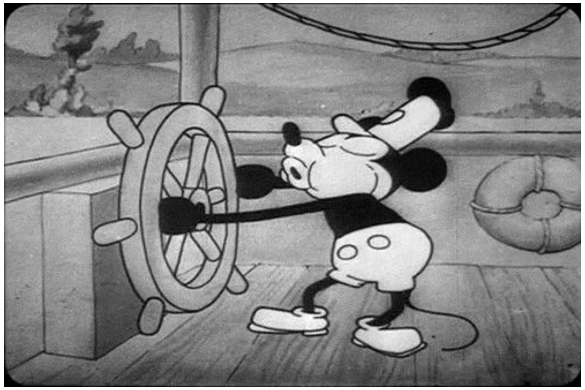
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



09 December 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

Lecture 25:

Unit 27:
Combined loading (Ch. 8)

09 December 2025

General information

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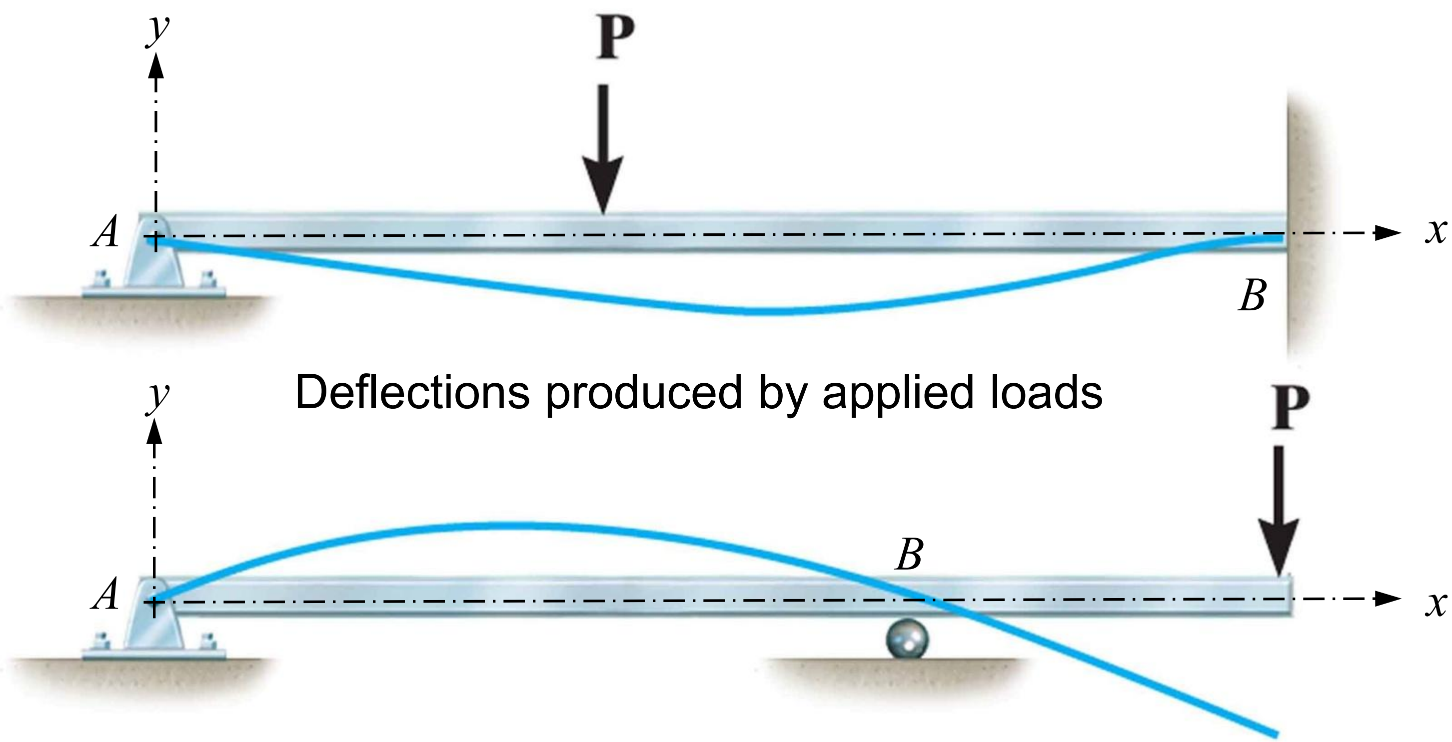
Deflection of beams and shafts

The elastic curve

We can study how beams and shafts deflect by knowing **both**:

a) Distribution of bending moments (V-M diagrams), and

b) Material & geometrical properties of components



Bending deformation of straight beams

The elastic curve

Radius of curvature is computed by:

$$\frac{1}{\rho} = \frac{\dfrac{d^2 y}{dx^2}}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{2/3}}$$

Therefore,

$$\frac{\dfrac{d^2 y}{dx^2}}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{2/3}} = \frac{M}{E \cdot I_{zz}}$$

Bending deformation of straight beams

The elastic curve

For small deformations:

$$\boxed{\frac{d^2 y}{dx^2} = \frac{M}{E \cdot I_{zz}}}$$

← ***Elastica*** equation

Important to remember!!

Bending deformation of straight beams

The elastic curve

For small deformations:

$$\boxed{\frac{d^2 y}{dx^2} = \frac{M}{E \cdot I_{zz}}}$$

Elastica equation

$$\frac{d}{dx}\left(E \cdot I_{zz} \frac{d^2 y}{dx^2}\right) = V(x)$$

Shear force

$$\frac{d^2}{dx^2}\left(E \cdot I_{zz} \frac{d^2 y}{dx^2}\right) = w(x)$$

Applied load

Bending deformation of straight beams

The elastic curve

$$E \cdot I_{zz} \frac{d^4 y}{dx^4} = w(x)$$

Applied load

$$E \cdot I_{zz} \frac{d^3 y}{dx^3} = V(x)$$

Shear force

$$\boxed{E \cdot I_{zz} \frac{d^2 y}{dx^2} = M \quad \longleftarrow \quad \textit{\textbf{Elastica}} \text{ equation}}$$

Bending deformation of straight beams

The elastic curve

$$\frac{w}{EI} = \frac{d^4 y}{dx^4}$$ Load function – deflection

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$ Shear function – deflection

$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$ Moment function – *elastica*

$$\theta = \frac{dy}{dx}$$ Slope – deflection

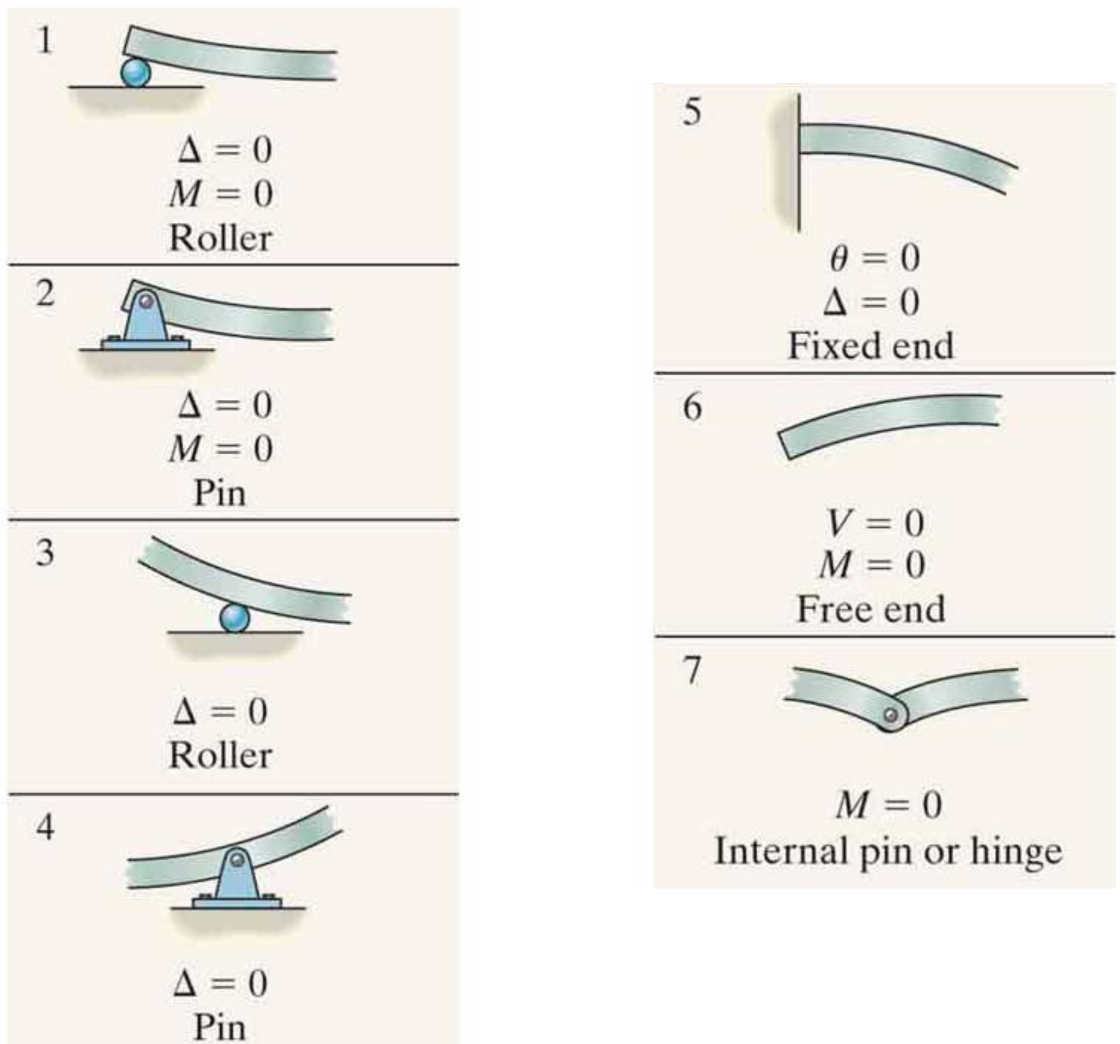
$$y = f(x)$$ Deflection

Bending deformation of straight beams

Boundary and continuity conditions

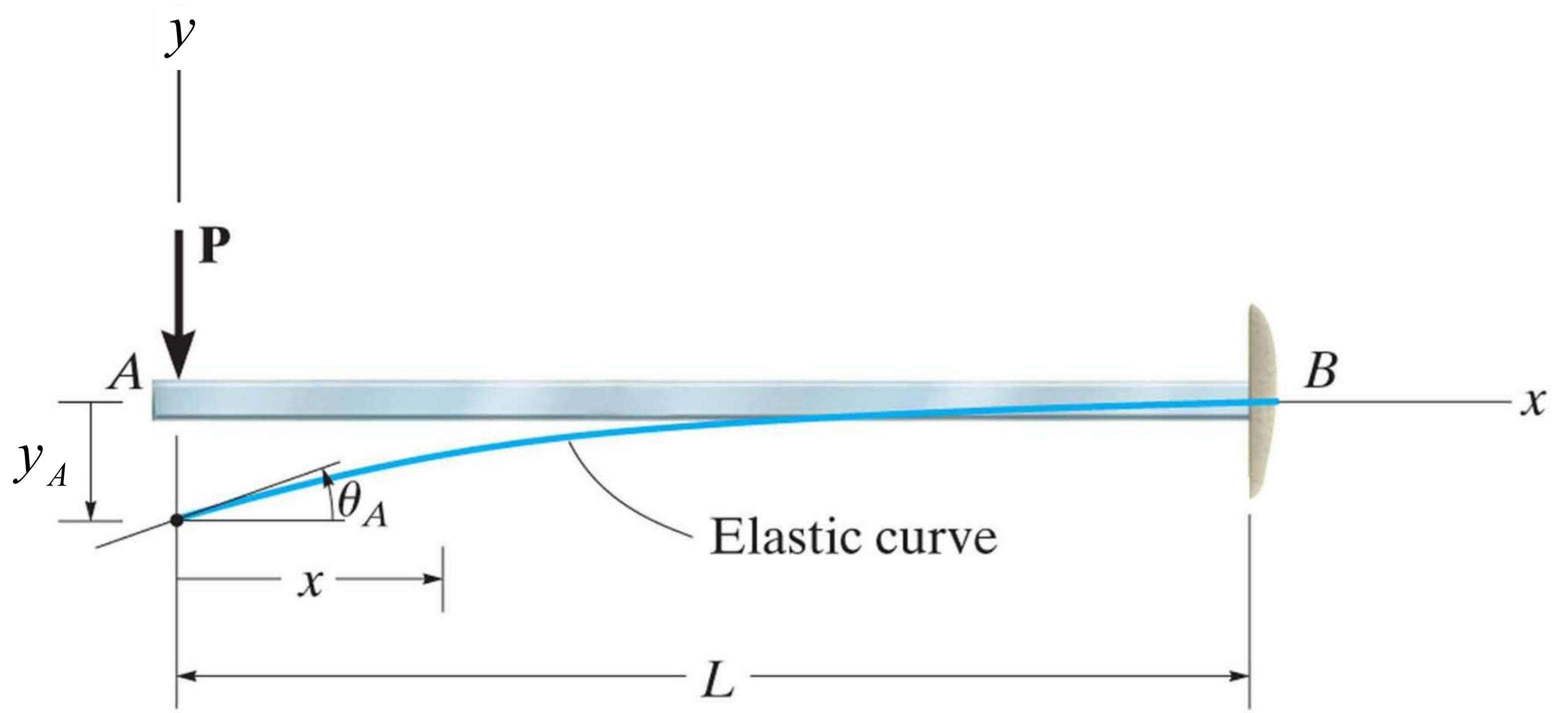
Δ = displacement

θ = slope of displacement



Bending deformation of straight beams: example A

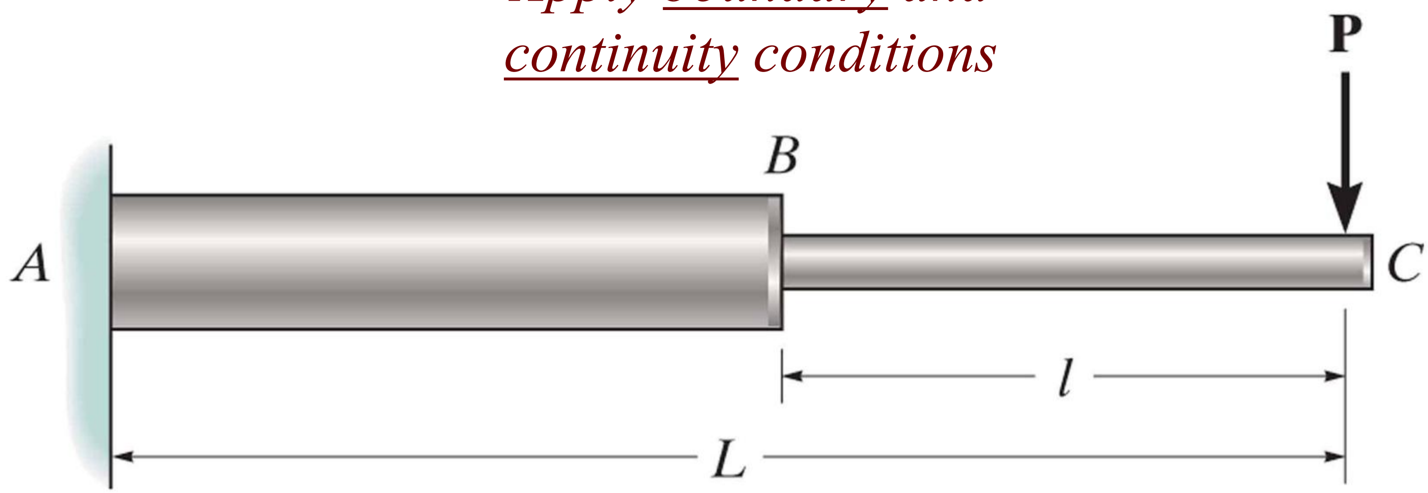
The cantilever shown is subjected to a vertical load P at it end. Determine the equation of the deformation (elastic) curve. $E{\cdot}I$ is constant.



Bending deformation of straight beams: example B

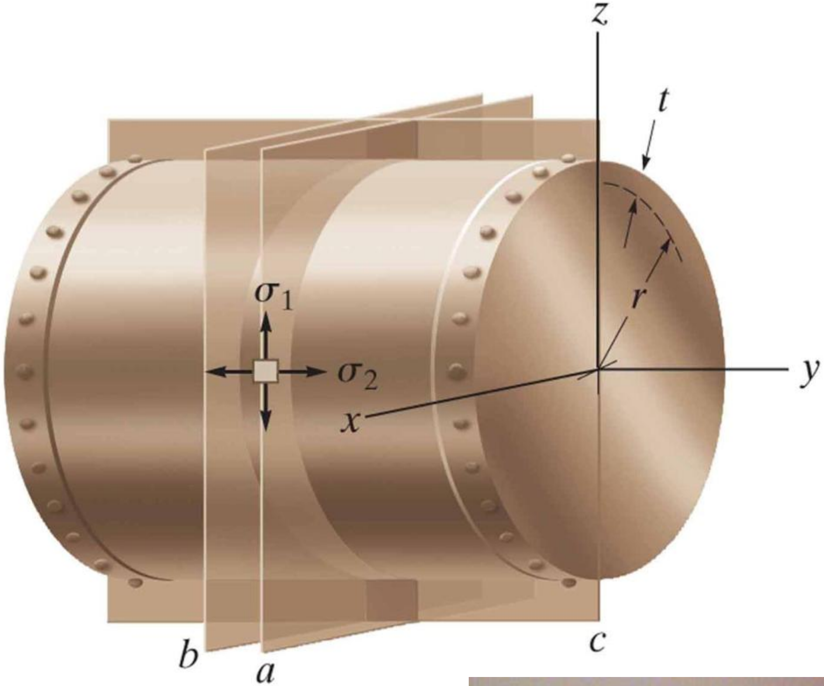
The beam is made of two rods and is subjected to the concentrated load **P**. Determine the maximum deflection of the beam if the moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.

Apply boundary and continuity conditions



Combined loading

Thin-wall vessels: 2D state of stresses

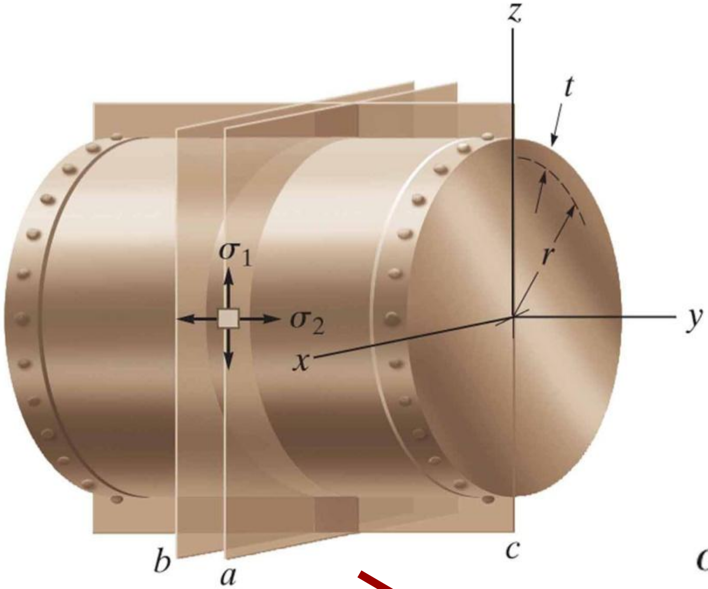


Thin-wall condition:

$$\frac{r}{t} \geq 10$$

Combined loading

Thin-wall vessels: 2D state of stresses

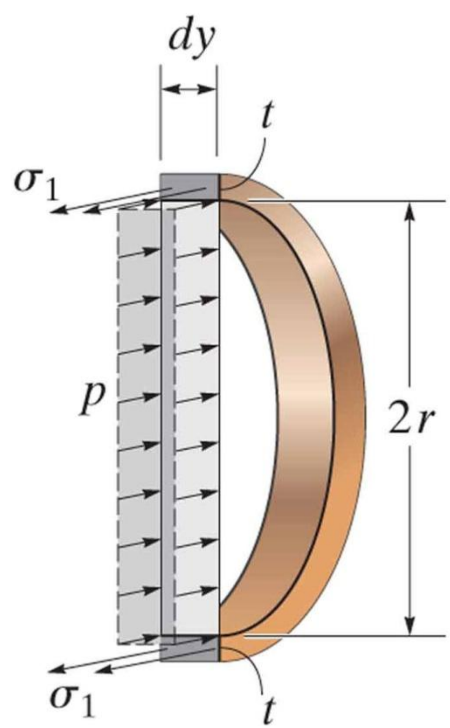


$$\sum F_x = 0 \quad \Rightarrow$$

$$2\,\sigma_1 (t \cdot dy) - p\,(2\,r\,dy) = 0$$

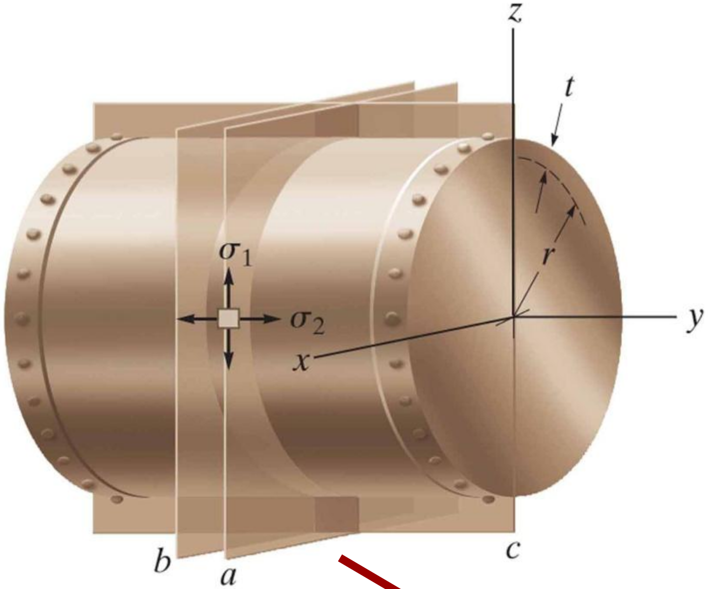
$$\Rightarrow \quad \boxed{\sigma_1 = \frac{P \cdot r}{t}}$$

Tangential *component of stresses*



Combined loading

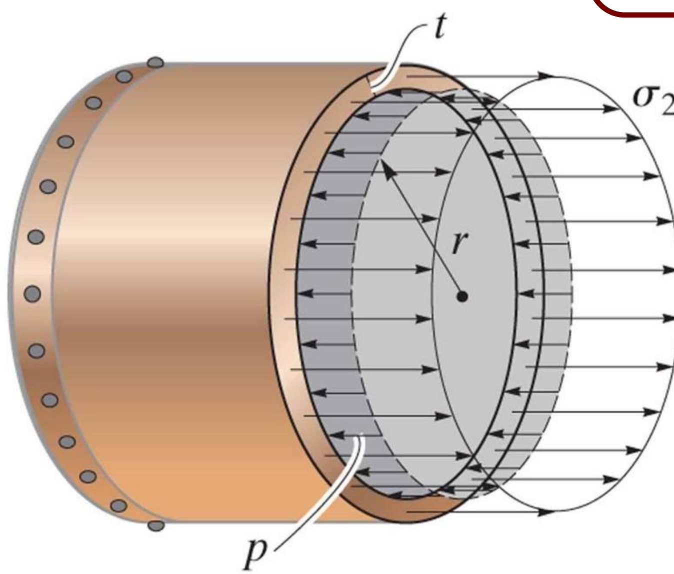
Thin-wall vessels: 2D state of stresses



$$\sum F_y = 0 \quad \Rightarrow$$

$$\sigma_2 \left(2\pi \cdot r \cdot t\right) - p\left(\pi \cdot r^2\right) = 0$$

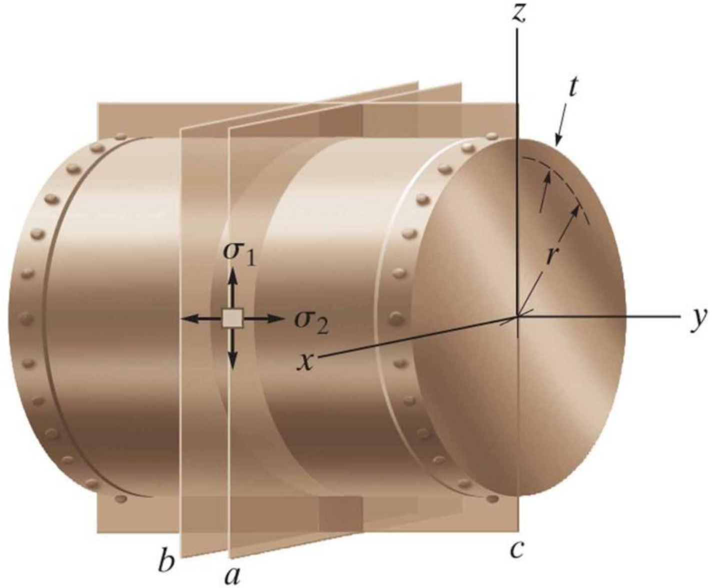
$$\Rightarrow \quad \boxed{\sigma_2 = \frac{P \cdot r}{2t}}$$



Longitudinal *component of stresses*

Combined loading

Thin-wall vessels: 2D state of stresses

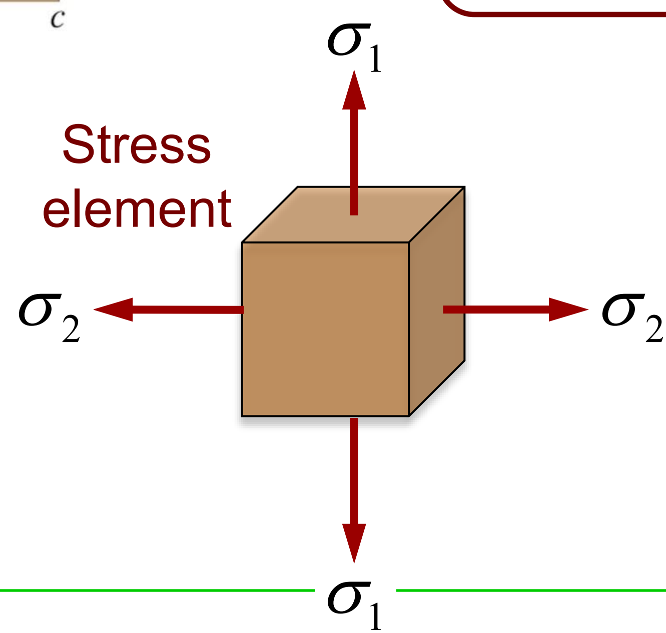


$$\sigma_1 = \frac{P \cdot r}{t}$$

__Radial__ component of stresses

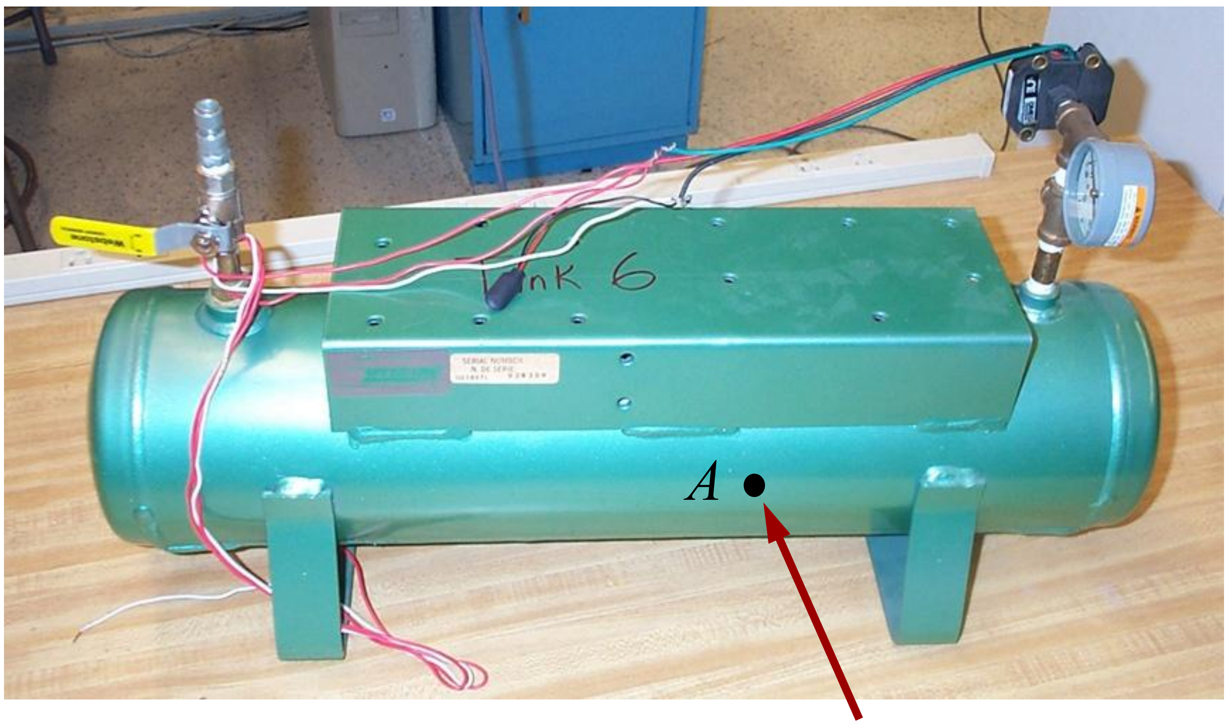
$$\sigma_2 = \frac{P \cdot r}{2t}$$

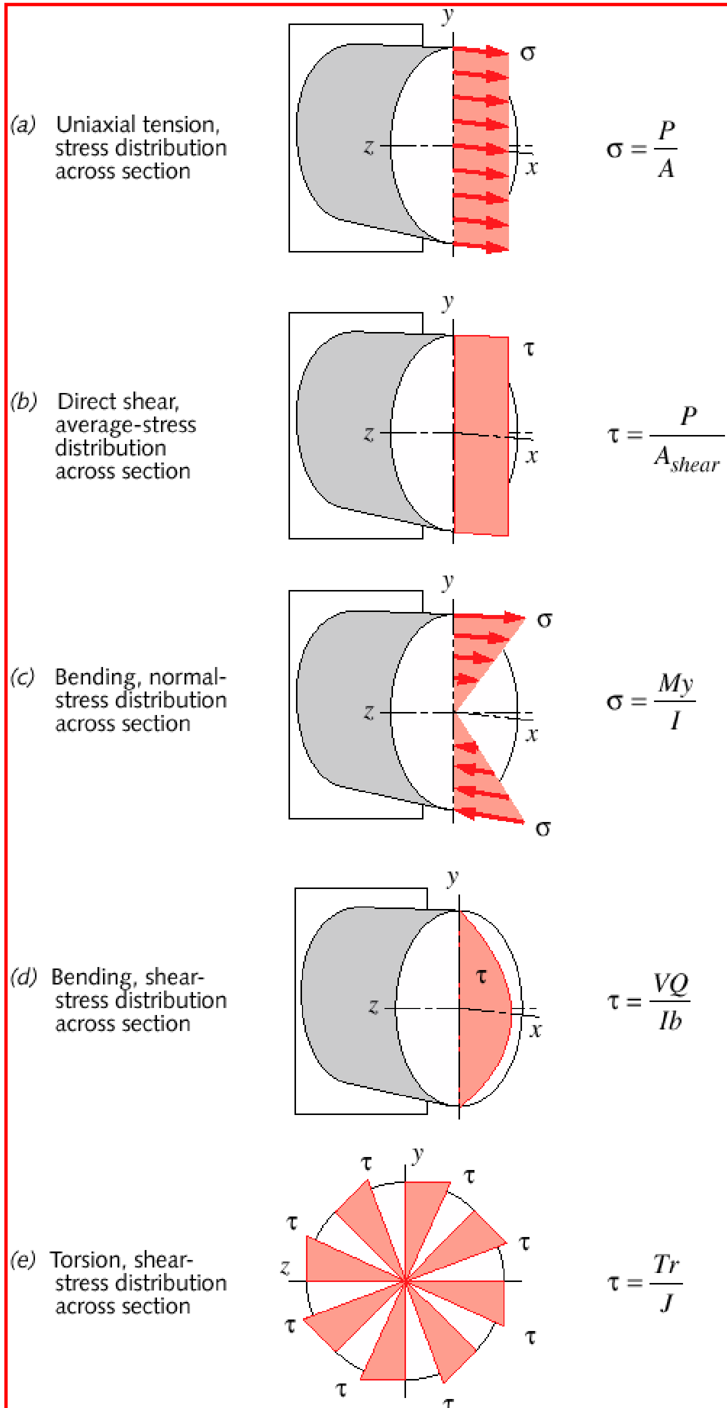
__Longitudinal__ component of stresses



Combined loading: thin-wall vessels: example A

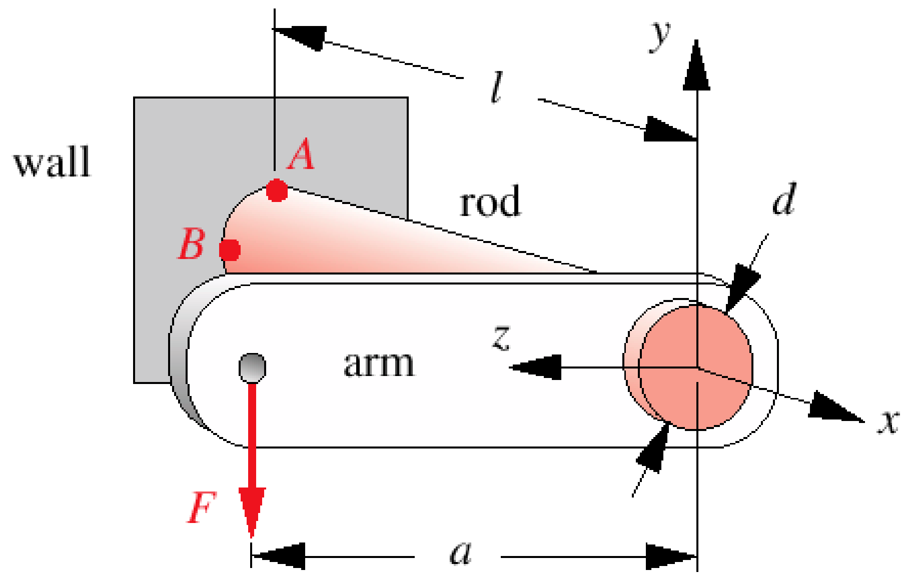
The tank of an air compressor is subjected to an internal pressure of 90 psi. If the internal diameter of the tank is 6.0 in, and the wall thickness is 0.10 in, determine the stress components acting at point A. Draw a volume (stress) element of the material at this point, and show the results on the element.





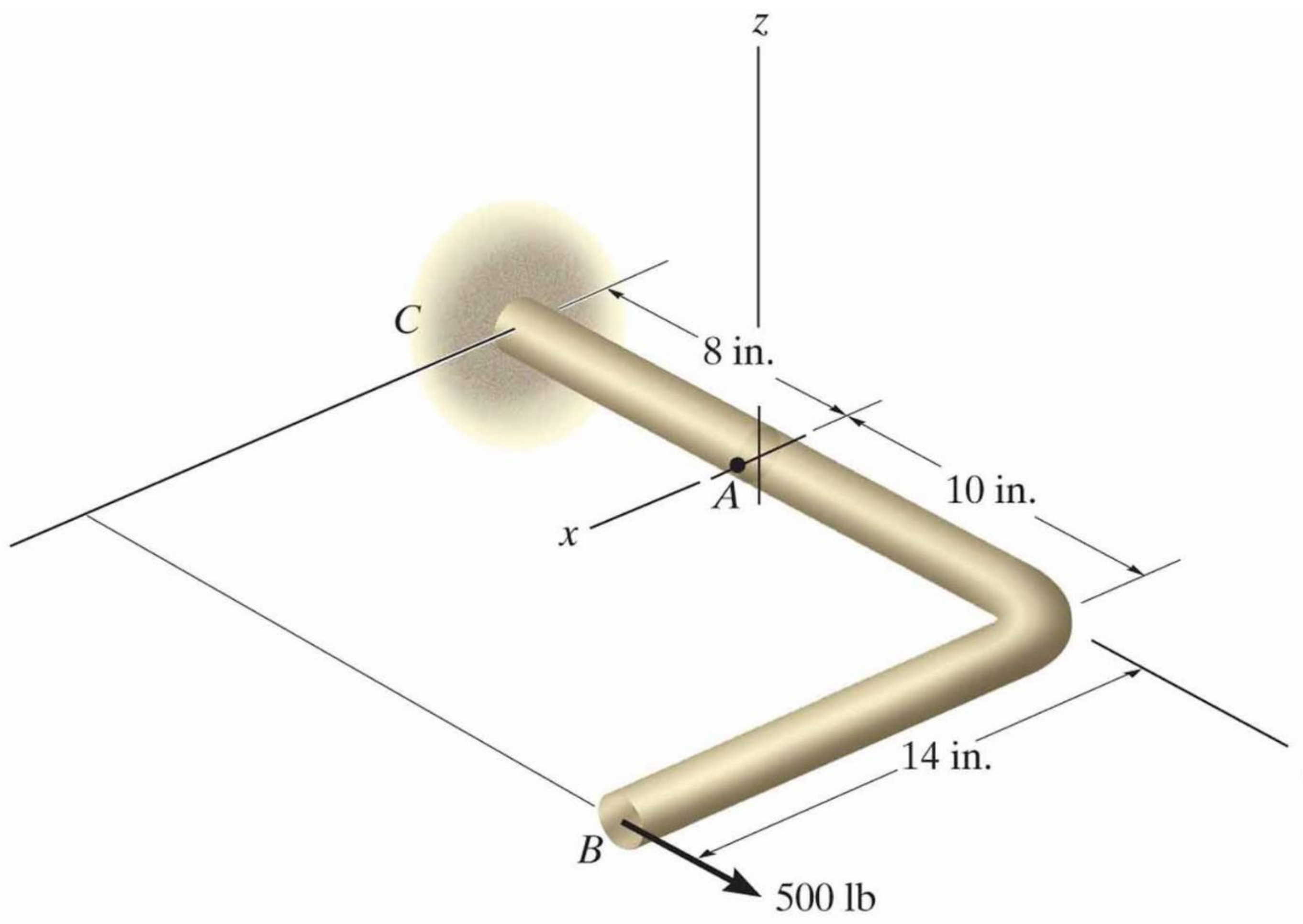
Combined loading

Find the most highly stressed locations on the bracket shown. Draw volume (stress) elements at points A and B



Combined loading: example B

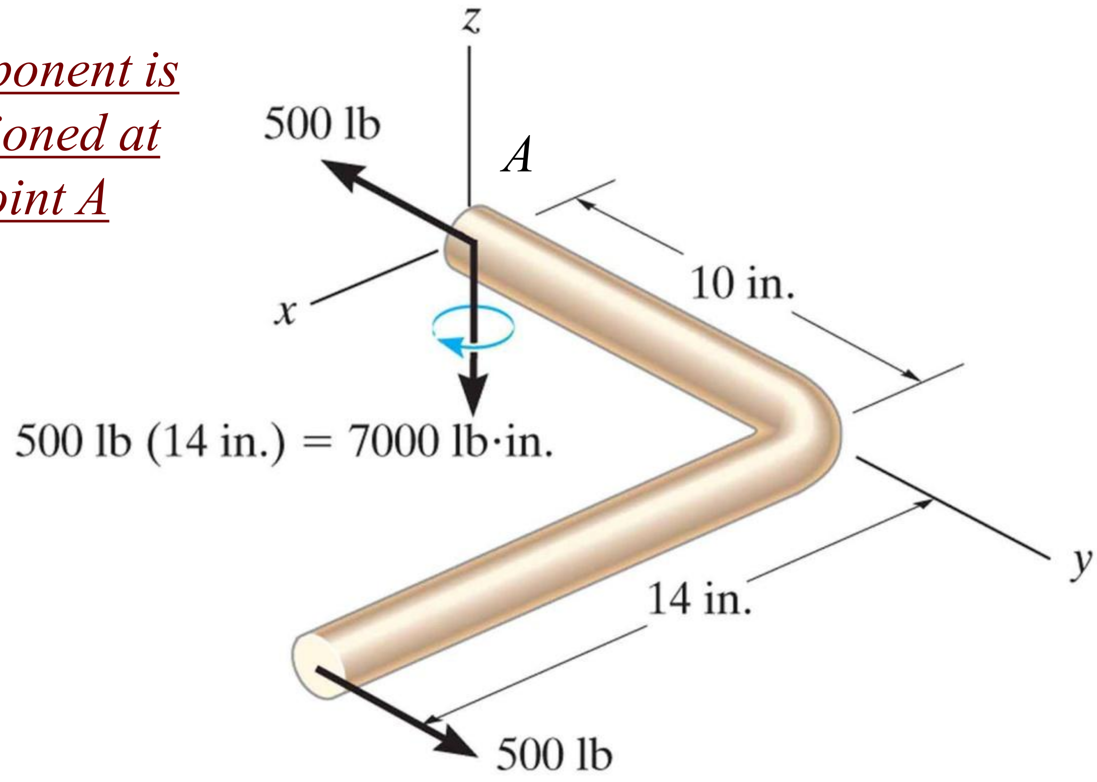
The solid rod shown has a radius of 0.75 in. If it is subjected to the force of 500 lbf, determine the state of stress at point A.



Combined loading: example B

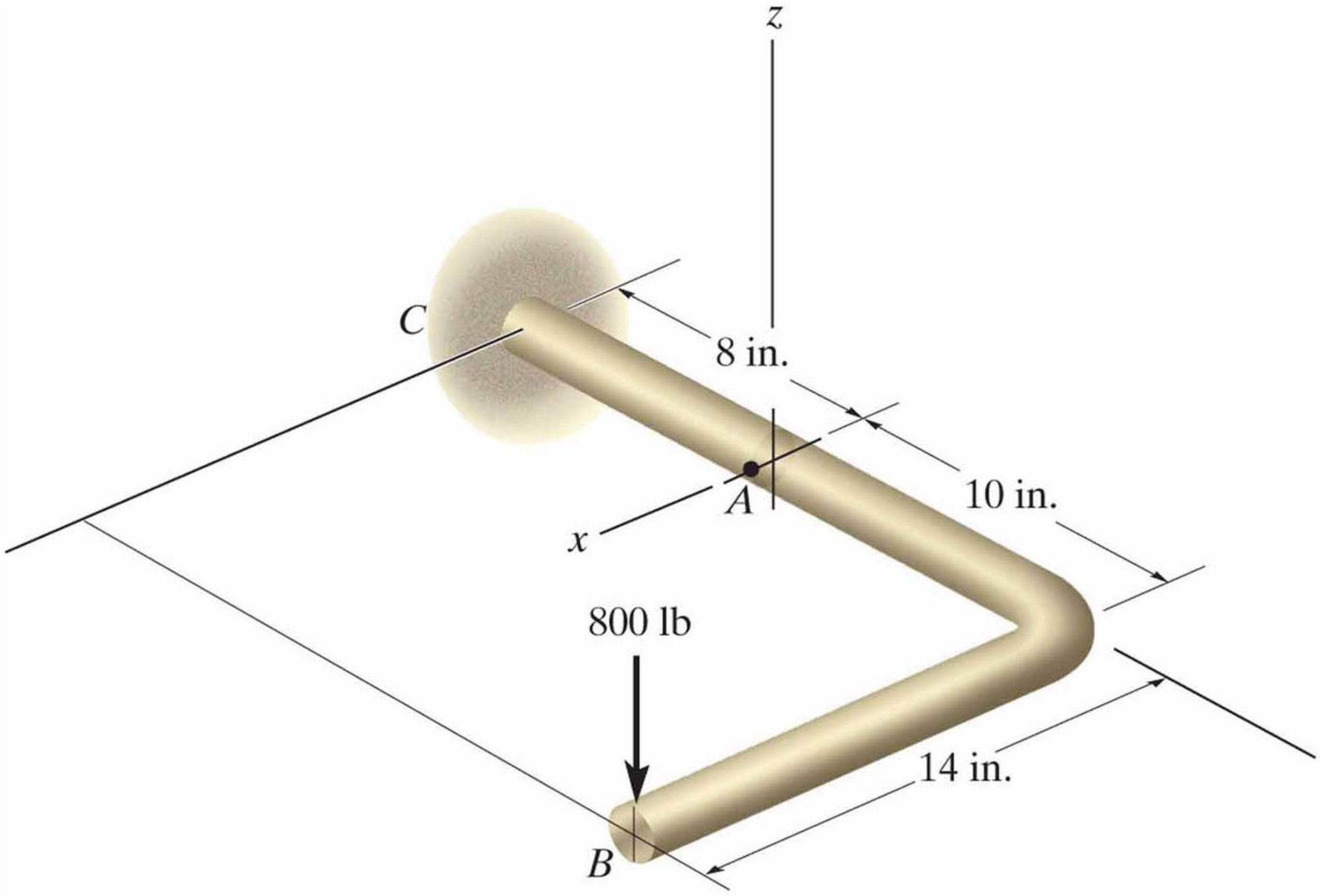
The solid rod shown has a radius of 0.75 in. If it is subjected to the force of 500 lbf, determine the state of stress at point A.

Component is sectioned at point A



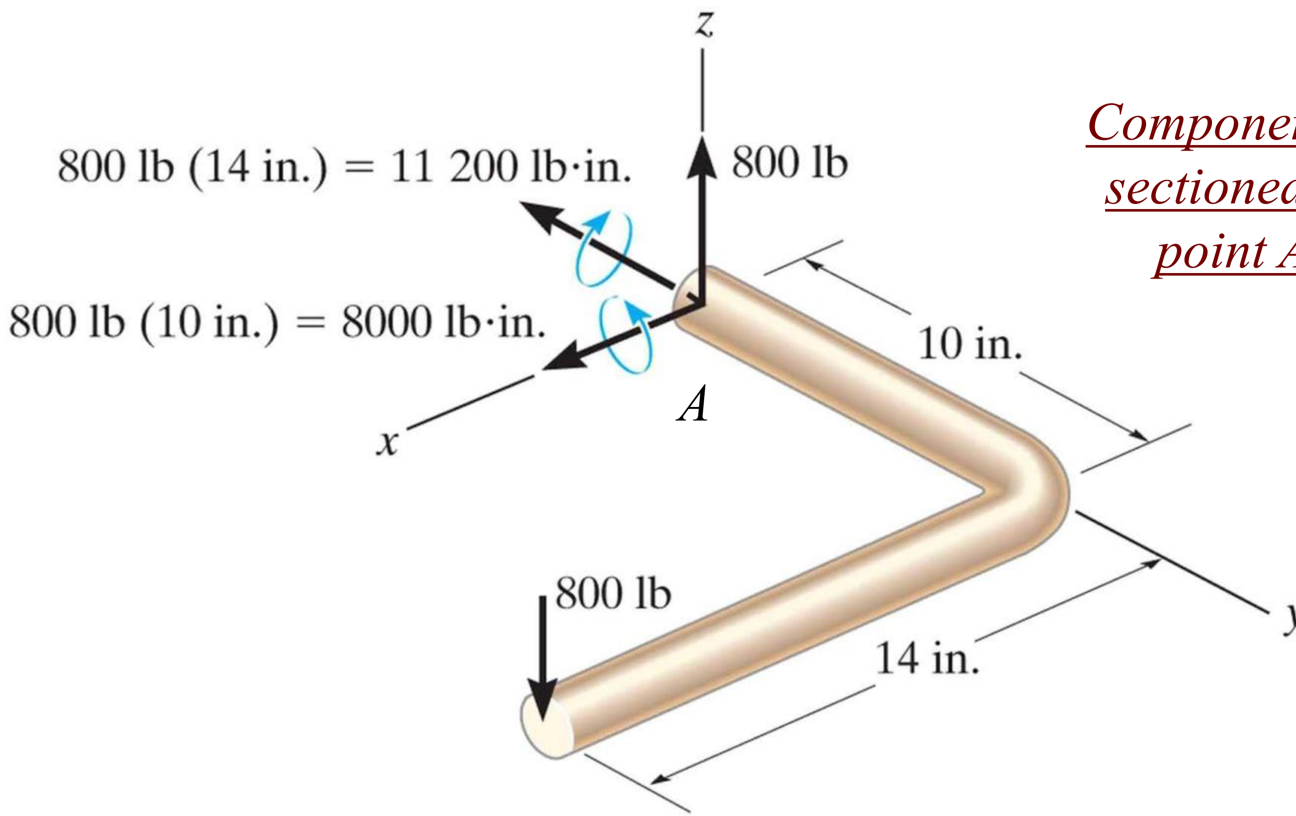
Combined loading: example C

The solid rod shown has a radius of 0.75 in. If it is subjected to the force of 800 lbf, determine the state of stress at point A.



Combined loading: example C

The solid rod shown has a radius of 0.75 in. If it is subjected to the force of 800 lbf, determine the state of stress at point A.



Component is sectioned at point A

Reading assignment

- Chapters 8 and 12 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course