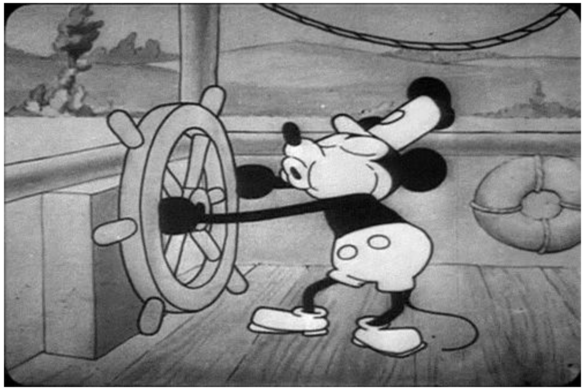
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



10 December 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

Lecture 26:
Unit 24, 25:
Principal Stresses. Mohr's circle

10 December 2025

General information

Instructor: Cosme Furlong
HL-152
(508) 831-5126
Email: cfurlong @ wpi.edu
http://www.wpi.edu/~cfurlong/es2502.html

Graduate Assistants:

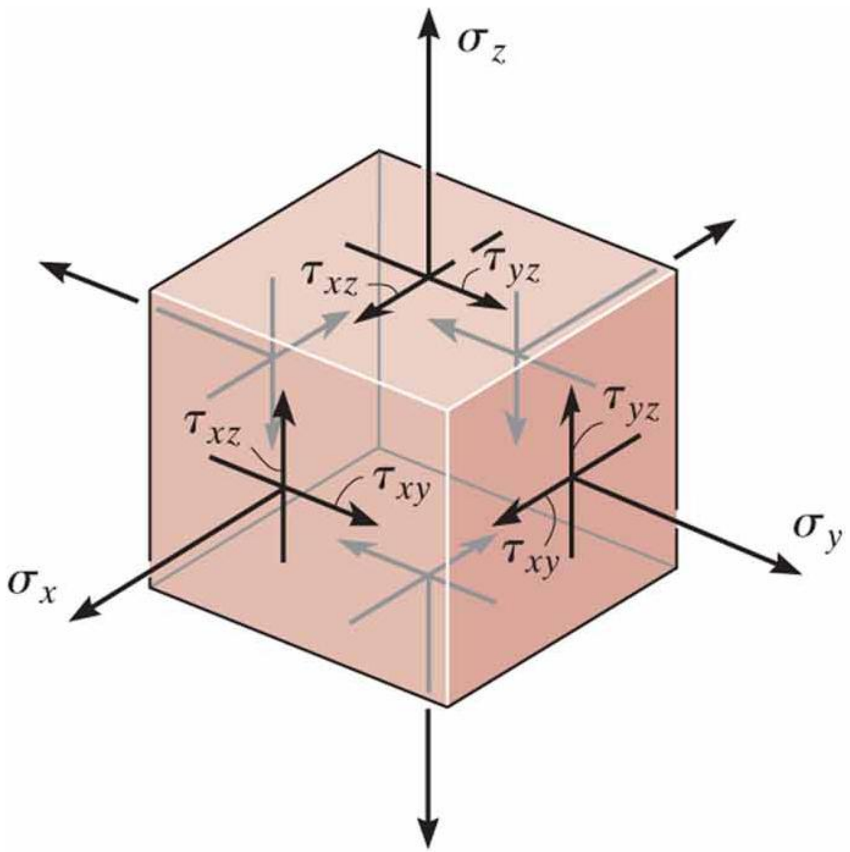
→ Hamed Ghavami (TA)
Email: sghavami @ wpi.edu

→ Jay Patil (GA)
Email: jpatil1 @ wpi.edu

→ Mikayla Almeida (GA)
mpalmeida @ wpi.edu

Stress at a point

General state of stress. Stress cube in 3D



Notation

σ Normal Stress

τ Shear Stress

τ_{xy}

Face

Direction

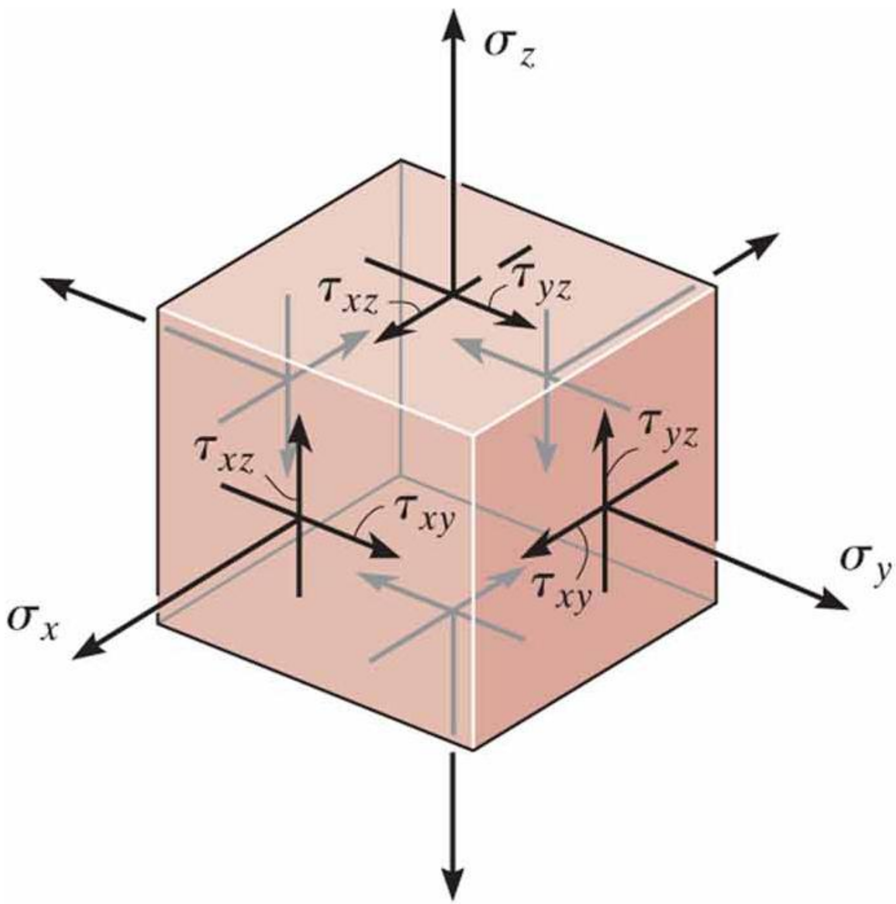
Equilibrium conditions require that:

$$\tau_{xy} = \tau_{yx}\,,\ \tau_{xz} = \tau_{zx}\,,\ \tau_{yz} = \tau_{zy}$$

Why?

Stress at a point

General state of stress. Stress cube in 3D



There are 9 components of stress.

Equilibrium conditions are used to reduce the number of stress components to 6:

$$\tau_{xy} = \tau_{yx}$$

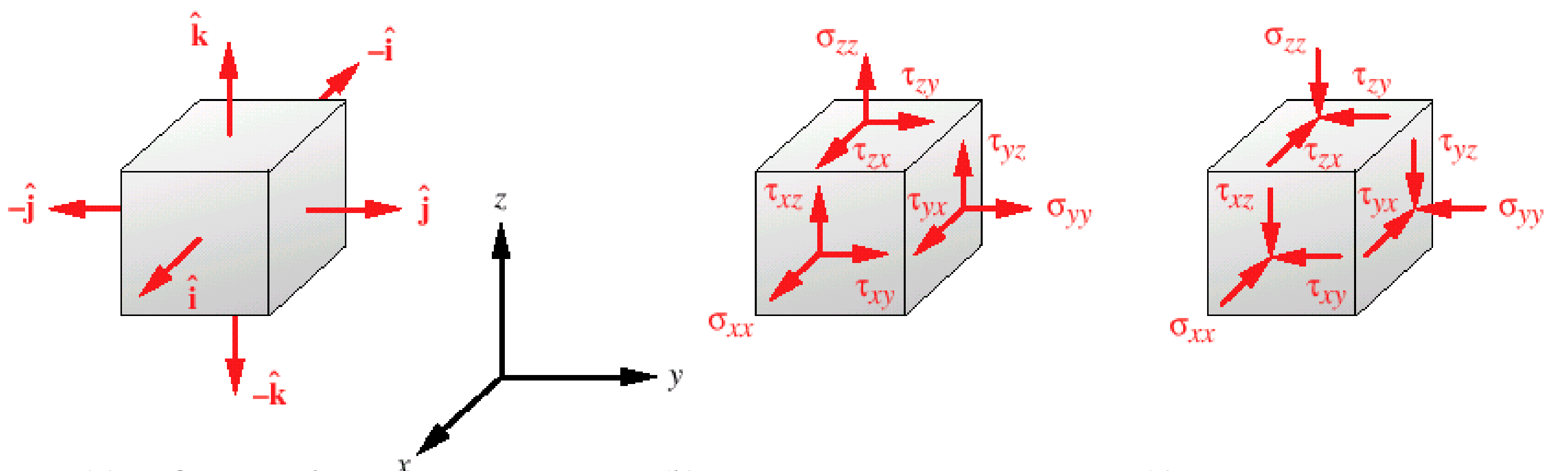
$$\tau_{xz} = \tau_{zx}$$

$$\tau_{yz} = \tau_{zy}$$

Stress tensor
Cauchy stress tensor

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

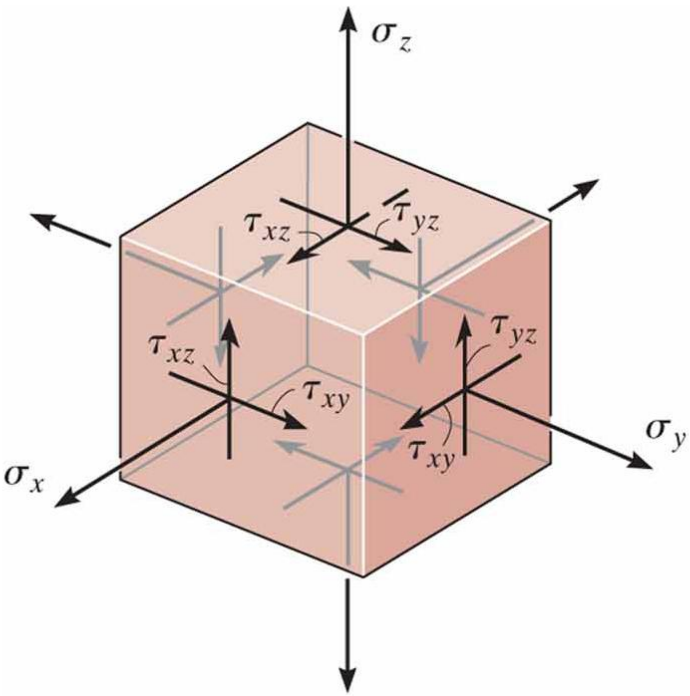
Tensors are quantities that are *invariant* to coordinate transformations (i.e., translations & rotations)



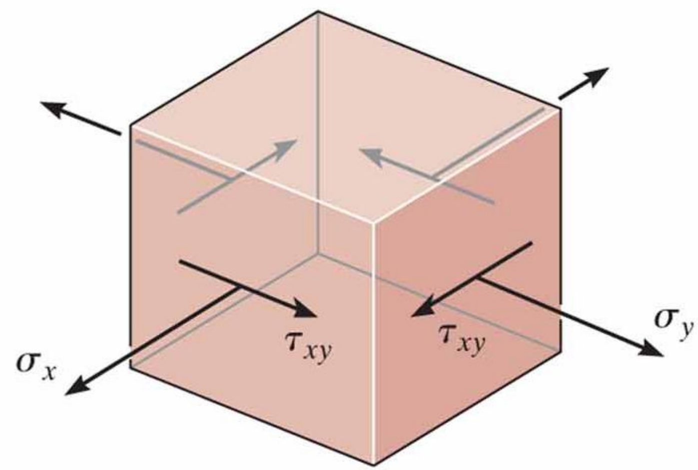
(a) Surface normals (b) Positive stress components (c) Negative stress components

Stress at a point
Plane stress

General state of stress

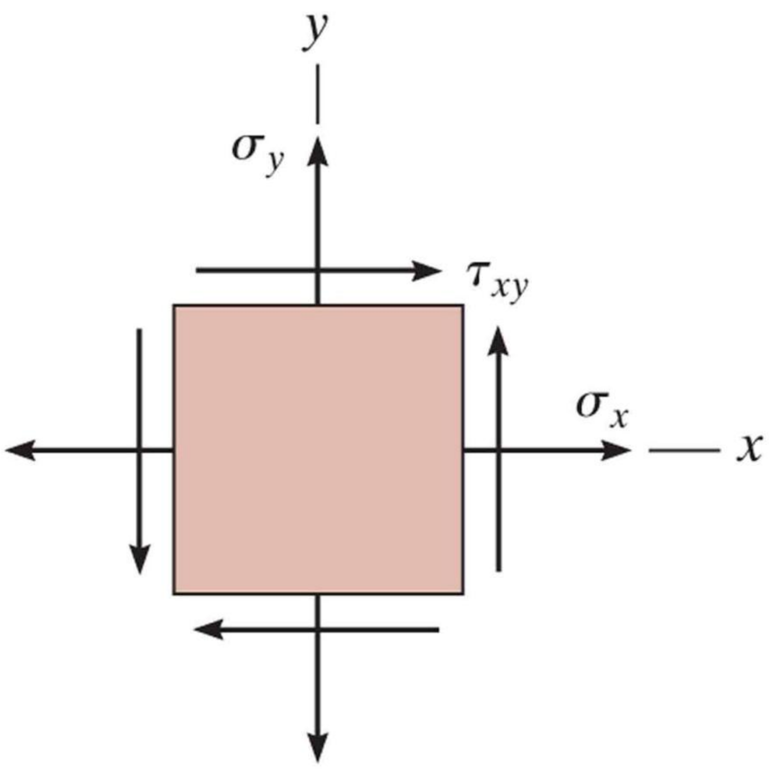


Pane stress



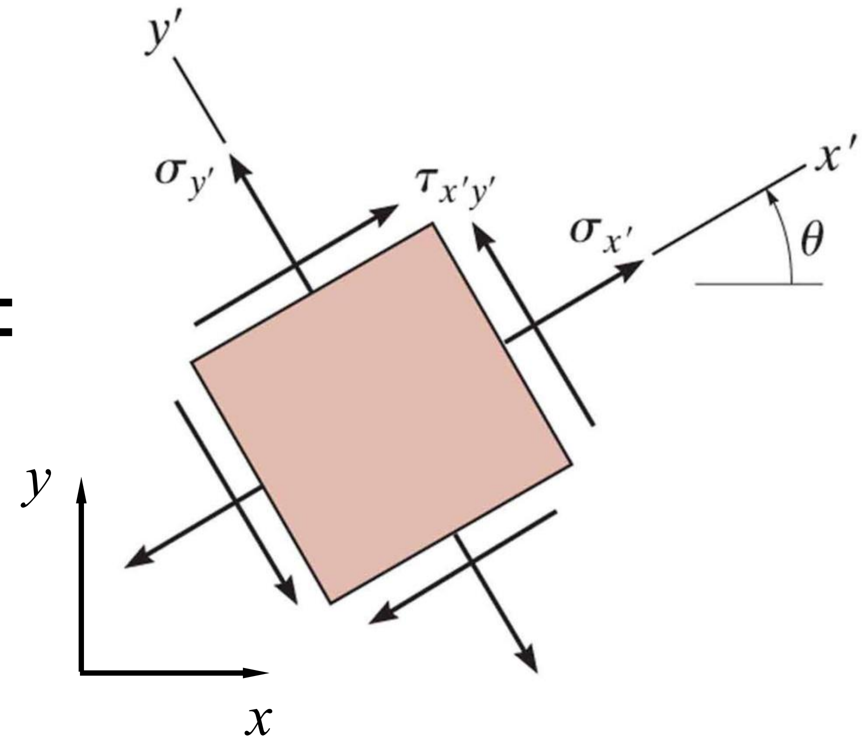
Plane stress transformation (rotation)

Stress cube in 2D



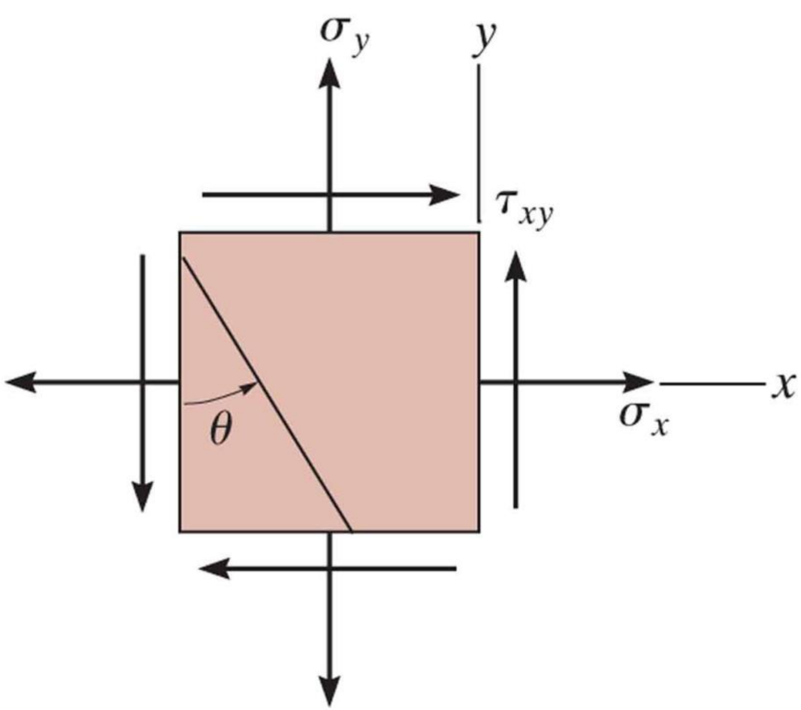
=

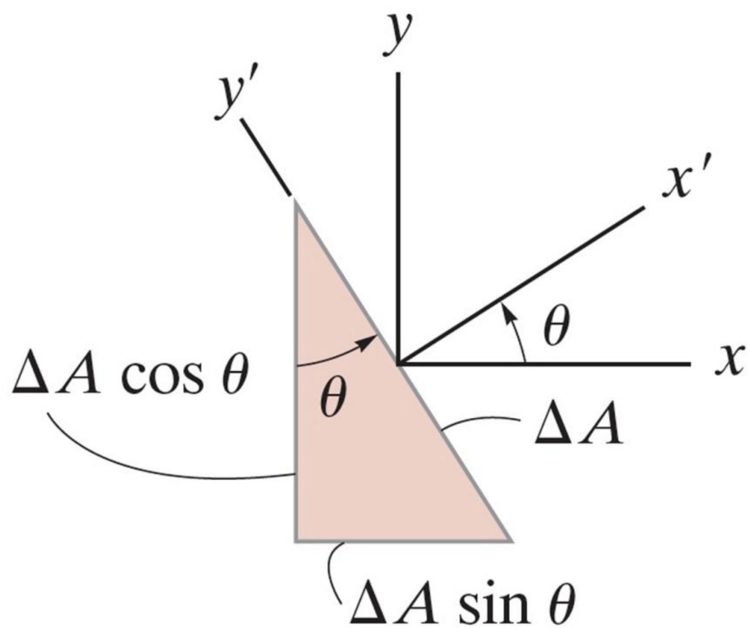
Rotate cube in 2D while **keeping resultant forces the same**



Plane stress transformation (rotation)

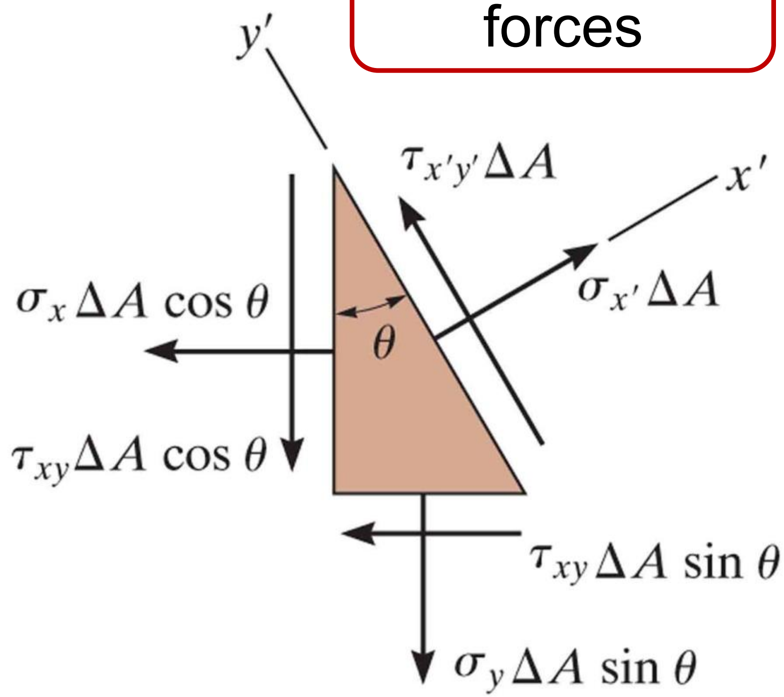
Introduce plane rotated at angle θ, therefore defining area ΔA



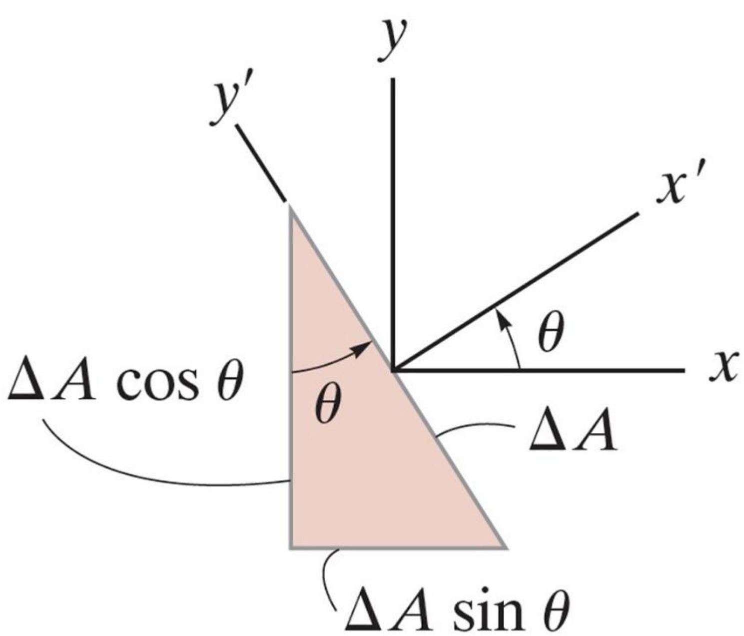


Plane stress transformation (rotation)

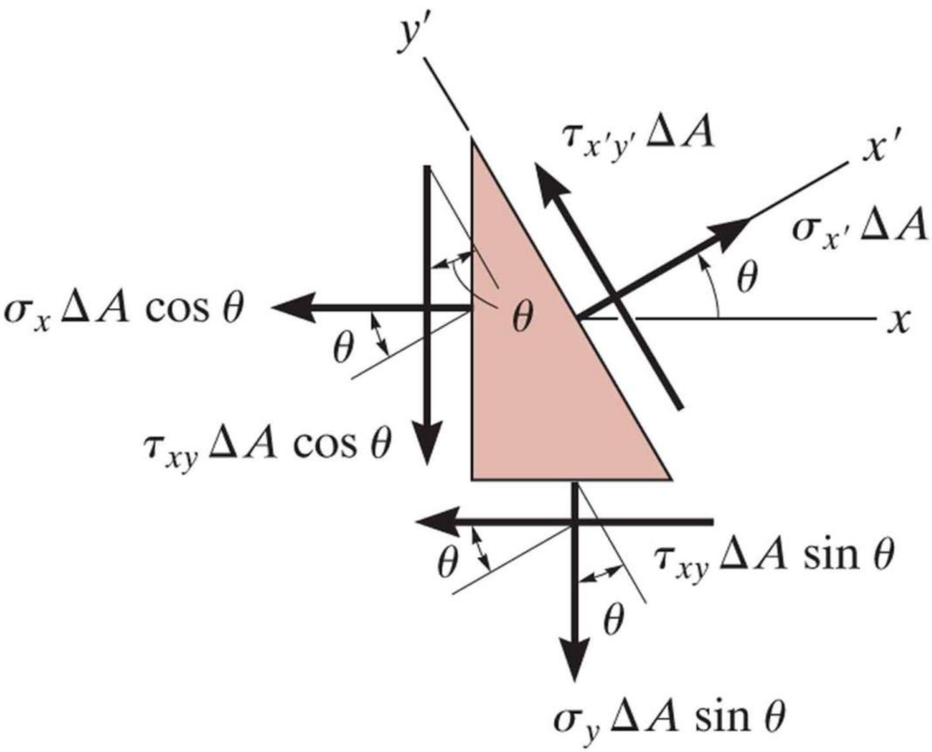
Corresponding forces



Plane rotated at angle θ defining area ΔA



Plane stress transformation (rotation)

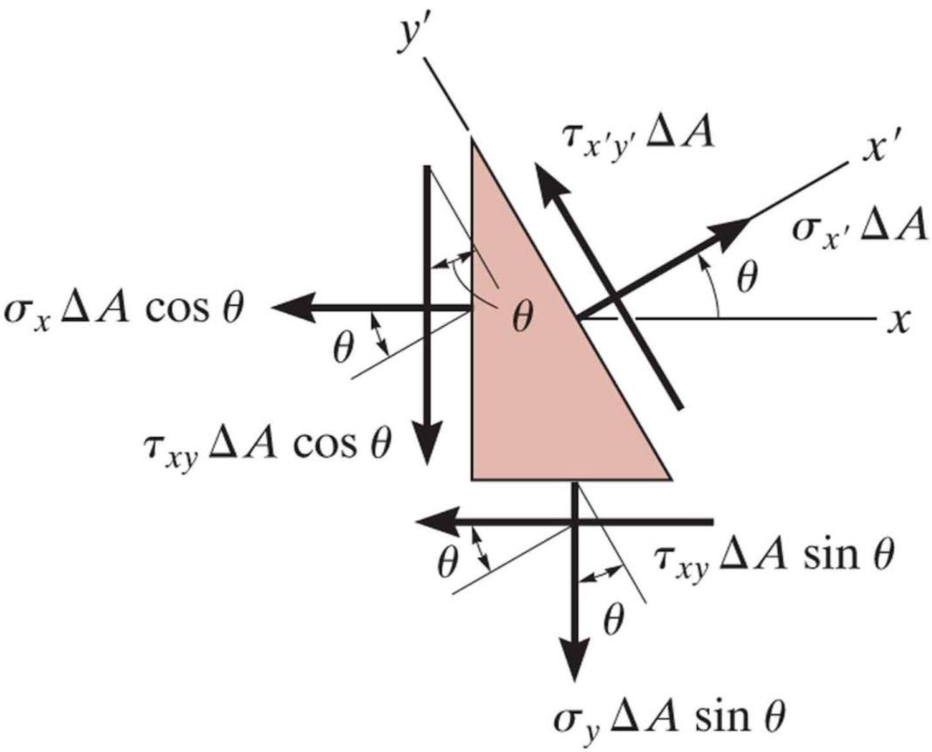


Apply equilibrium conditions:

(1) $\sum F_{x'} = 0\,;\quad +\nearrow$

(2) $\sum F_{y'} = 0\,;\quad +\nwarrow$

Plane stress transformation (rotation)

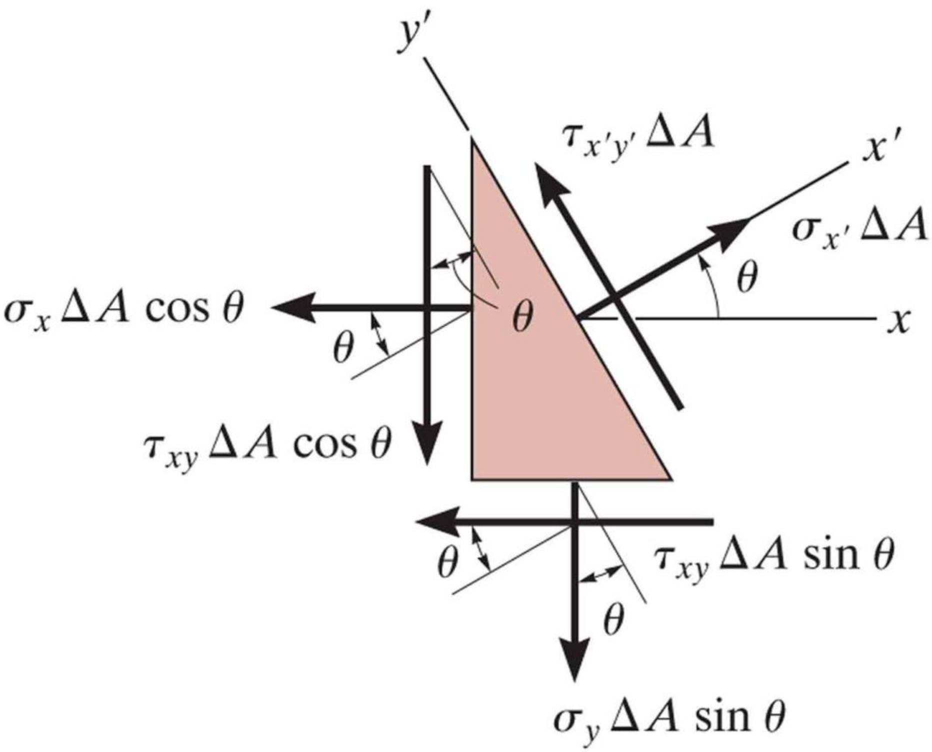


First equilibrium condition:

$$(1)\quad \sum F_{x'} = 0\,; \quad +\nearrow$$

$$\begin{aligned}\sigma_{x'}\Delta A &- (\tau_{xy}\Delta A\sin\theta)\cos\theta \\ &- (\sigma_y\Delta A\sin\theta)\sin\theta \\ &- (\tau_{xy}\Delta A\cos\theta)\sin\theta \\ &- (\sigma_x\Delta A\cos\theta)\cos\theta = 0\end{aligned}$$

Plane stress transformation (rotation)

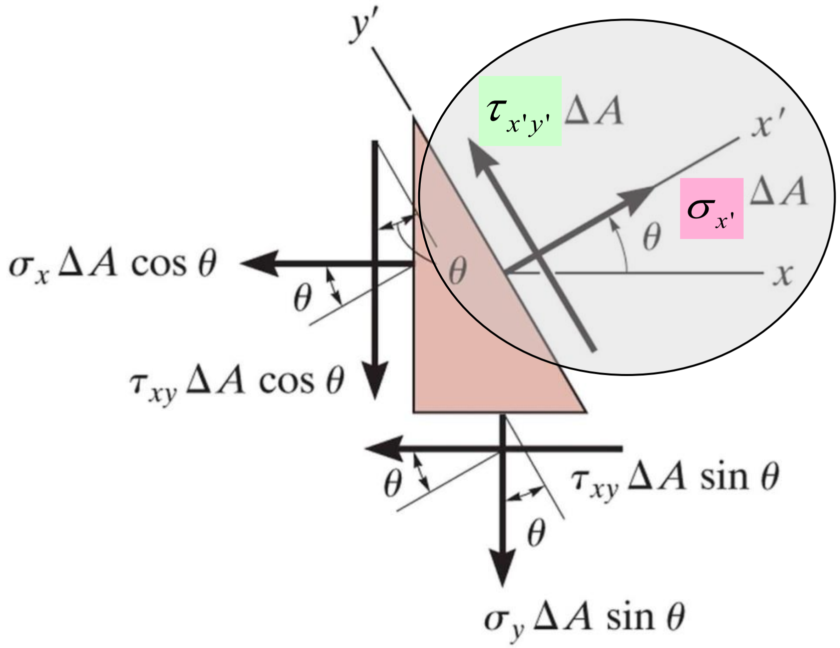


Second equilibrium condition:

$$(2) \quad \sum F_{y'} = 0\,; \quad +\nwarrow$$

$$\tau_{x'y'}\Delta A + (\tau_{xy}\Delta A\sin\theta)\sin\theta$$
$$-(\sigma_y\Delta A\sin\theta)\cos\theta$$
$$-(\tau_{xy}\Delta A\cos\theta)\cos\theta$$
$$+(\sigma_x\Delta A\cos\theta)\sin\theta = 0$$

Plane stress transformation (rotation)



Transformation equations become:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

and by setting $\theta = \theta + 90^o \Rightarrow$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

In-plane *principal stresses*

In-plane principal ***normal*** *stresses* can be obtained by:

$$\frac{d}{d\theta}\sigma_{x'} = \frac{d}{d\theta}\left(\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta+\tau_{xy}\sin 2\theta\right)=0$$

resulting in an angle of:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}$$

which is substituted into transformation equation to lead to the **principal stresses (Max/Min)**:

$$\sigma_{1,2\,(Max,Min)} = \frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}{}^2}$$

Maximum and Minimum in-plane shear stress

Max/Min in-plane shear stresses can be obtained by:

$$\frac{d}{d\theta}\tau_{x'y'} = \frac{d}{d\theta}\left(-\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right) = 0$$

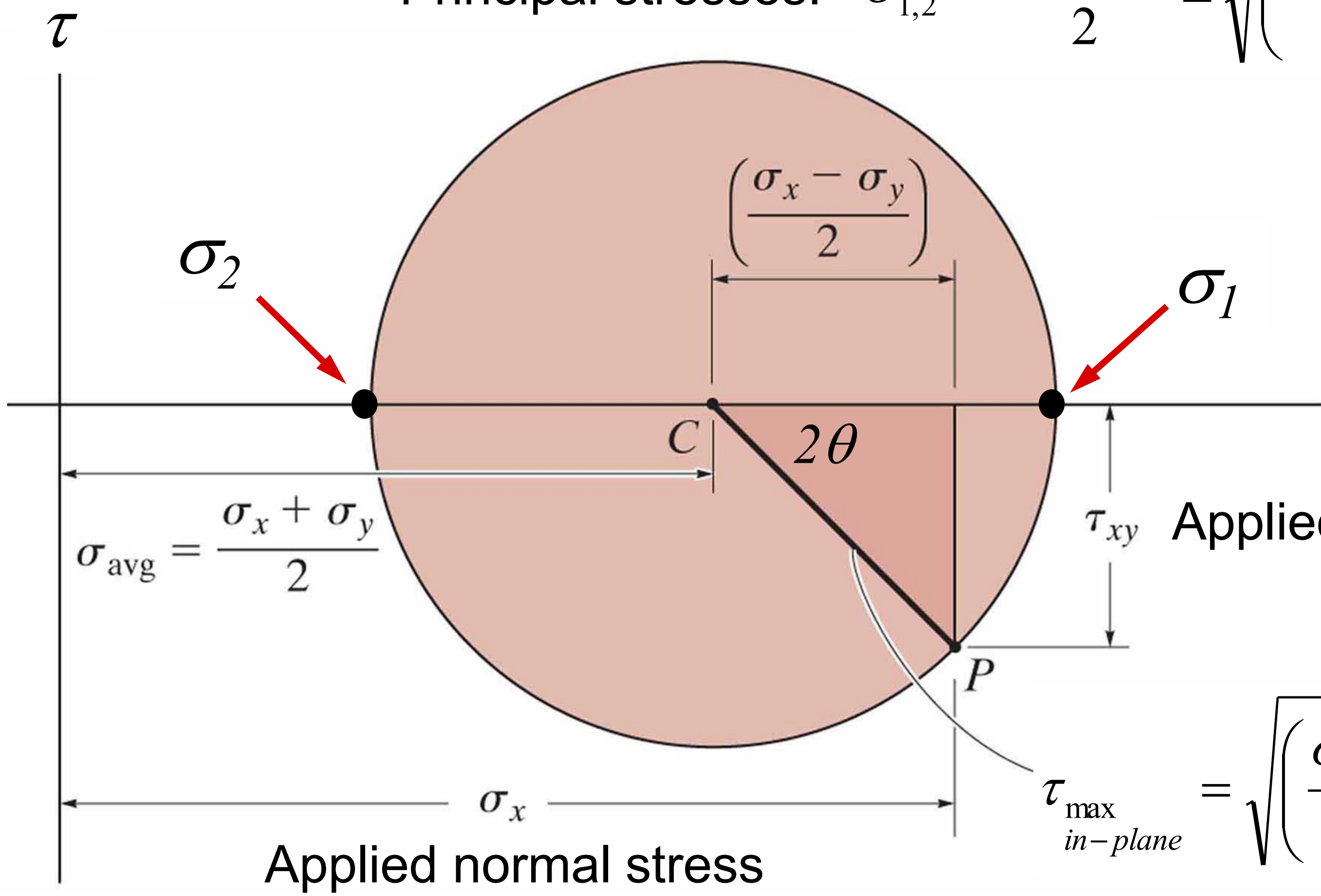
resulting in an angle of: $\tan 2\theta_s = \dfrac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$

which is substituted into transformation equation to lead to the **maximum in-plane shear stress (Max/Min)**:

$$\tau_{\substack{Max/Min \\ in-plane}} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

Mohr's circle (developed by Otto Mohr in 1882)

Principal stresses: $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$



Applied shear stress

$$\tau_{\substack{\max \\ in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$

Maximum shear

Principal and maximum shear stresses can be used to study failure of components

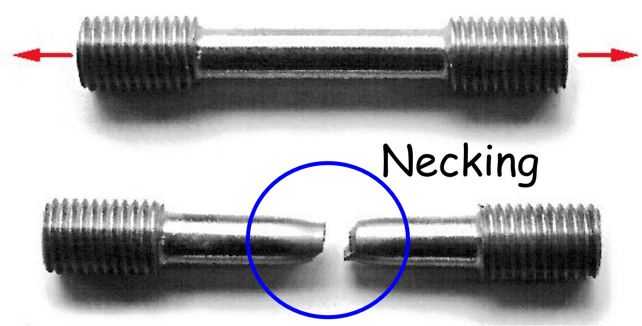


FIGURE 2-3

A Tensile Test Specimen of Mild, Ductile Steel After Fracture

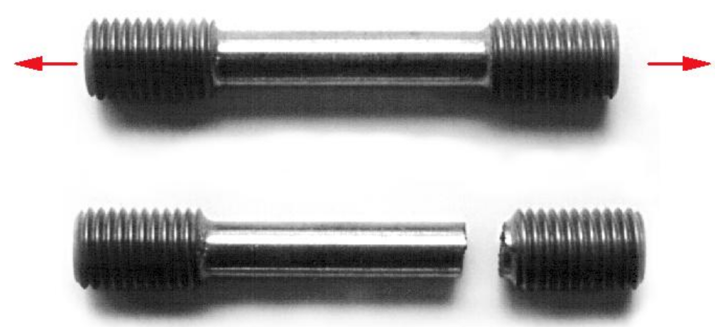


FIGURE 2-5

A Tensile Test Specimen of Brittle Cast Iron After Fracture

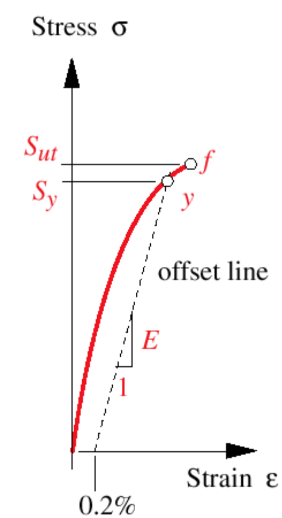


FIGURE 2-4

Stress-Strain Curve of a Brittle Material

Principal and maximum shear stresses can be used to study failure of components

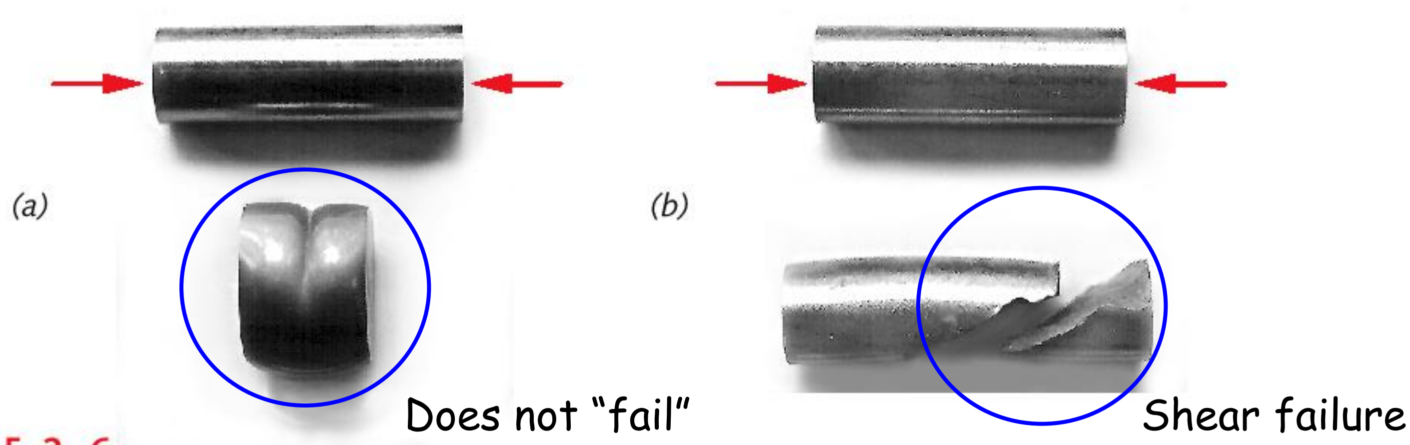


FIGURE 2-6

Compression Test Specimens After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

Even materials: same behavior in tension and in compression.

Principal and maximum shear stresses can be used to study failure of components

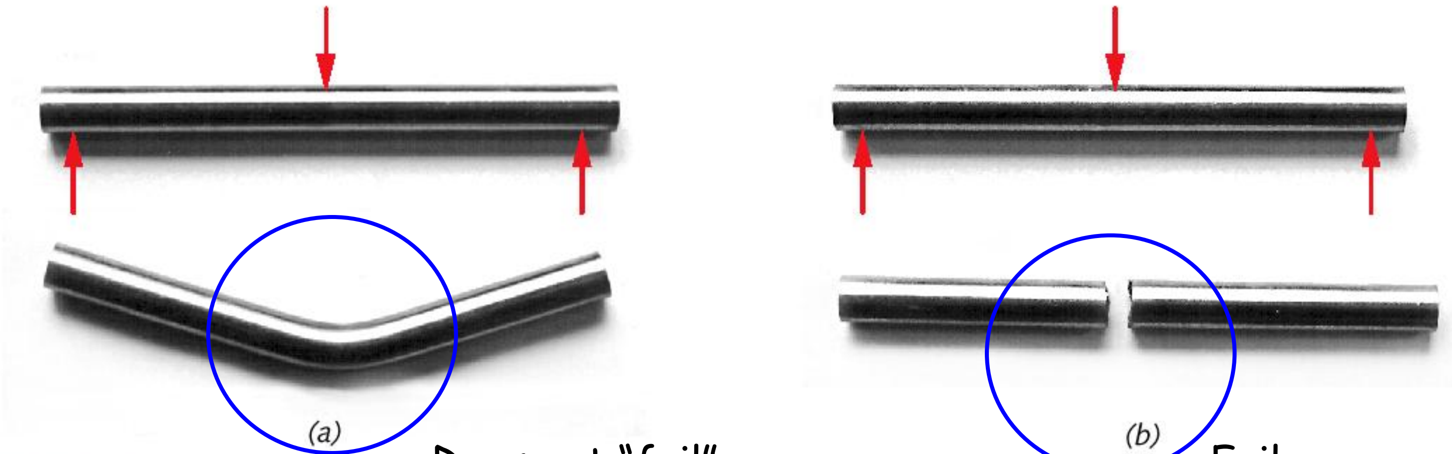


FIGURE 2-7

Bending Test Specimens After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

Principal and maximum shear stresses can be used to study failure of components

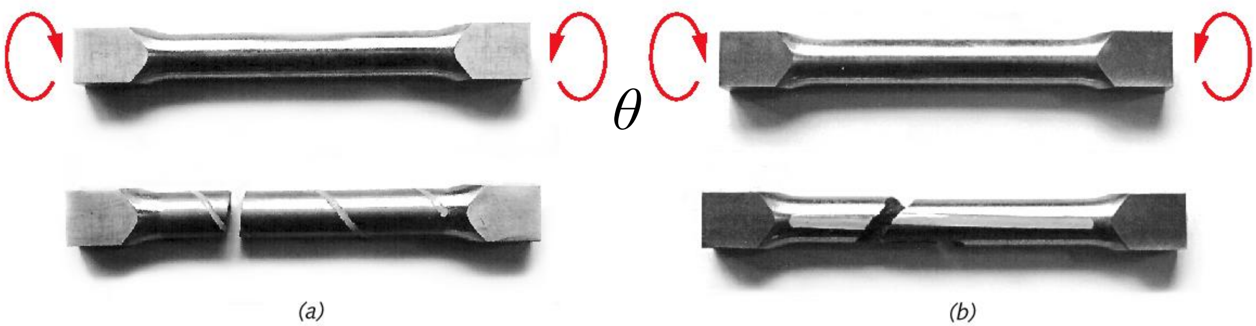


FIGURE 2-8

Torsion Test Specimens After Failure (a) Ductile Steel (b) Brittle Cast Iron

Steels: $S_{us} = 0.80\, S_{ut}$

Other ductile mtls.: $S_{us} = 0.75\, S_{ut}$

Note: $S_{sy} = 0.58\, S_{y}$

Stress-strain relation (torsion):

$$\tau = \frac{Gr\theta}{l_o}$$

Modulus of rigidity:

$$G = \frac{E}{2(1+\nu)}$$

Table 2-1

Poisson's Ratio ν

Material	ν
Aluminum	0.34
Copper	0.35
Iron	0.28
Steel	0.28
Magnesium	0.33
Titanium	0.34

Ultimate shear strength (torsion):

$$S_{us} = \frac{T_{(break)}r}{J}$$

Not uniform stress distribution; (in some cases, thin-walled tubes are preferred for this test, why?)

Reading assignment

- Chapter 9 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course