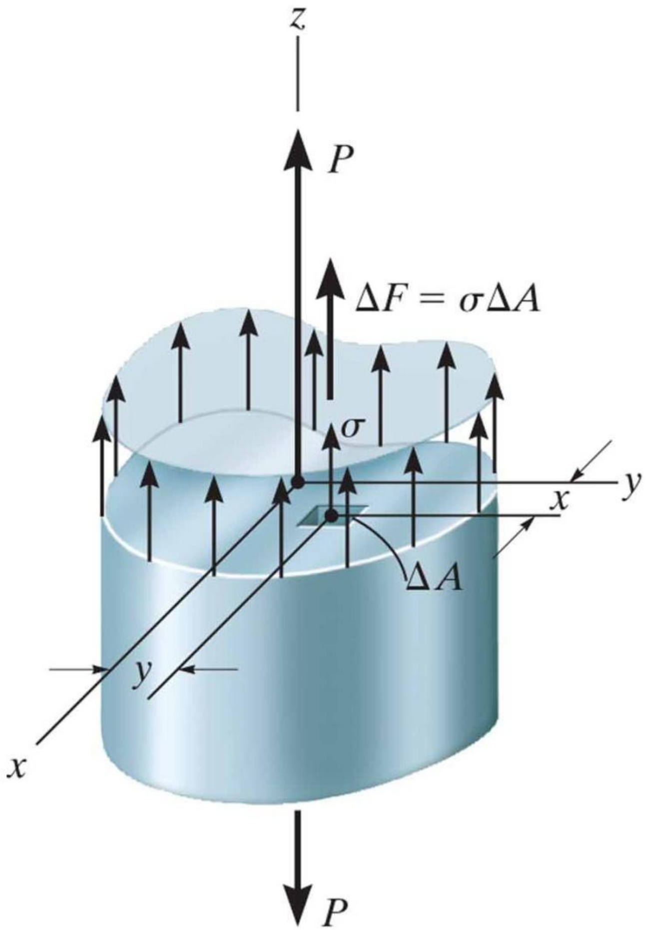


Figure: 02-01-B-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Average normal stress in an axially loaded bar

Internal distribution of forces



$$+\uparrow F_{Rz} = \sum F_z$$

$$\int dF = \int_A \sigma \, dA$$

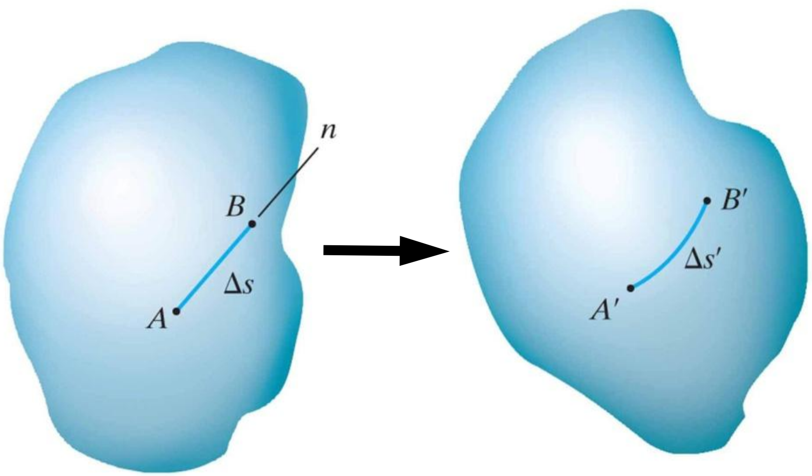
$$P = \sigma A$$

Average normal stress:

$$\sigma = \frac{P}{A}$$

Strain: definition: change in length per unit length

Normal strain



Undeformed body
(a)

Deformed body
(b)

Average normal strain:

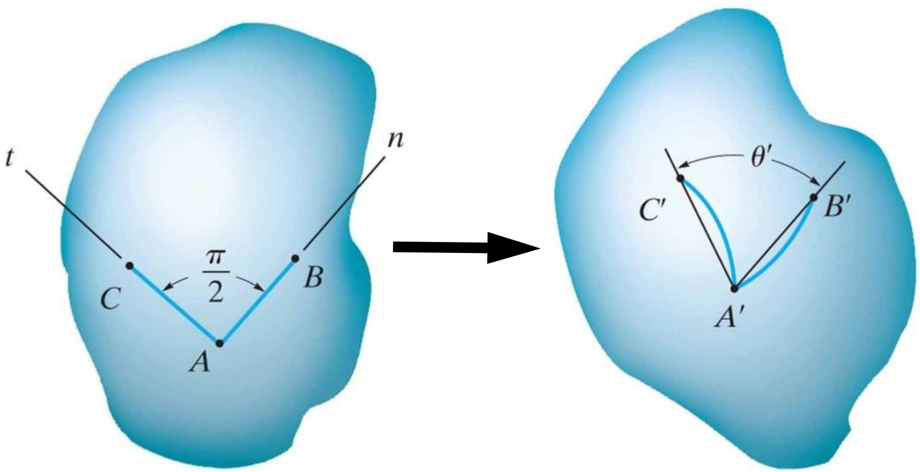
$$\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

Normal strain:

$$\varepsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$

Strain: definition: change in length per unit length

Shear strain



Undeformed body
(a)

Deformed body
(b)

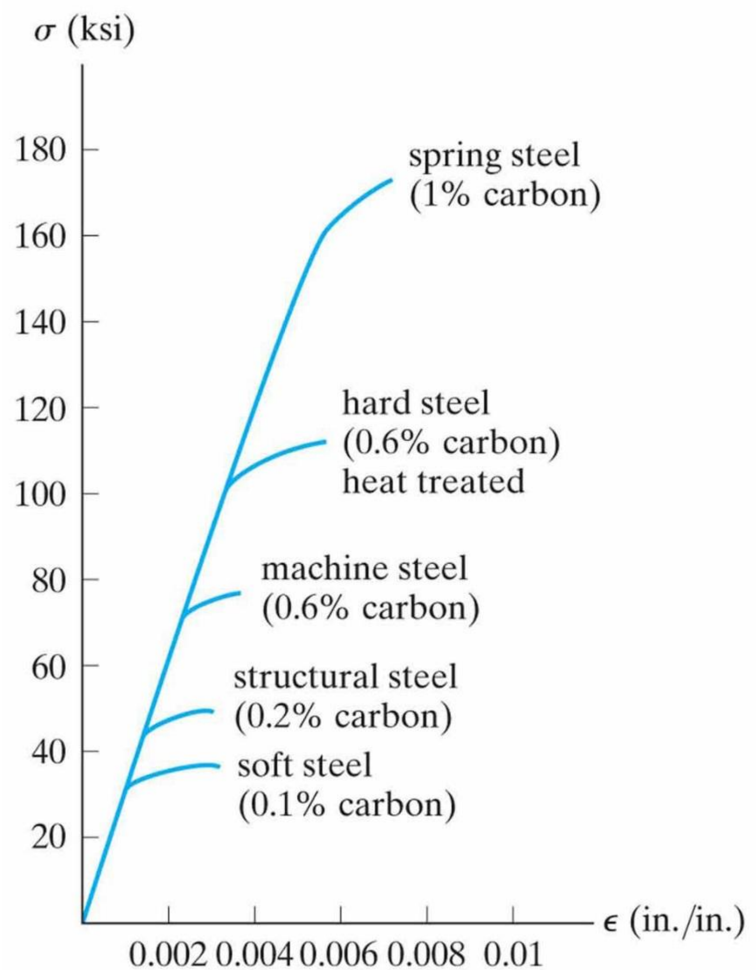
Shear strain:

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \ along \ n \\ C \to A \ along \ t}} \theta'$$

Stress ↔ Strain: Hook's Law

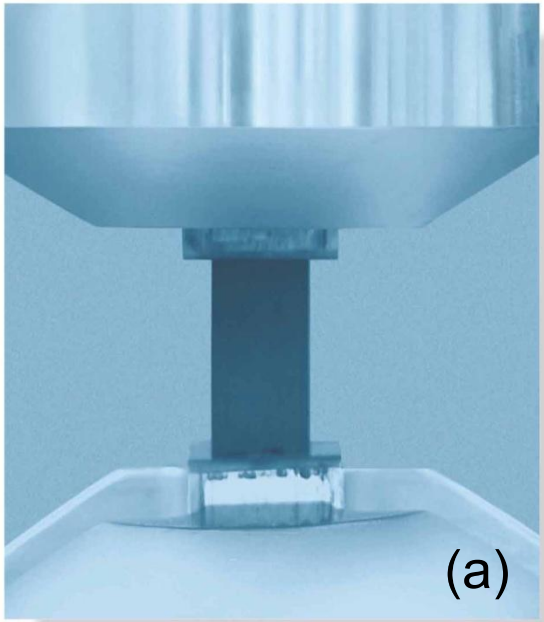
$$\sigma = E \cdot \varepsilon$$

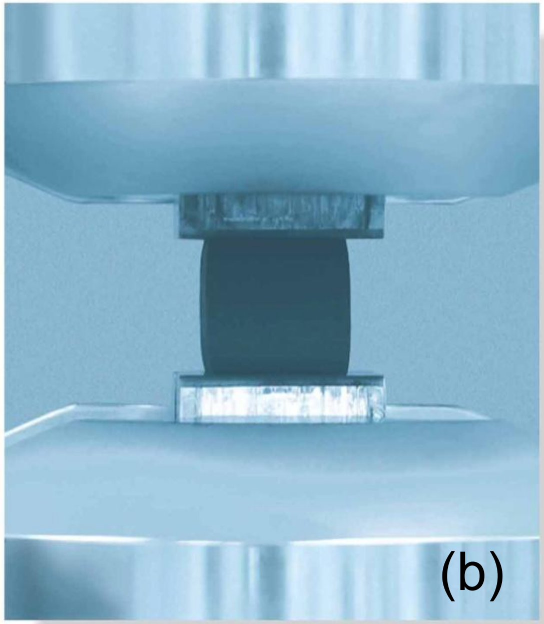
E = Elastic modulus (aka)



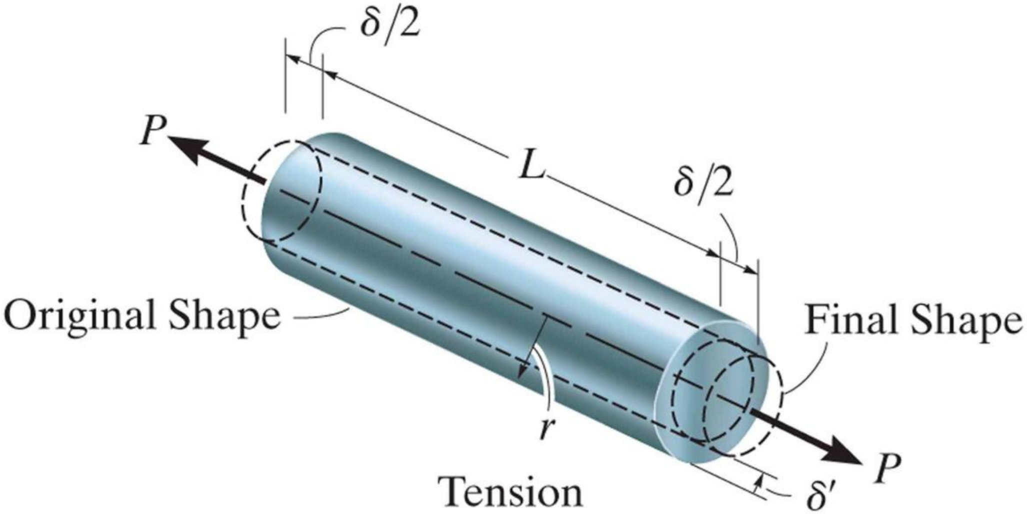
Remember: E is nearly the same for different classes of steels !!

Poisson's ratio:





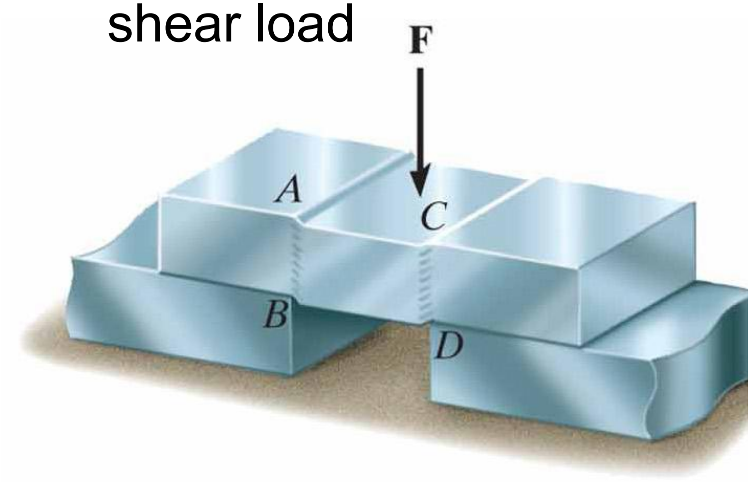
Poisson's ratio: $\nu = -\dfrac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}}$



Average *direct shear* stress

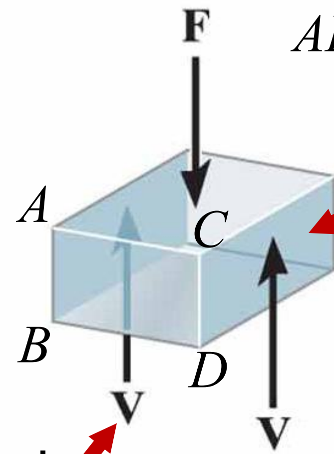
$$\tau_{avg} = \frac{V}{A_V}$$

Bar subjected to shear load



(a)

ABCD section

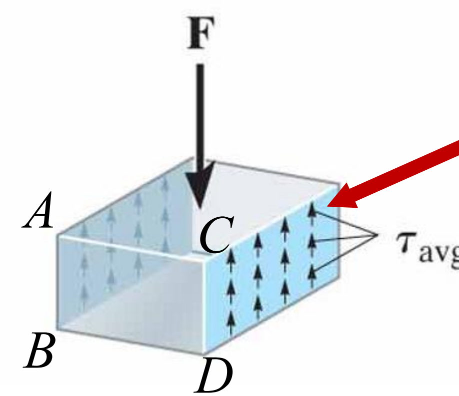


A_V

(Area under shear)

(Internal loading)

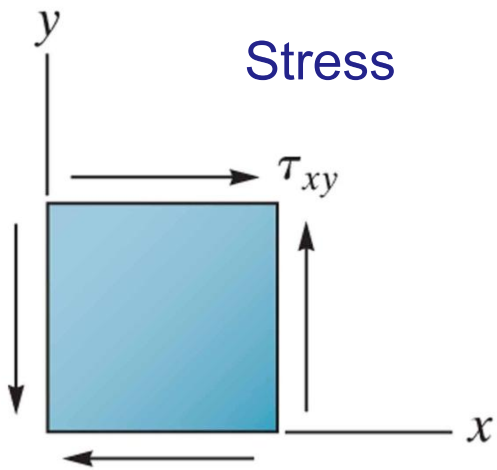
(b)

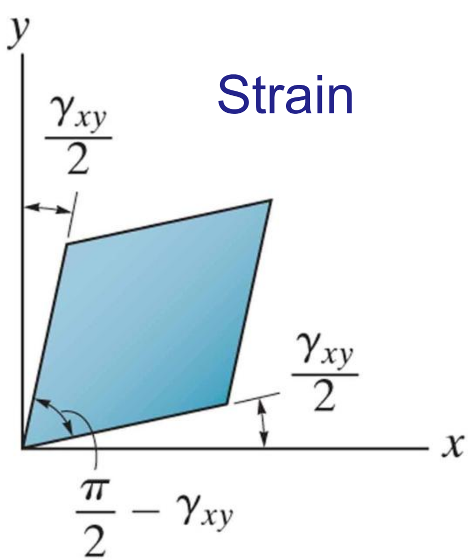


A_V

Shear stress ↔ strain

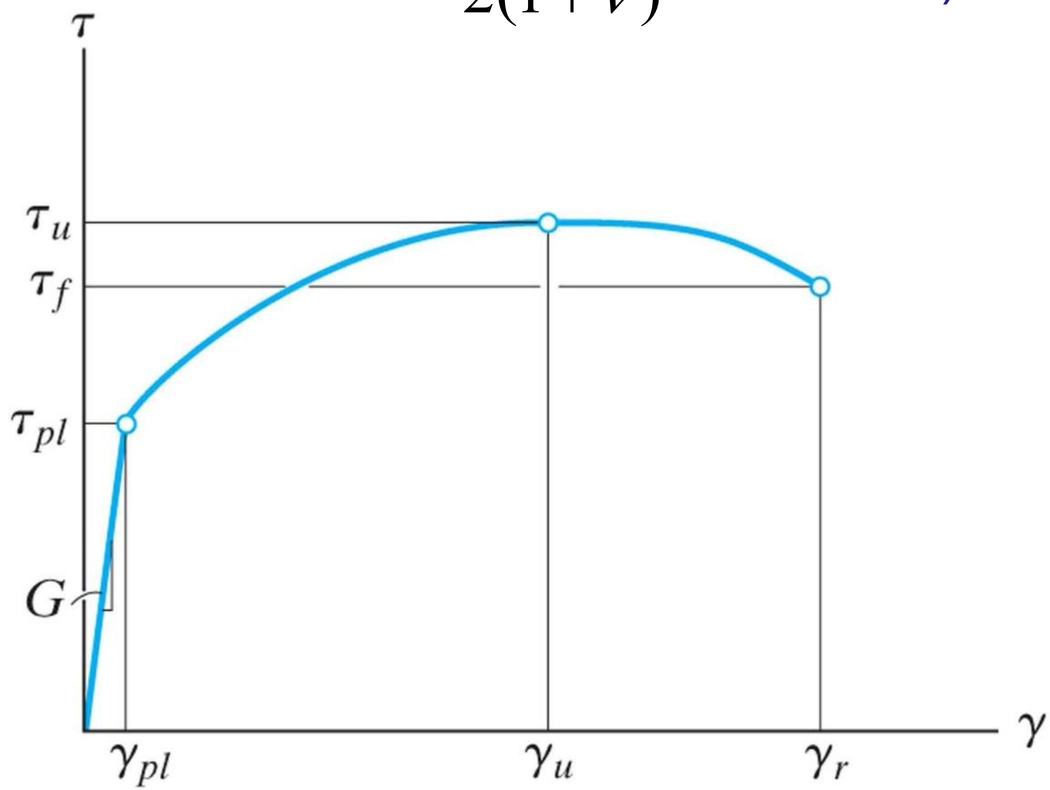
Pure shear





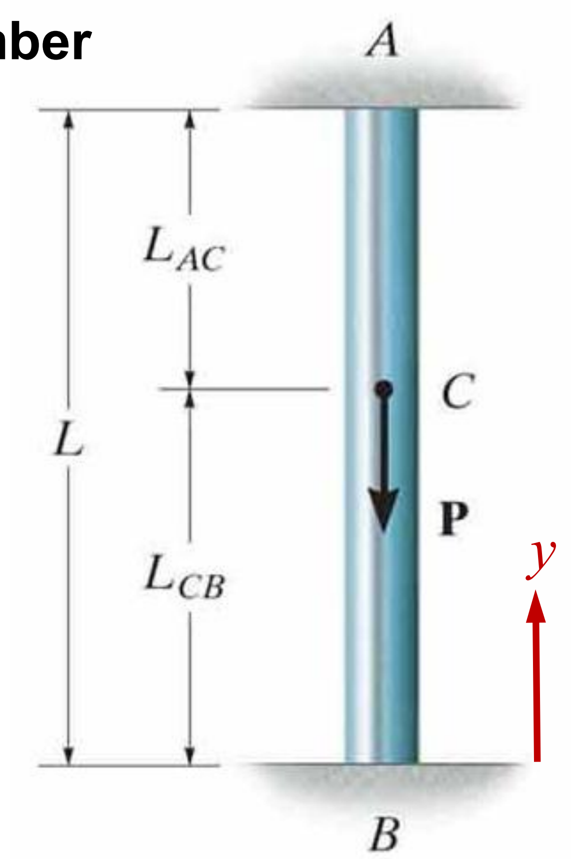
Hook's law for shear: $\tau = G\,\gamma$

with $G = \dfrac{E}{2(1+\nu)}$ (shear modulus)



Statically indeterminate axially loaded member

Axially loaded member



Additional equations are obtained by applying:

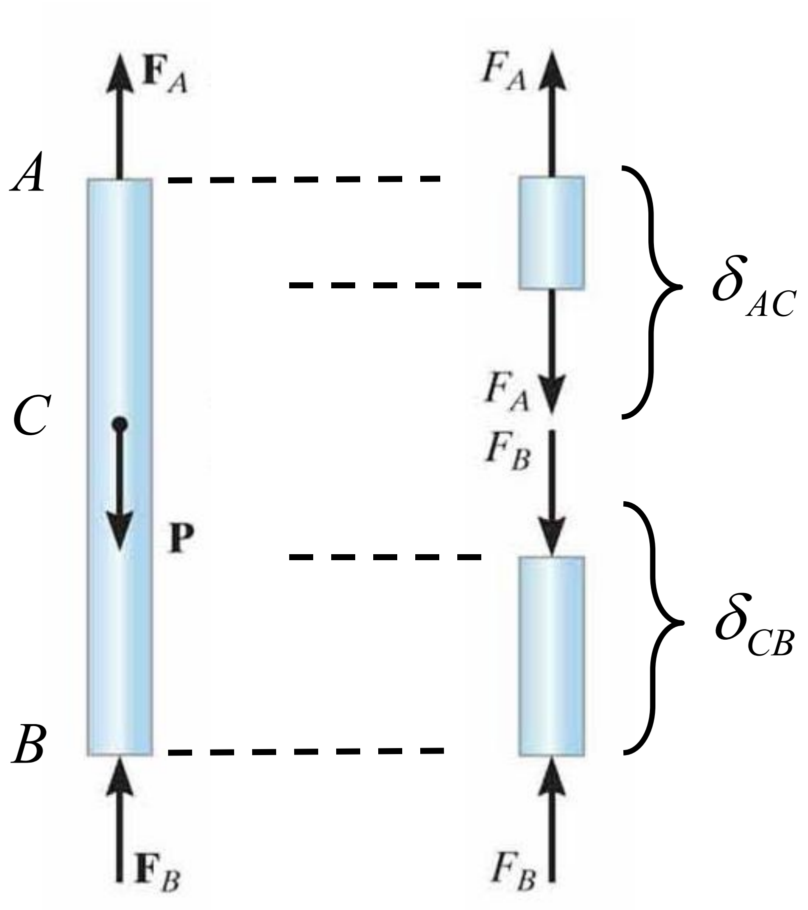
Compatibility or kinematic equations

↑

Load-displacement equations

$$\delta_{A/B} = 0$$

Statically indeterminate axially loaded member



***Compatibility or kinematic equations*:**

$$\frac{F_A\, L_{AC}}{A\,E} - \frac{F_B\, L_{CB}}{A\,E} = 0 \qquad (2)$$

Thermal stresses: uniaxial effects

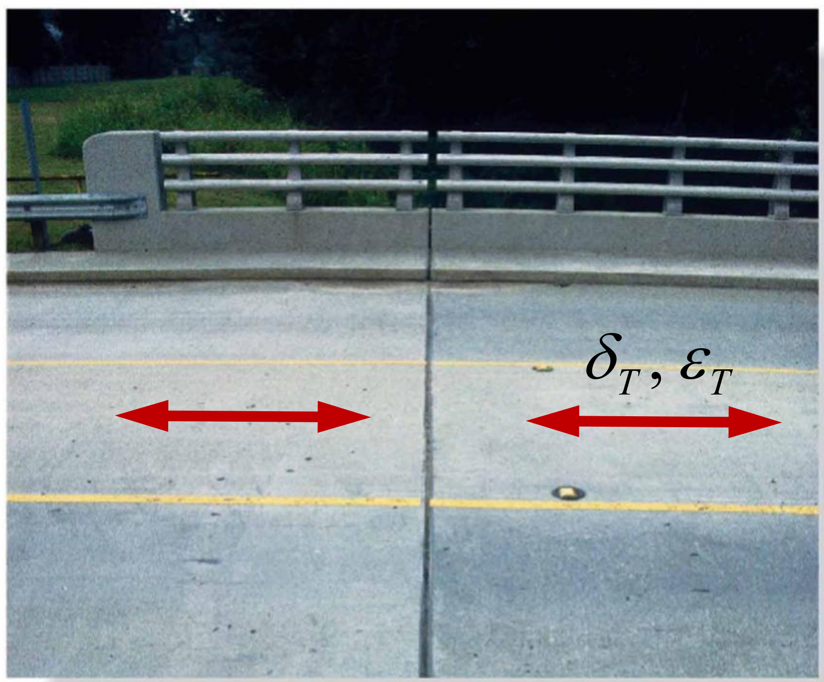
$$\varepsilon_T = \alpha \, \Delta T \quad \longrightarrow \quad \delta_T = \varepsilon_T \, L = \alpha \, \Delta T \cdot L$$

(Thermal strains) (Thermal deformations)

α = linear coefficient of thermal expansion, 1/°C, 1/°F

ΔT = temperature differential

L = original length of component



Torsion formula

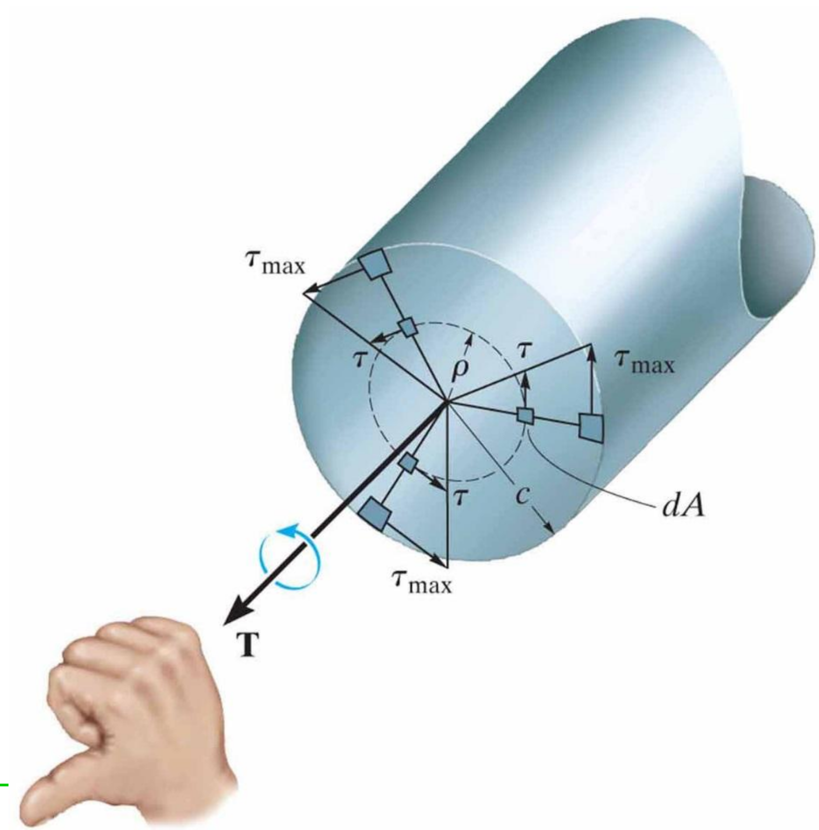
According to Hook's law (linear elasticity):

$$(\tau = G \cdot \gamma)$$

Torsion formula for stresses:

(linear elastic)

$$\tau_{max} = \frac{T\,c}{J} \quad and \quad \tau = \tau(\rho) = \frac{T\,\rho}{J}$$



Differential Force:

$$dF = \tau \cdot dA$$

Differential Torque:

$$dT = \rho\,(\tau \cdot dA)$$

The flexure formula

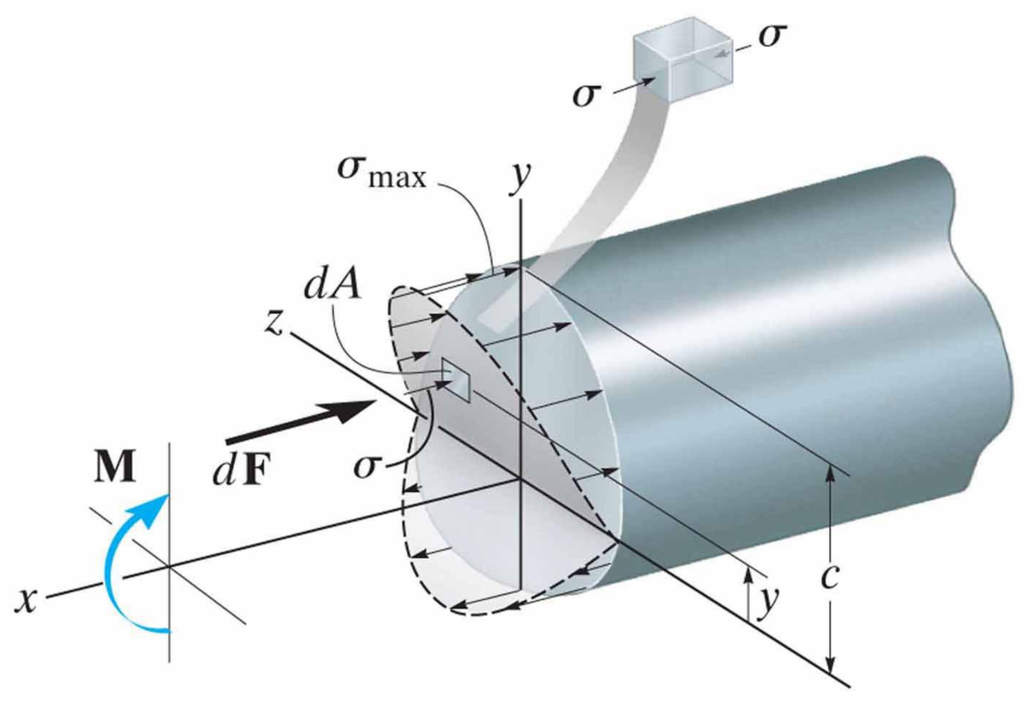
Resultant internal moment:

$$M = \frac{\sigma_{x\,Max}}{c} \int_A y^2 \, dA$$

$$\boxed{\sigma_x = \frac{M\,y}{I_{zz}}}$$

$$I_{zz} = \int_A y^2 \, dA$$

Area moment of inertia wrt to *z*-axis



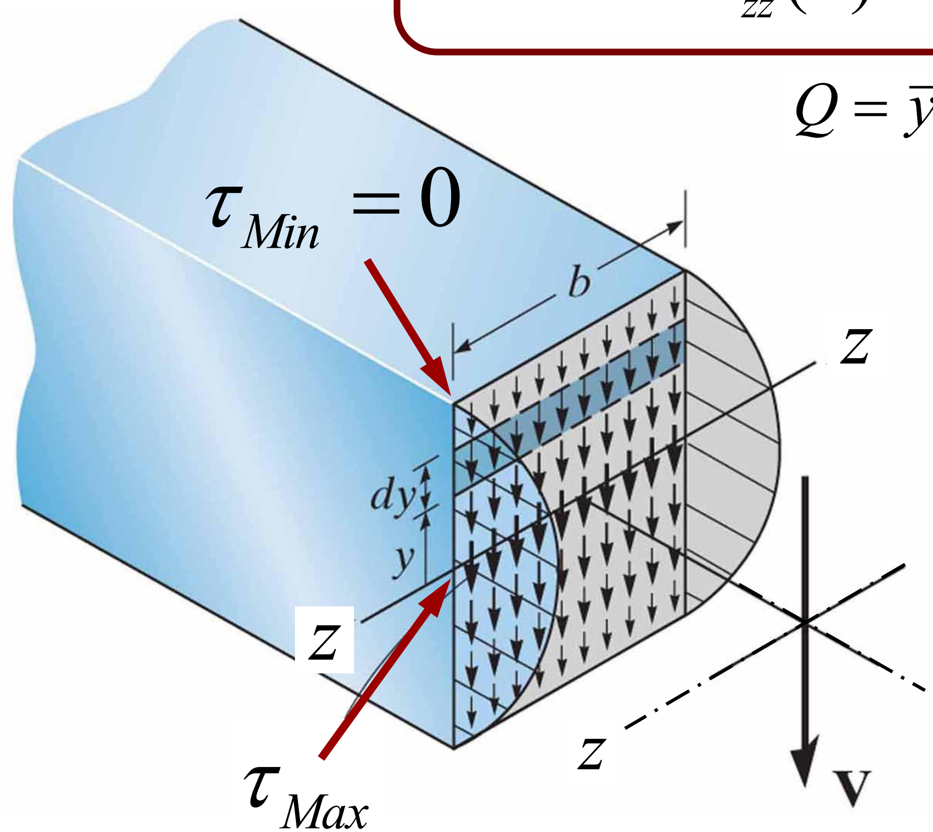
Shear formula

Observed in components subjected to bending loads

$$\boxed{\tau(x,y)=\frac{V(x)\cdot Q(y)}{I_{zz}(x)\cdot t(y)}}$$

Important to remember!!

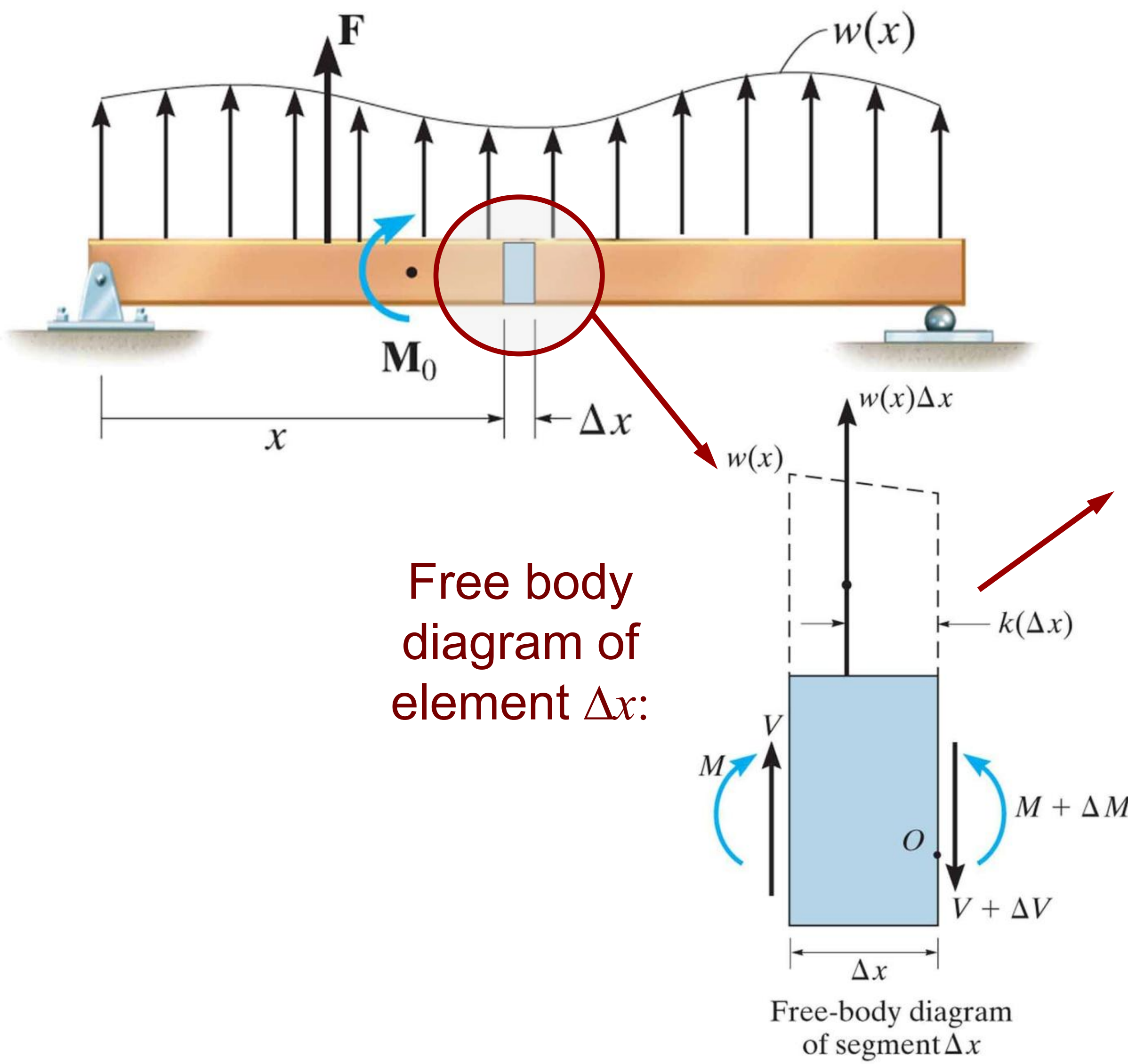
$$Q=\bar{y}'A'$$



Internal distribution of shear stresses:

$$\tau_{xy}=\tau_{yx}$$

Shear and bending diagrams: regions with distributed load



Free body diagram of element Δx:

Important to remember!!

$$\frac{dV}{dx} = w(x)\,;$$

$$\frac{dM}{dx} = V(x)$$

Bending deformation of straight beams

The elastic curve

$$\frac{w}{EI} = \frac{d^4 y}{dx^4}$$ Load function – deflection

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$ Shear function – deflection

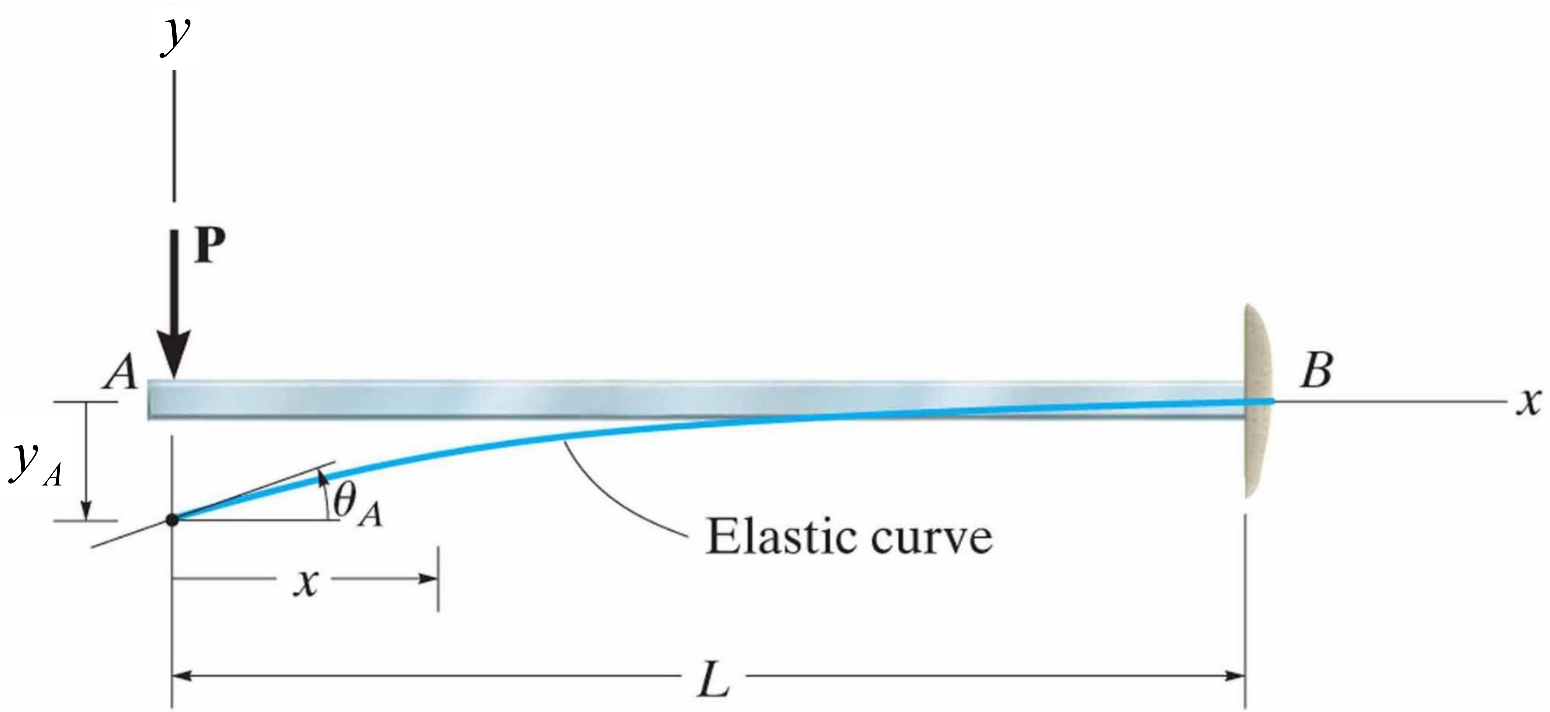
$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$ Moment function – *elastica*

$$\theta = \frac{dy}{dx}$$ Slope – deflection

$$y = f(x)$$ Deflection

Bending deformation of straight beams: example A

The cantilever shown is subjected to a vertical load P at it end. Determine the equation of the deformation (elastic) curve. $E{\cdot}I$ is constant.



Bending deformation of straight beams

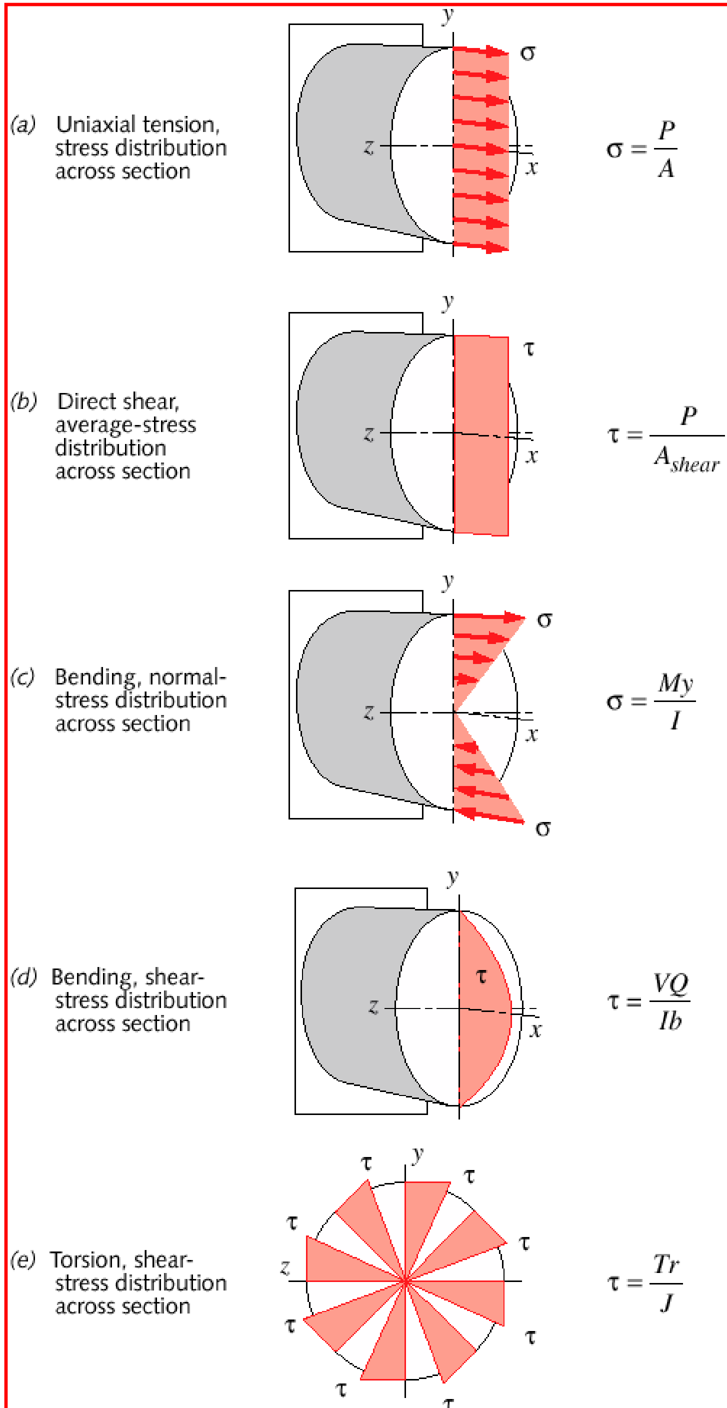
The elastic curve

For small deformations:

$$\boxed{\frac{d^2 y}{dx^2} = \frac{M}{E \cdot I_{zz}}}$$

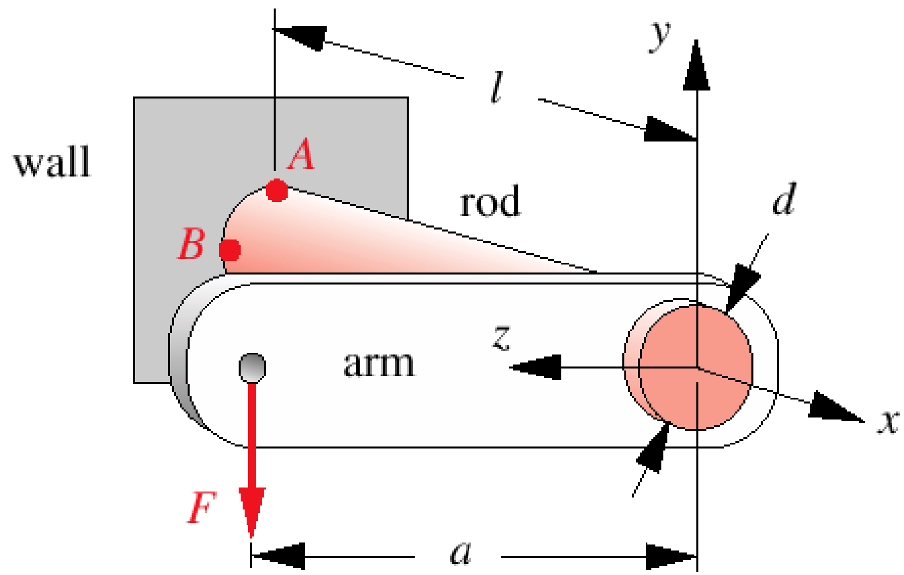
Elastica equation

Important to remember!!



Combined loading

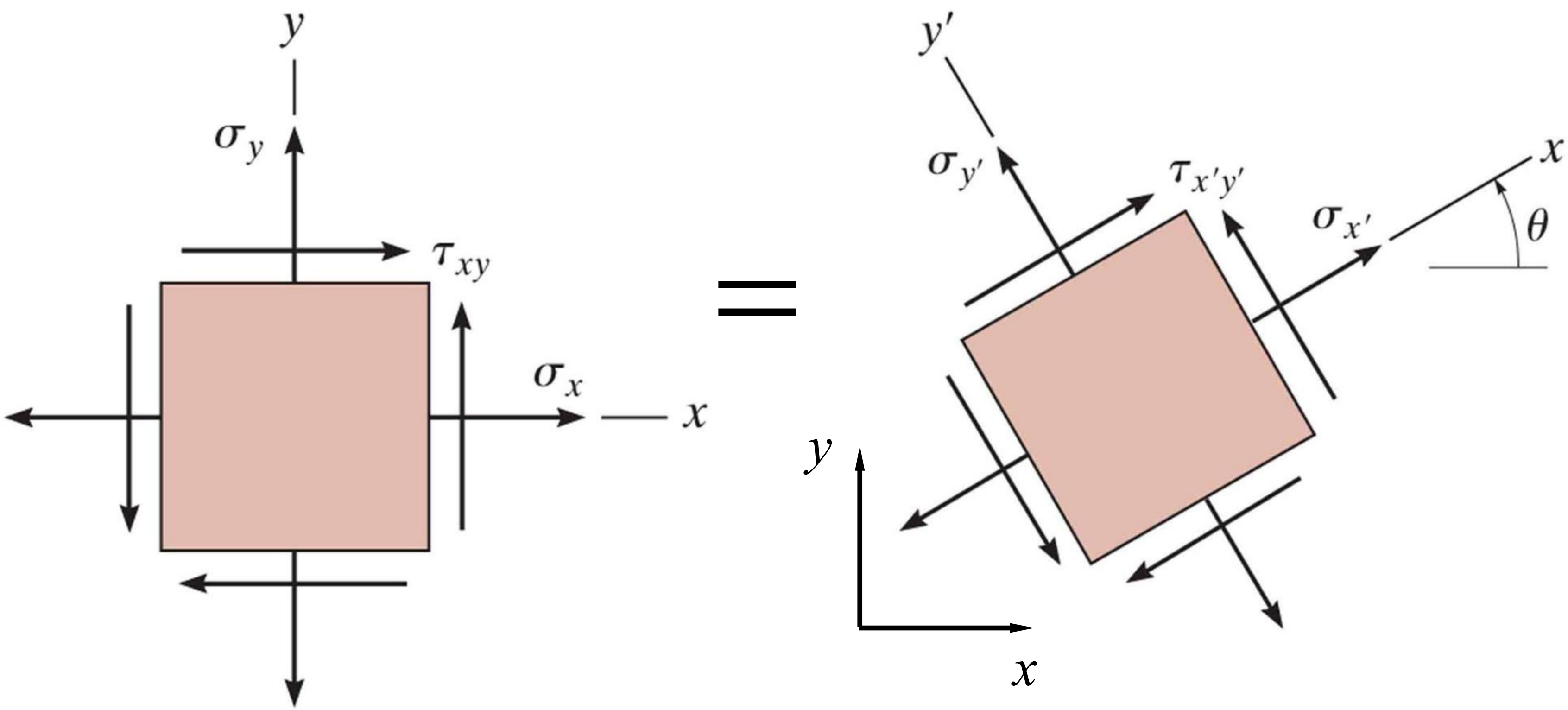
Find the most highly stressed locations on the bracket shown. Draw volume (stress) elements at points A and B



Plane stress transformation (rotation)

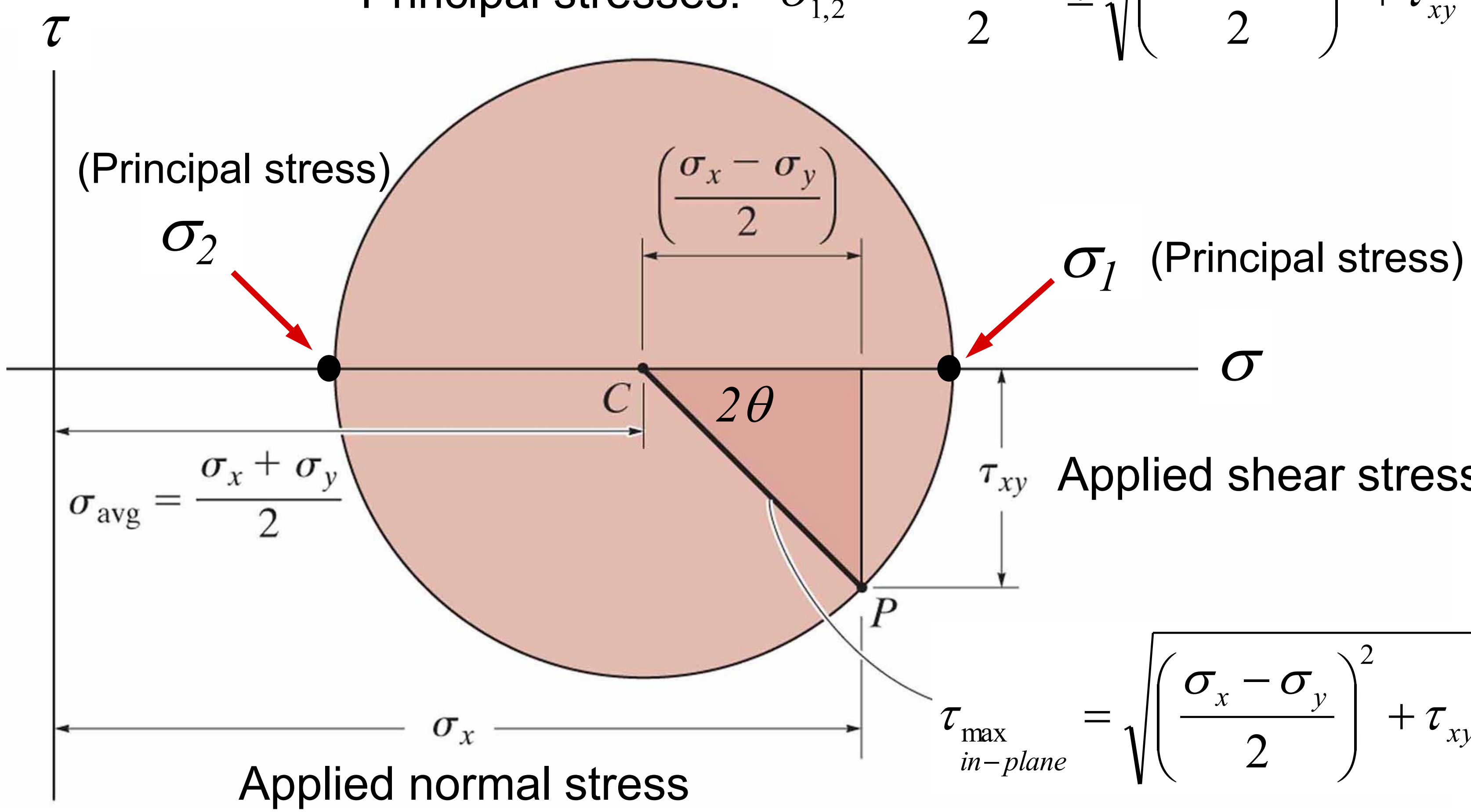
Stress cube in 2D

Rotate cube in 2D while **keeping resultant forces the same**



Mohr's circle (developed by Otto Mohr in 1882)

Principal stresses: $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$



$$\tau_{\substack{\max \\ in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$

Maximum shear

Thank you

and

Best wishes !!