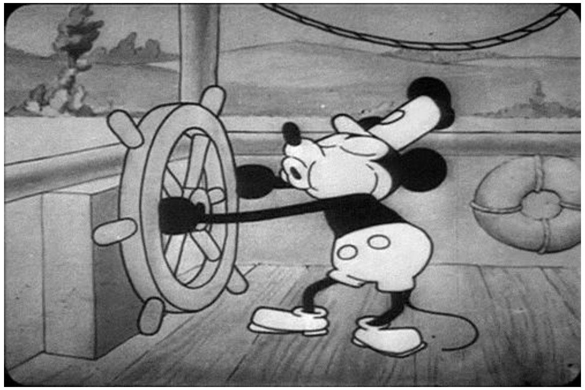
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



04 December 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

Lecture 24:
Unit 21: Bending of beams:
Deflection analysis (Ch.12, textbook)

04 December 2025

General information

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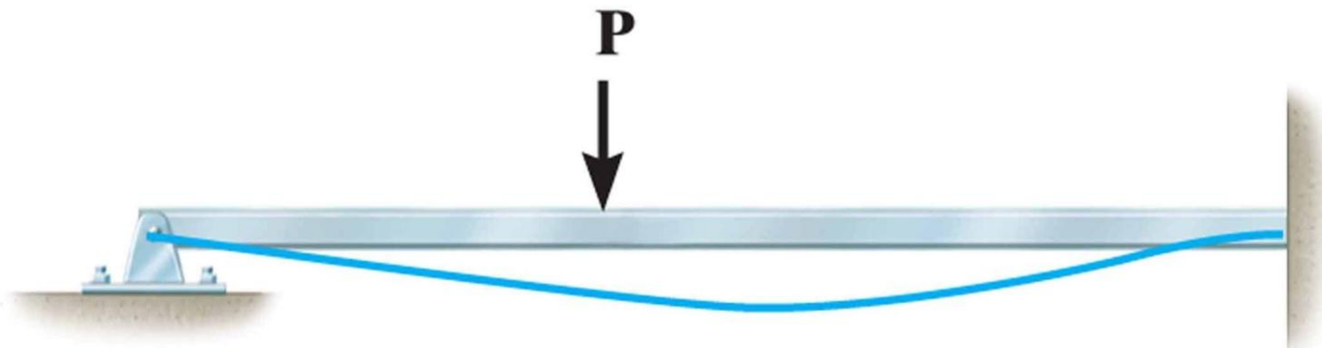
Deflection of beams and shafts

The elastic curve

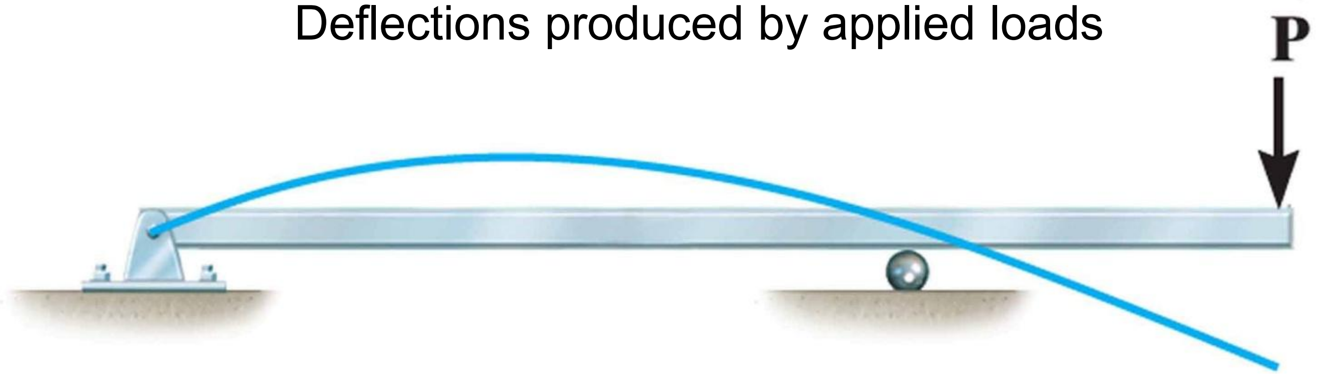
We can study how beams and shafts deflect by knowing **both**:

a) Distribution of bending moments (*V-M* diagrams), and

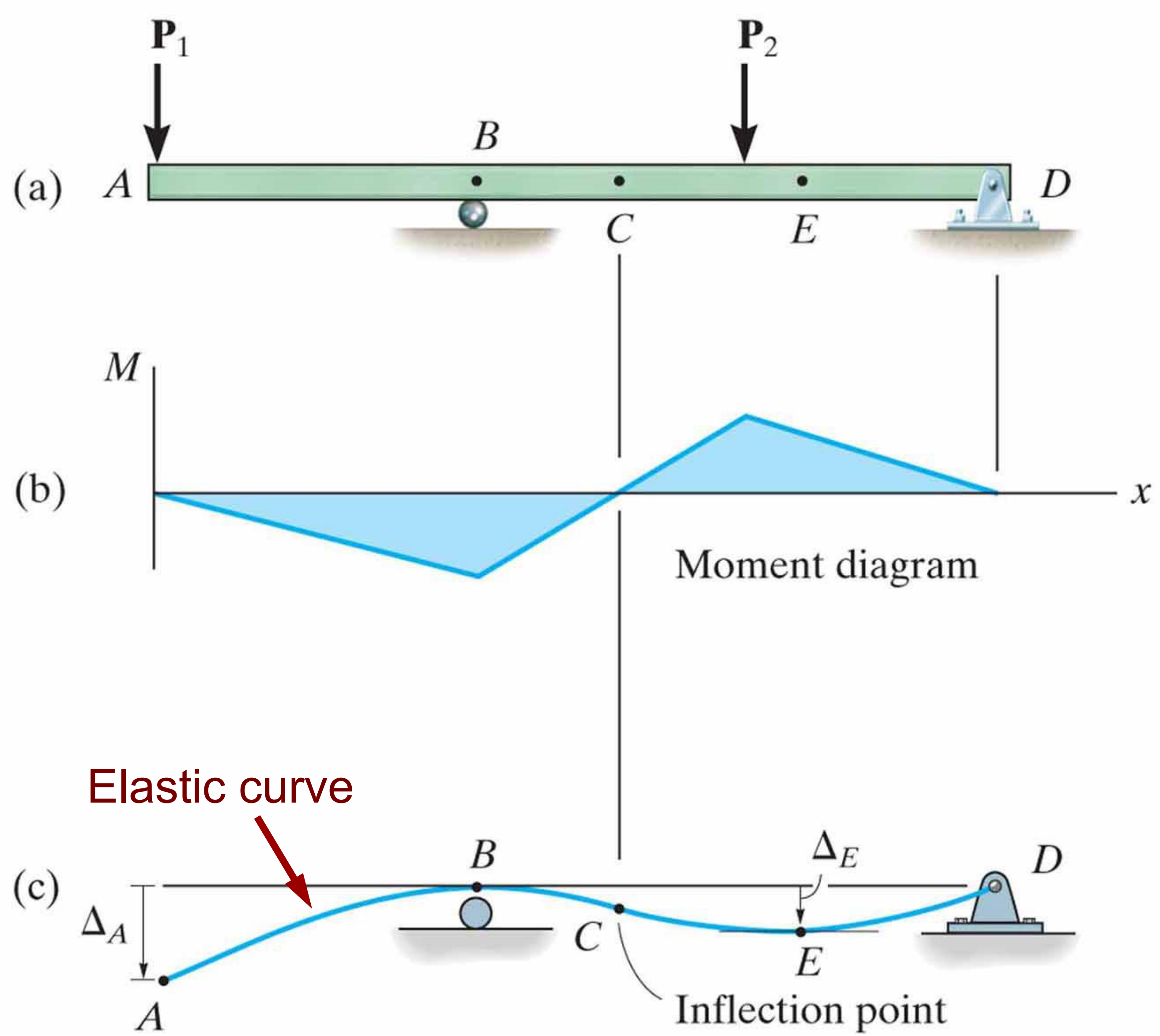
b) Material & geometrical properties of components



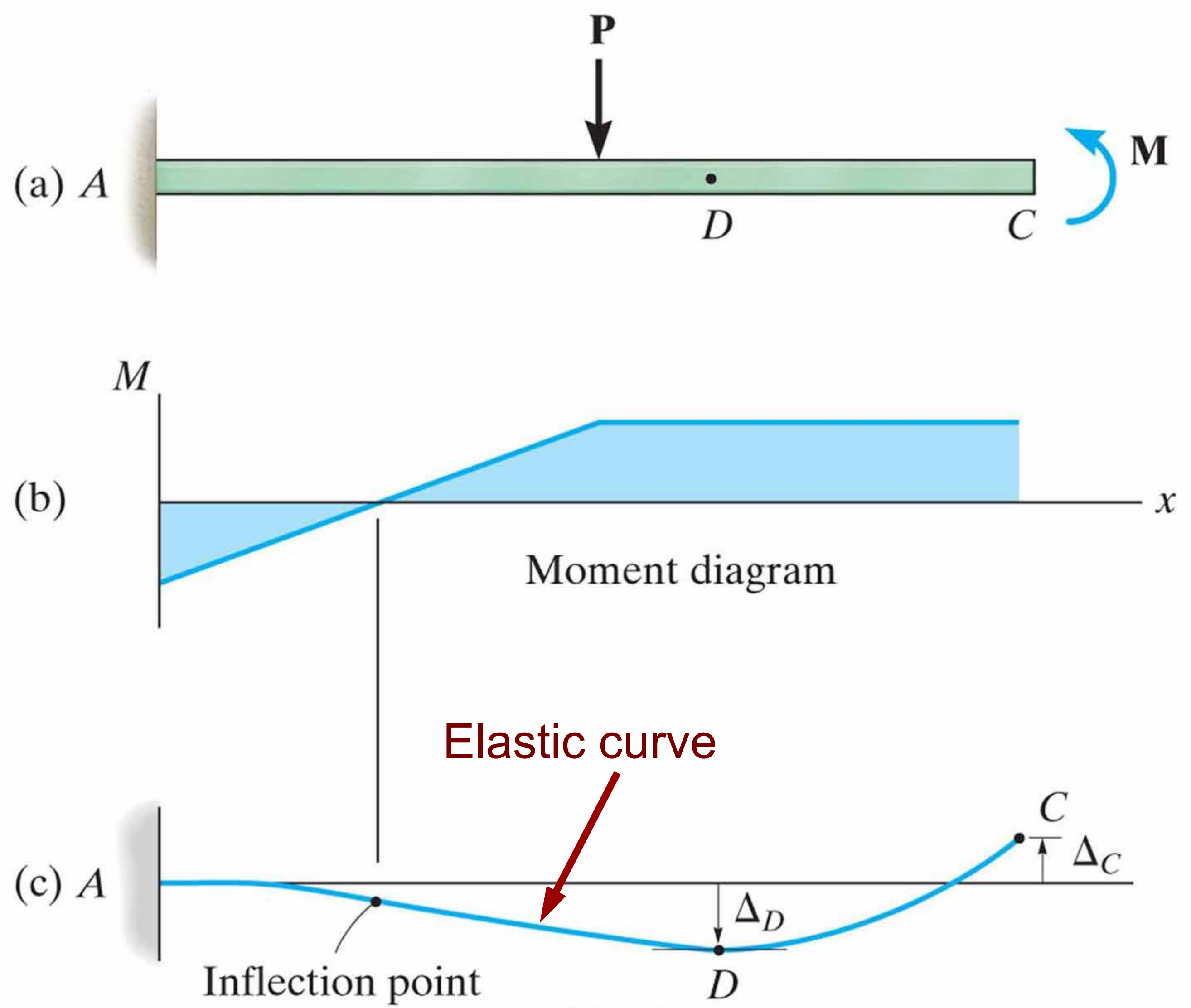
Deflections produced by applied loads



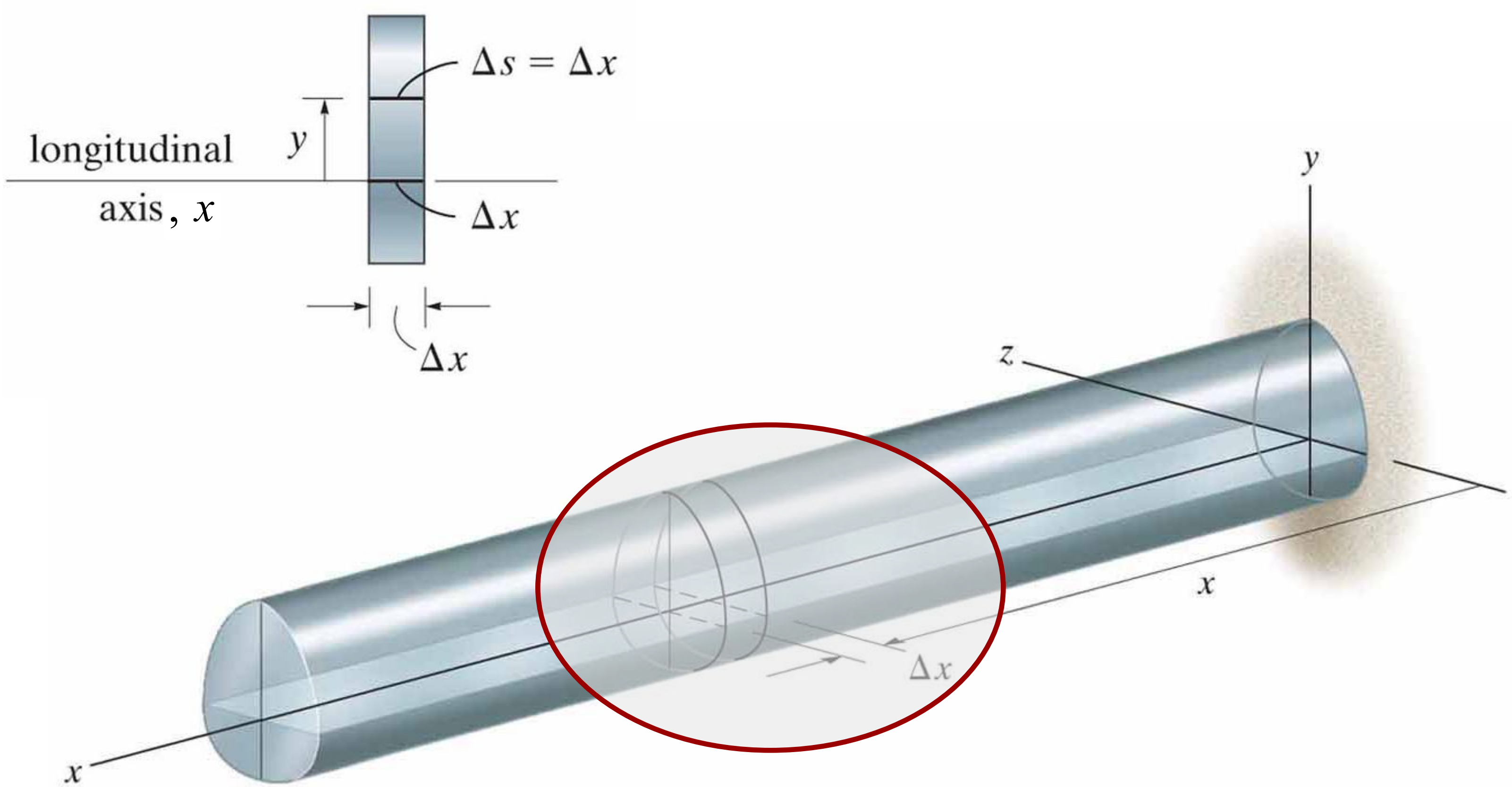
Deflection of beams and shafts



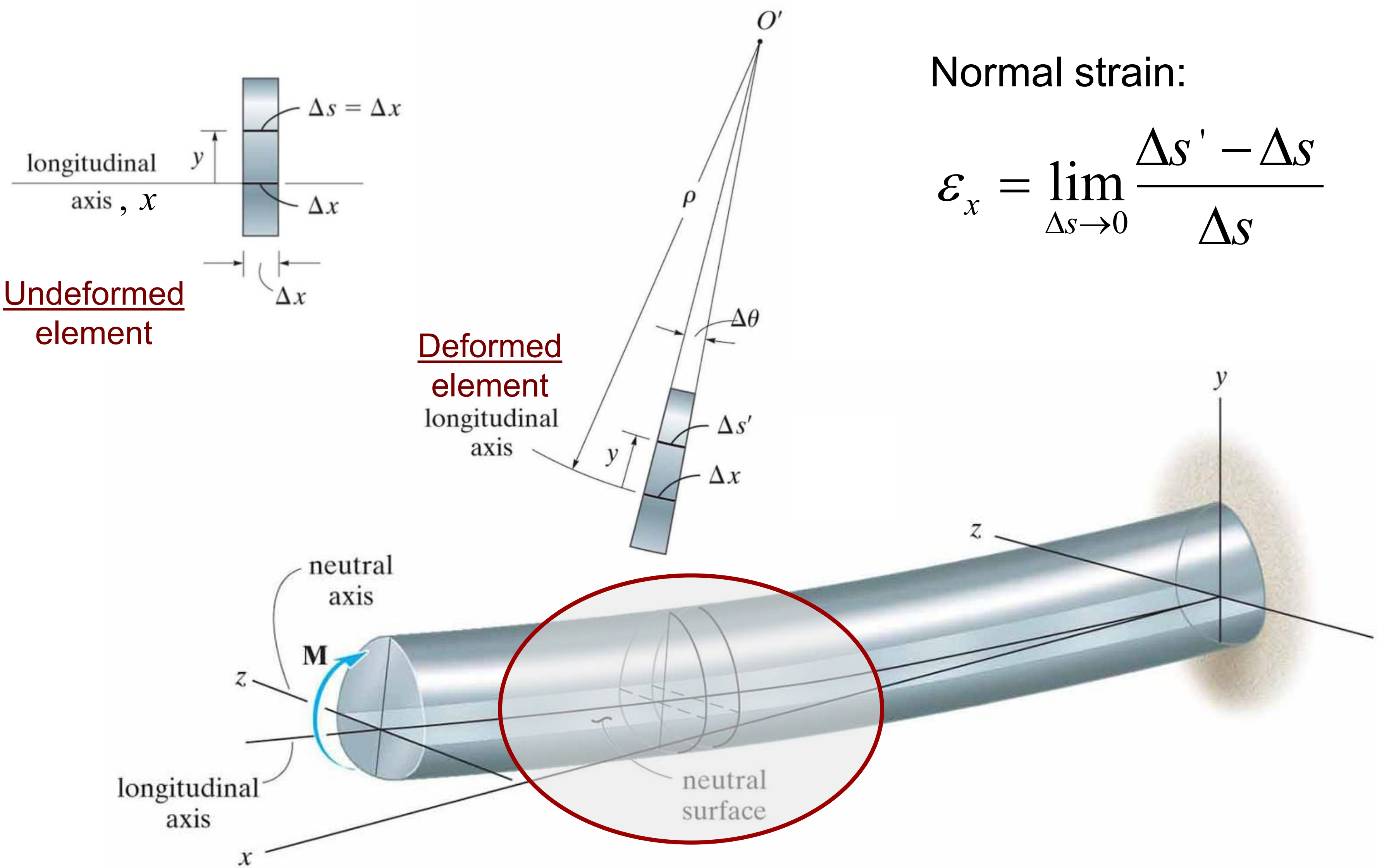
Deflection of beams and shafts



Bending deformation of straight beams



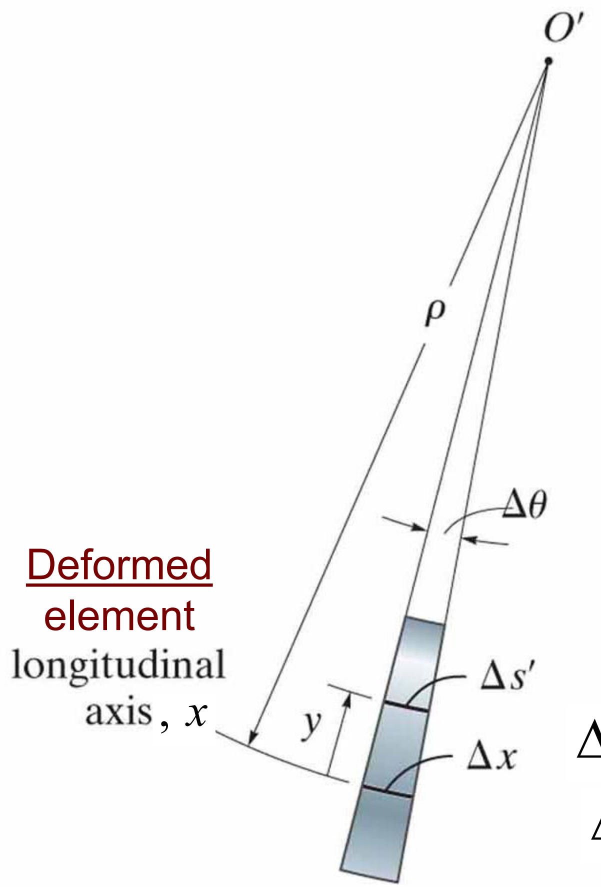
Bending deformation of straight beams



Normal strain:

$$\varepsilon_x = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

Bending deformation of straight beams



Normal strain:

$$\varepsilon_x = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\varepsilon_x = \lim_{\Delta \theta \to 0} \frac{(\rho - y) \cdot \Delta\theta - \rho \cdot \Delta\theta}{\rho \cdot \Delta\theta}$$

$$\Delta s' = (\rho - y) \cdot \Delta\theta$$

$$\Delta x = \Delta s = \rho \Delta\theta$$

$$\varepsilon_x = -\frac{y}{\rho} \rightarrow \rho = -\frac{y}{\varepsilon_x}$$

$$\rightarrow \frac{1}{\rho} = -\frac{\varepsilon_x}{y}$$

Bending deformation of straight beams

From before:

$$\frac{1}{\rho} = -\frac{\varepsilon_x}{y}$$

$$\rightarrow \quad \frac{1}{\rho} = \frac{M}{E \cdot I_{zz}}$$

Hook's law: $\varepsilon_x = \dfrac{\sigma_x}{E}$

Flexure formula: $\sigma_x = -\dfrac{M \cdot y}{I_{zz}}$

Bending deformation of straight beams

The elastic curve

Radius of curvature is computed by:

$$\frac{1}{\rho} = \frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{2/3}}$$

Therefore,

$$\frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{2/3}} = \frac{M}{E \cdot I_{zz}}$$

Bending deformation of straight beams

The elastic curve

For small deformations:

$$\frac{d^2y}{dx^2} = \frac{M}{E \cdot I_{zz}}$$

Elastica equation

Important to remember!!

Bending deformation of straight beams

The elastic curve

For small deformations:

$$\frac{d^2 y}{dx^2} = \frac{M}{E \cdot I_{zz}}$$

← ***Elastica*** equation

$$\frac{d}{dx}\left(E \cdot I_{zz} \frac{d^2 y}{dx^2} \right) = V(x)$$

Shear force

$$\frac{d^2}{dx^2}\left(E \cdot I_{zz} \frac{d^2 y}{dx^2} \right) = w(x)$$

Applied load

Bending deformation of straight beams

The elastic curve

$$E \cdot I_{zz} \frac{d^4 y}{dx^4} = w(x)$$

Applied load

$$E \cdot I_{zz} \frac{d^3 y}{dx^3} = V(x)$$

Shear force

$$E \cdot I_{zz} \frac{d^2 y}{dx^2} = M$$

← ***Elastica*** equation

Bending deformation of straight beams

The elastic curve

$$\frac{w}{EI} = \frac{d^4 y}{dx^4}$$ Load function – deflection

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$ Shear function – deflection

$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$ Moment function – *elastica*

$$\theta = \frac{dy}{dx}$$ Slope – deflection

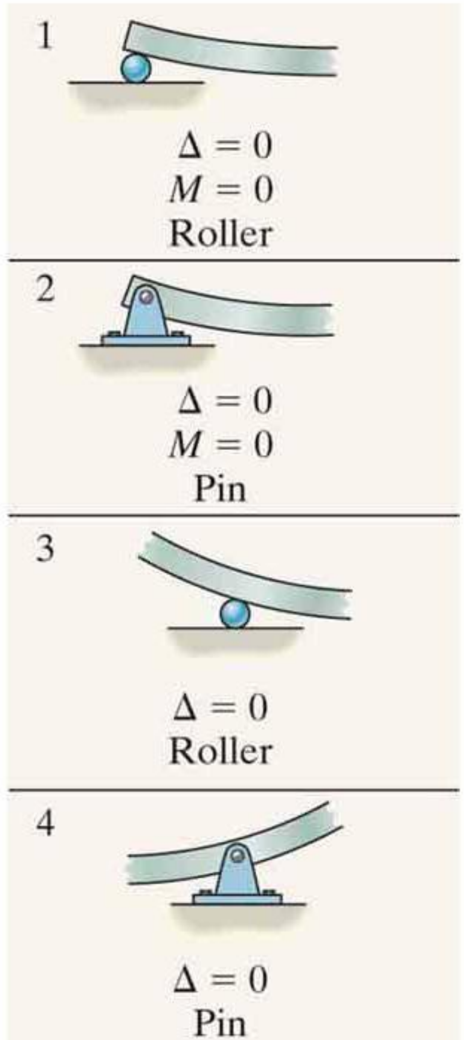
$$y = f(x)$$ Deflection

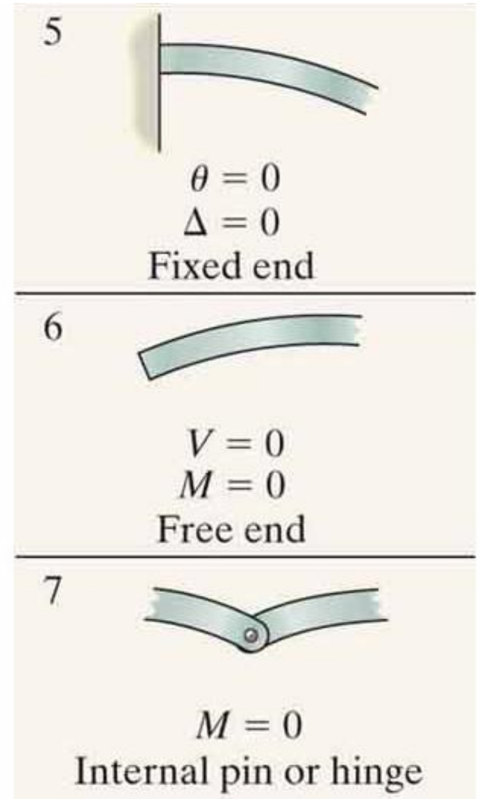
Bending deformation of straight beams

Boundary and continuity conditions

Δ = displacement

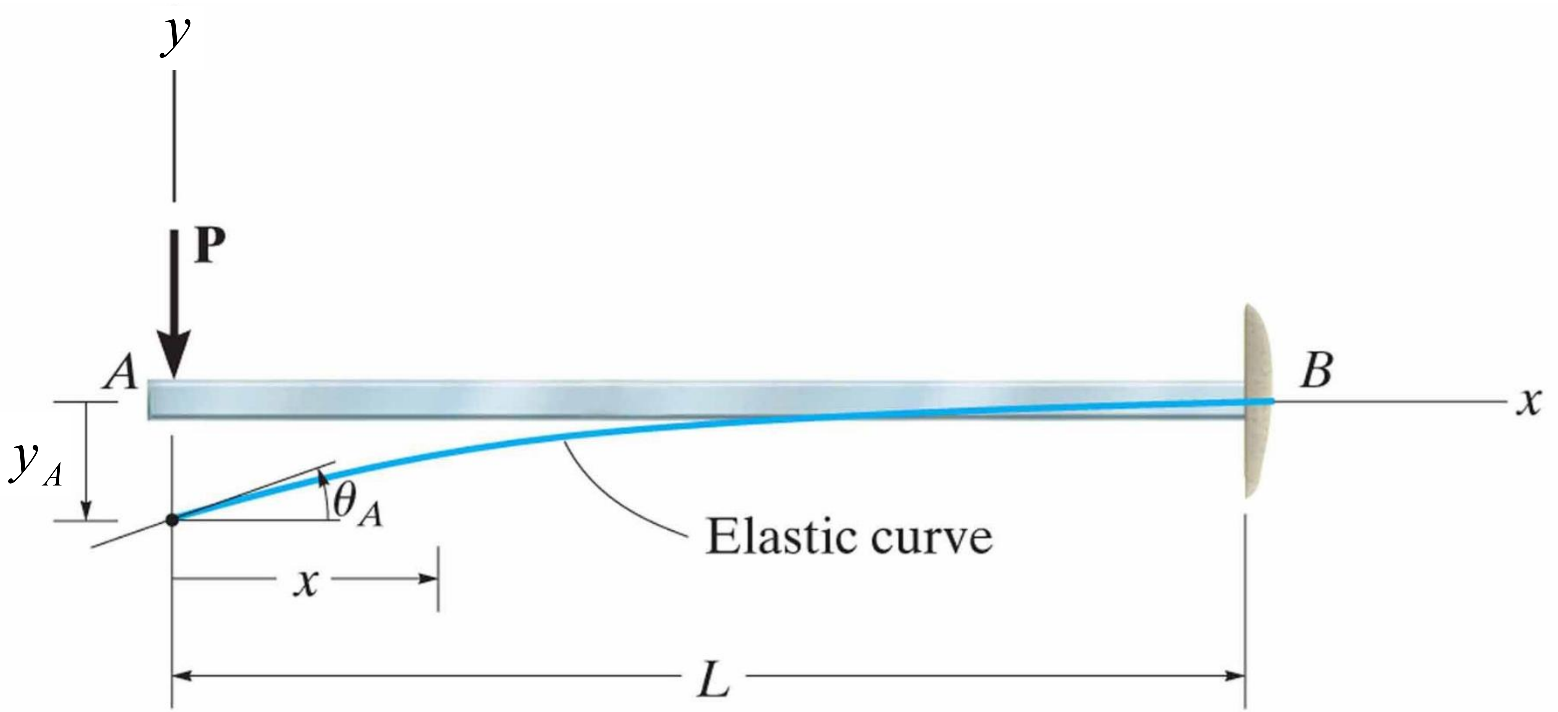
θ = slope of displacement





Bending deformation of straight beams: example A

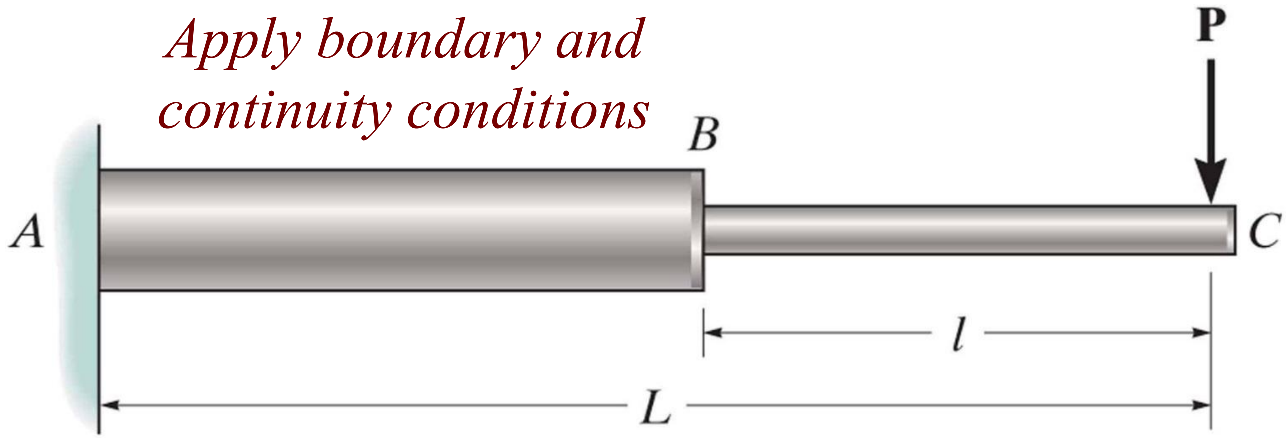
The cantilever shown is subjected to a vertical load P at it end. Determine the equation of the deformation (elastic) curve. $E \cdot I$ is constant.

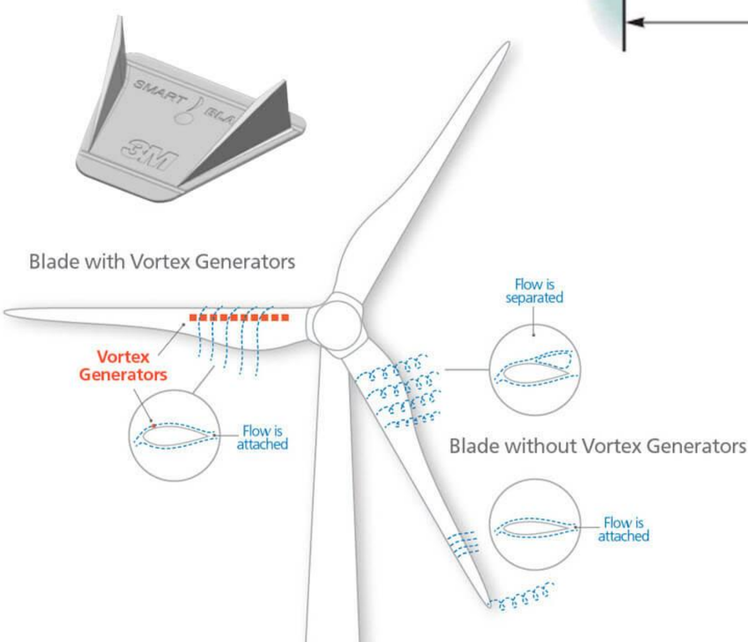


Bending deformation of straight beams: example B

The beam is made of two rods and is subjected to the concentrated load **P**. Determine the maximum deflection of the beam if the moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.

Apply boundary and continuity conditions





Reading assignment

- Chapter 12 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course