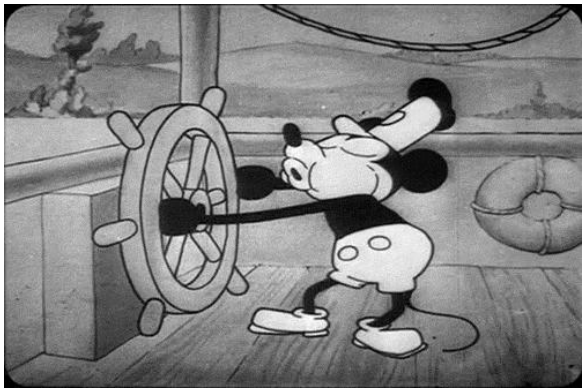
# WORCESTER POLYTECHNIC INSTITUTE
## MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS
## ES-2502, B'2025

**We will get started soon...**



**03 December 2025**

# WORCESTER POLYTECHNIC INSTITUTE
## MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS
## ES-2502, B'2025

## Lecture 23:
## Unit 18, 19: Bending of beams::
## *Bending of beams: transverse shear; section properties*

## 03 December 2025

# General information

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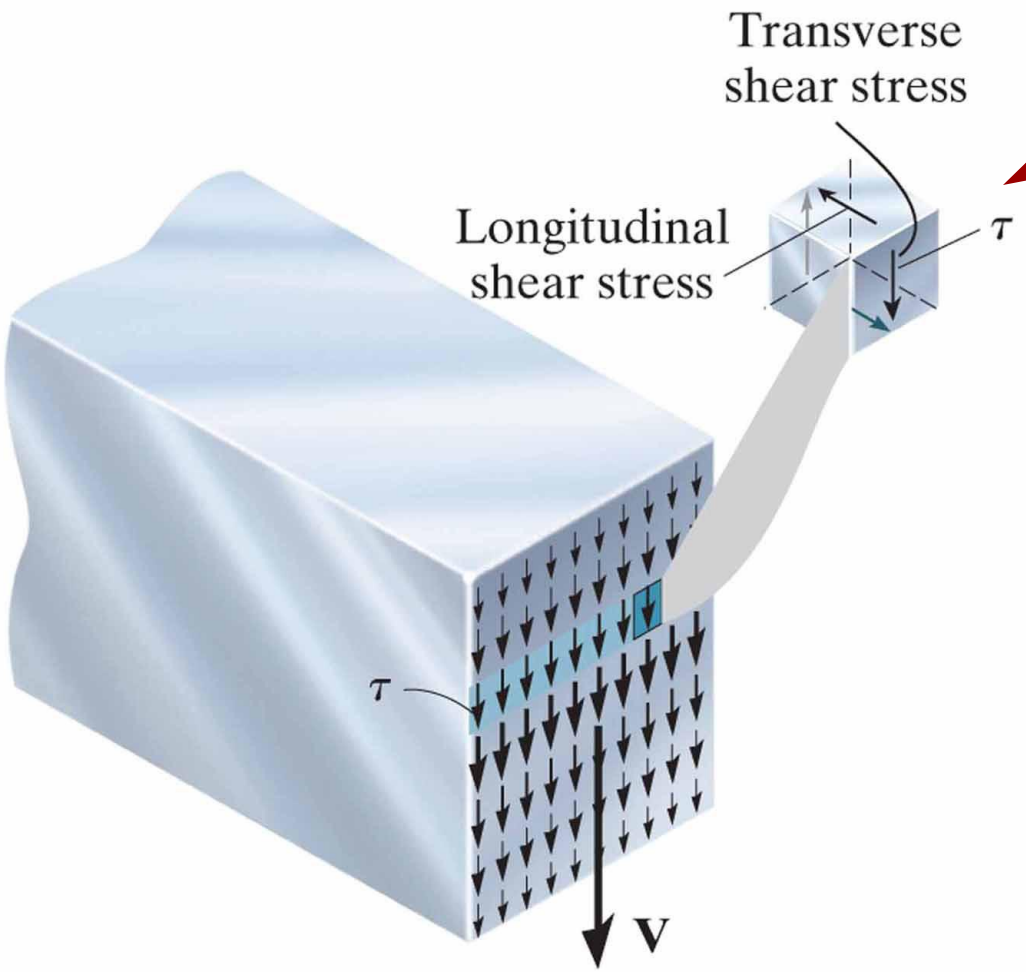
# Transverse shear: *produced by bending*

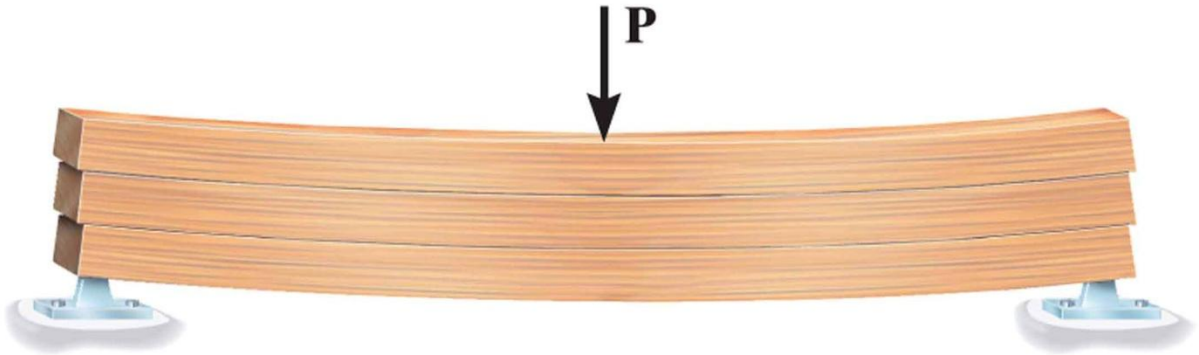
Observed in components subjected to bending loads



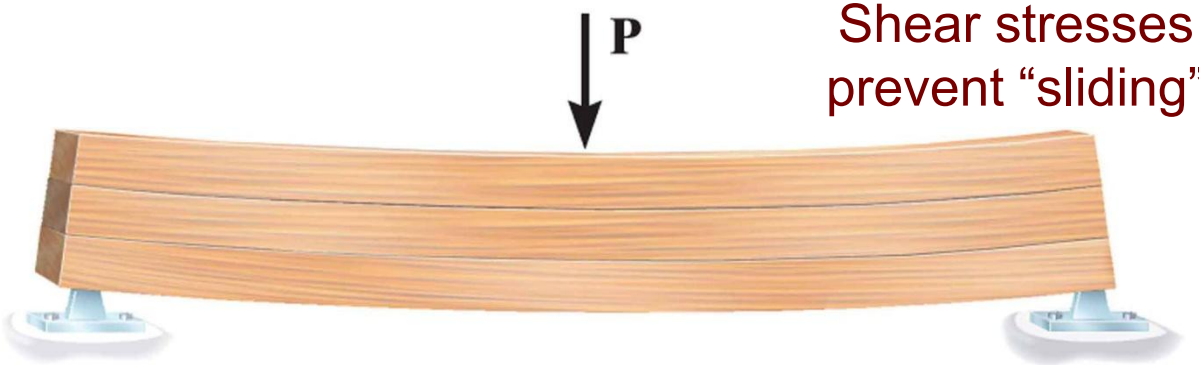

Differential stress element in equilibrium

# Transverse shear: *produced by bending*

Observed in components subjected to bending loads




Boards not bonded together

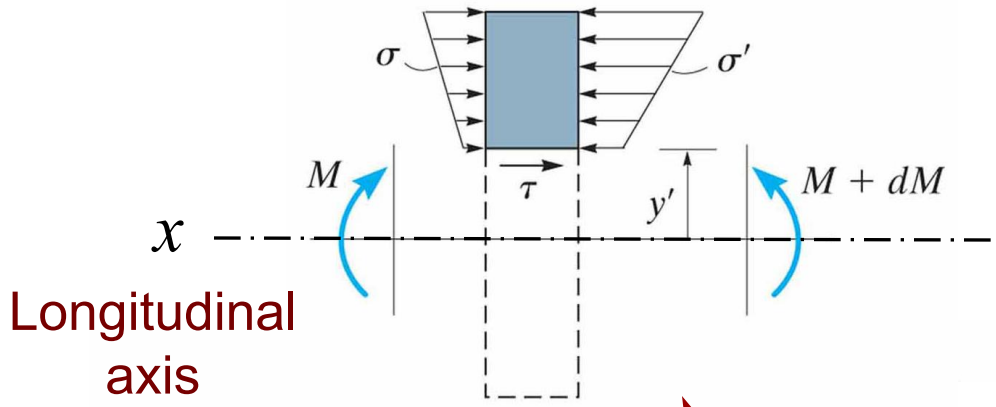



Shear stresses prevent "sliding"

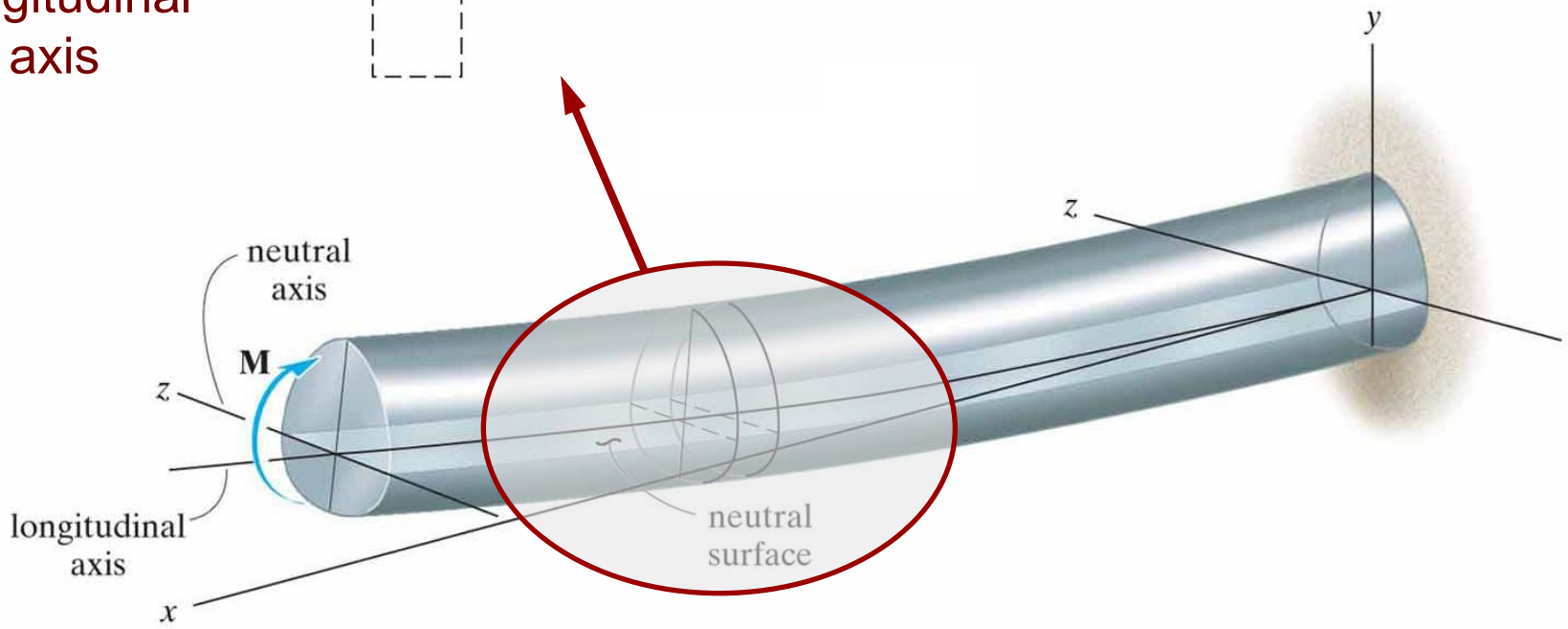
Boards bonded together

# Transverse shear: *produced by bending*

Observed in components subjected to bending loads




Longitudinal axis



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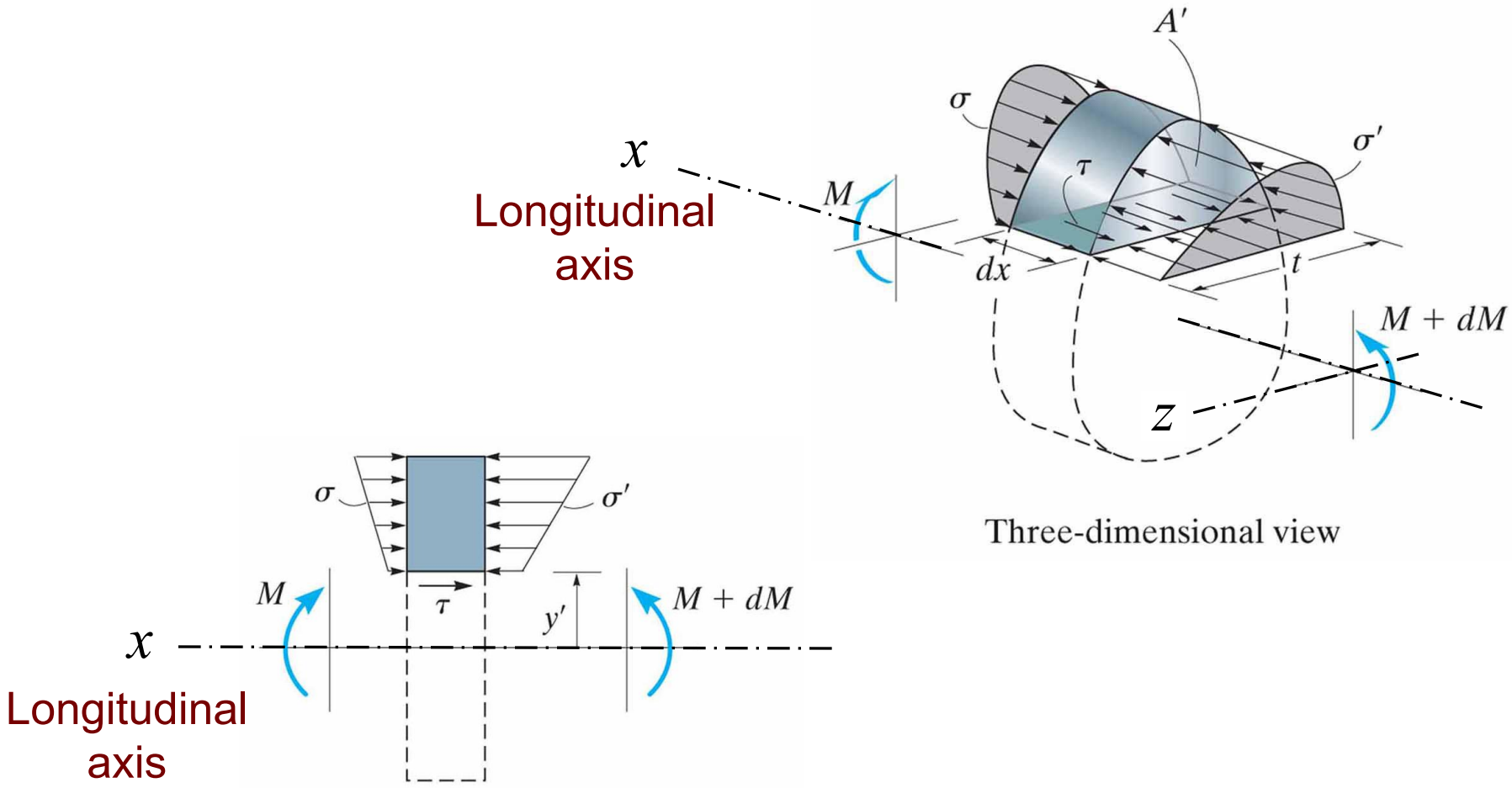

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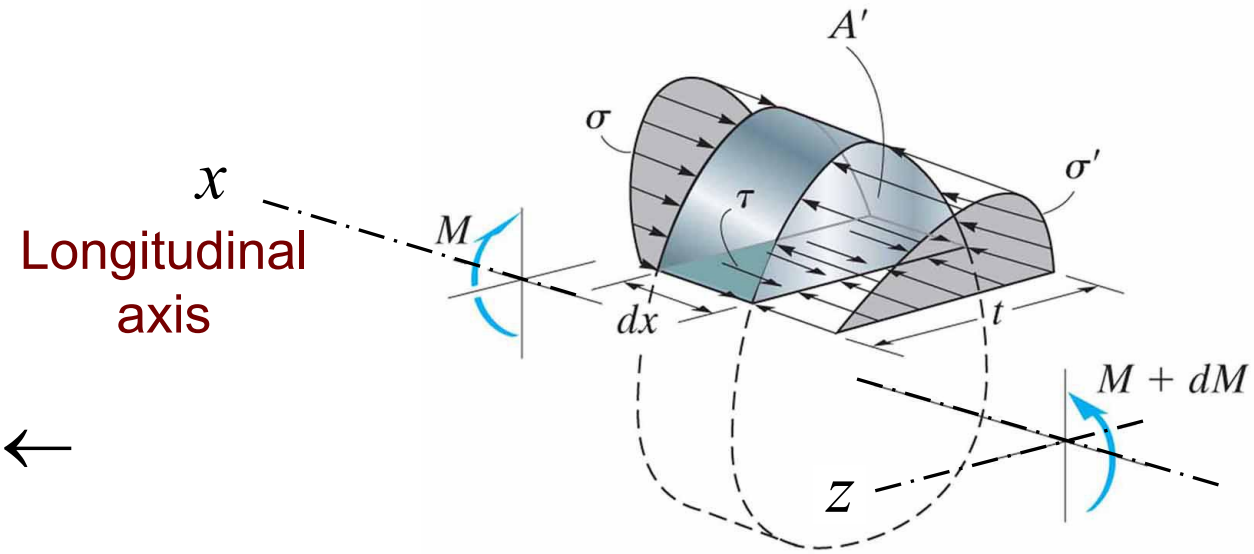


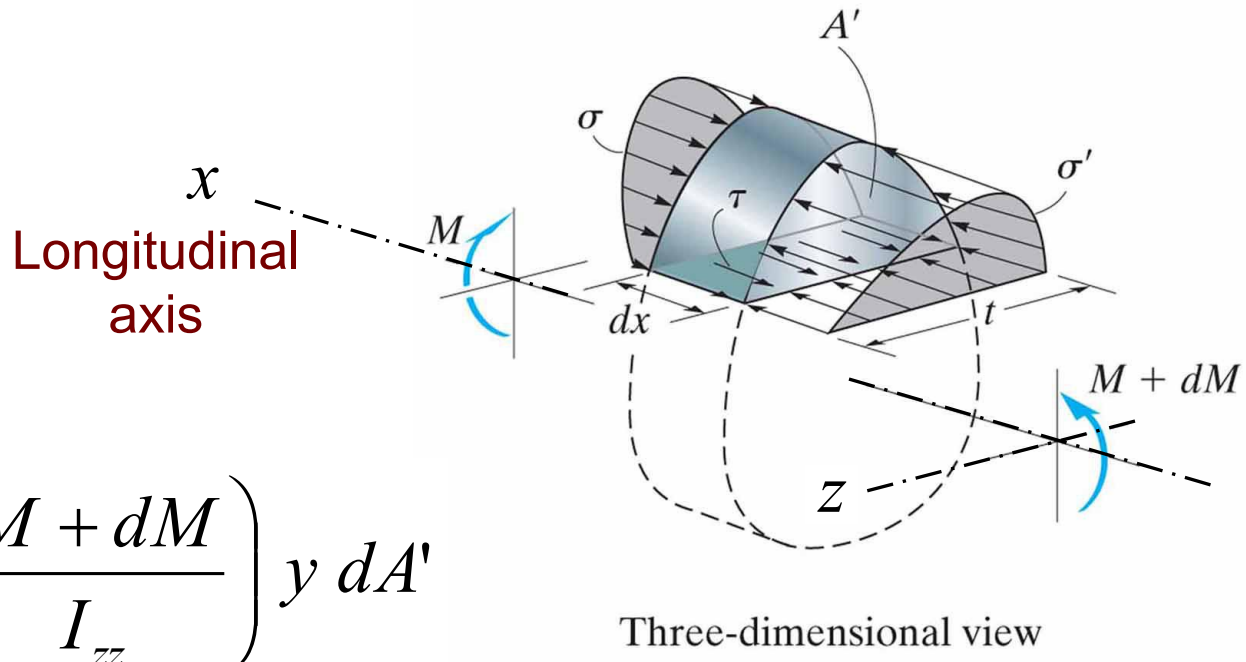

Three-dimensional view

$$\sum F_x = 0; \quad + \leftarrow$$

$$\int_{A'} \sigma' \, dA' - \int_{A'} \sigma \, dA' - \tau (t \cdot dx) = 0$$

# Transverse shear: *produced by bending*

Observed in components subjected to bending loads

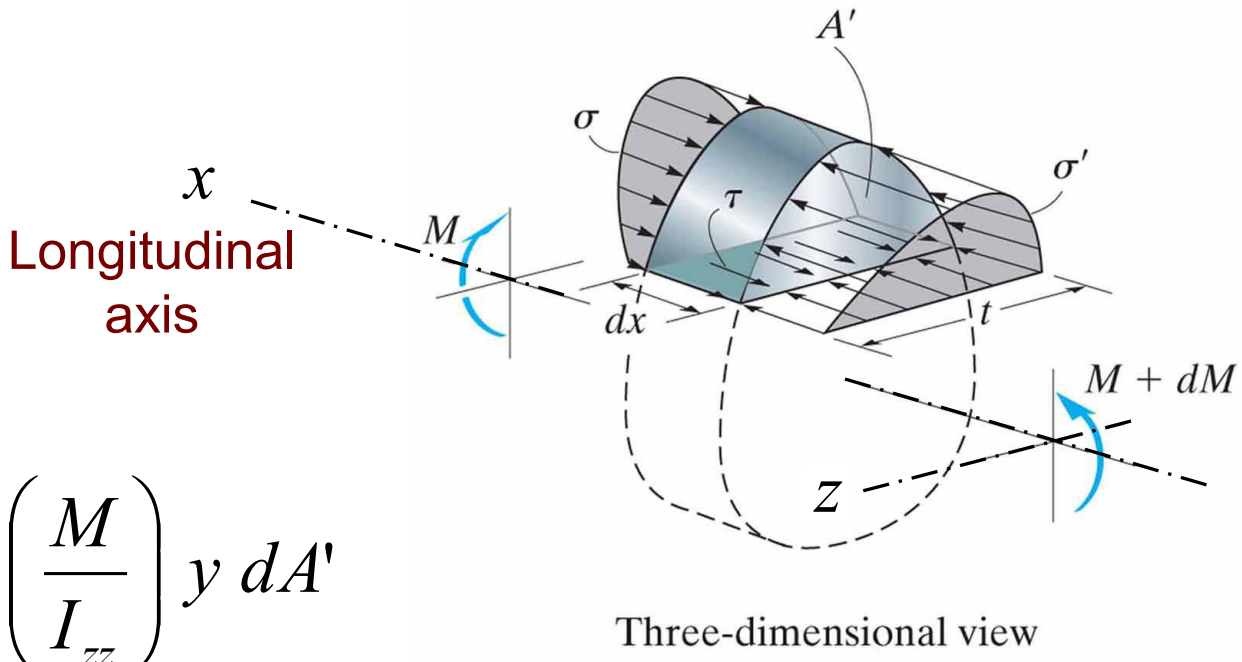



Three-dimensional view

$$\int_{A'} \sigma' \, dA' = \int_{A'} \left( \frac{M + dM}{I_{zz}} \right) y \, dA'$$

# Transverse shear: *produced by bending*

Observed in components subjected to bending loads




$$\int_{A'} \sigma \, dA' = \int_{A'} \left( \frac{M}{I_{zz}} \right) y \, dA'$$

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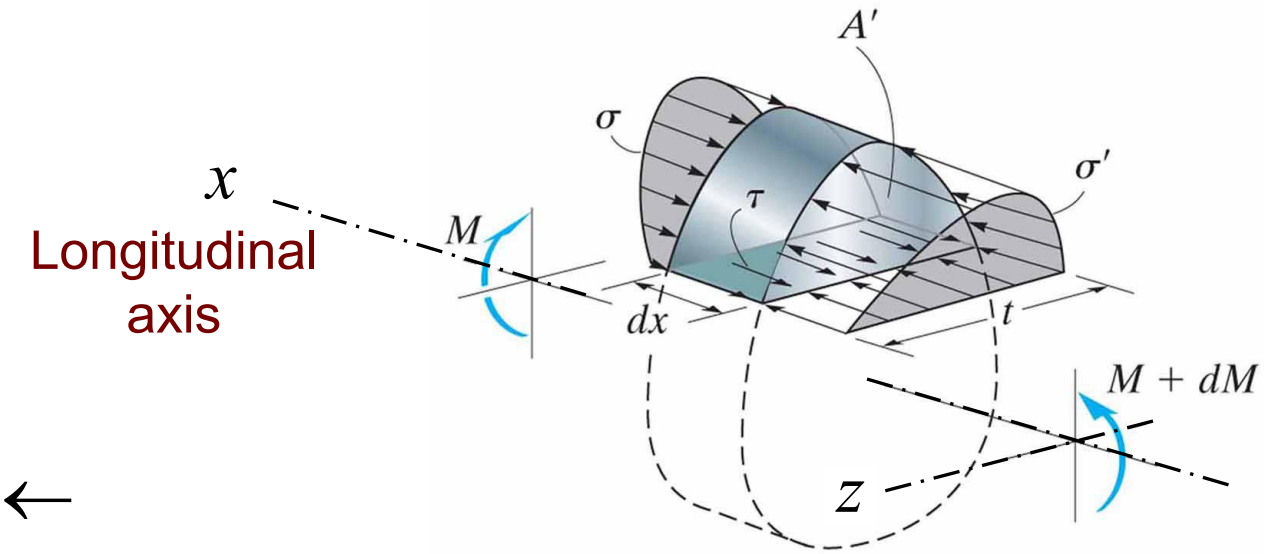
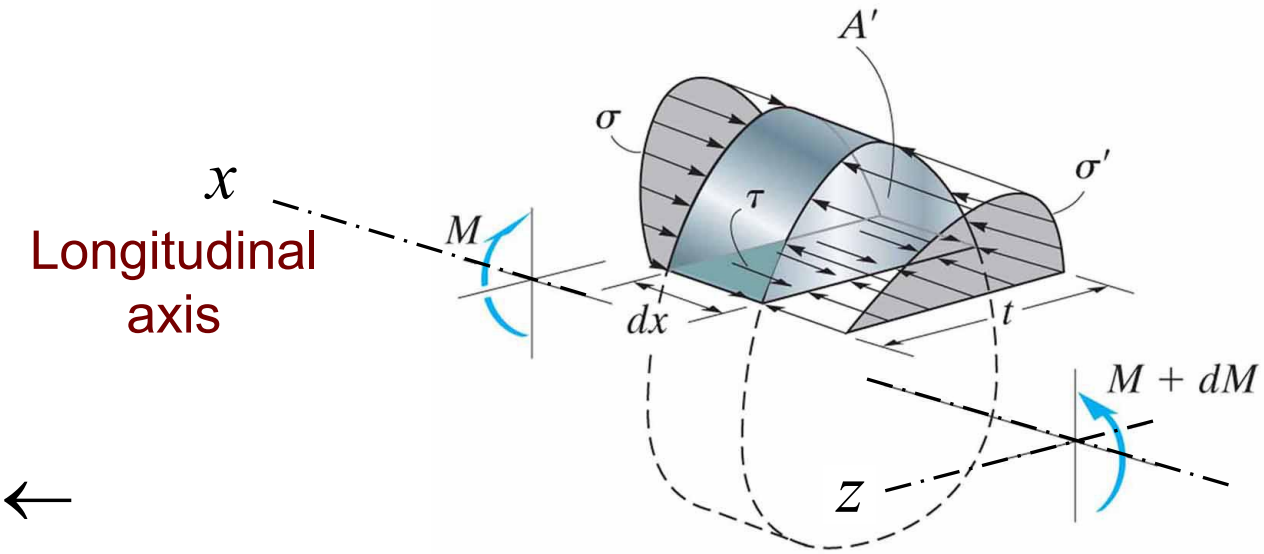



$$\sum F_x = 0\,; \quad + \leftarrow$$

$$\int_{A'} \left( \frac{M + dM}{I_{zz}} \right) y \, dA' - \int_{A'} \left( \frac{M}{I_{zz}} \right) y \, dA' - \tau \, (t \cdot dx) = 0$$

# Transverse shear: *produced by bending*

Observed in components subjected to bending loads



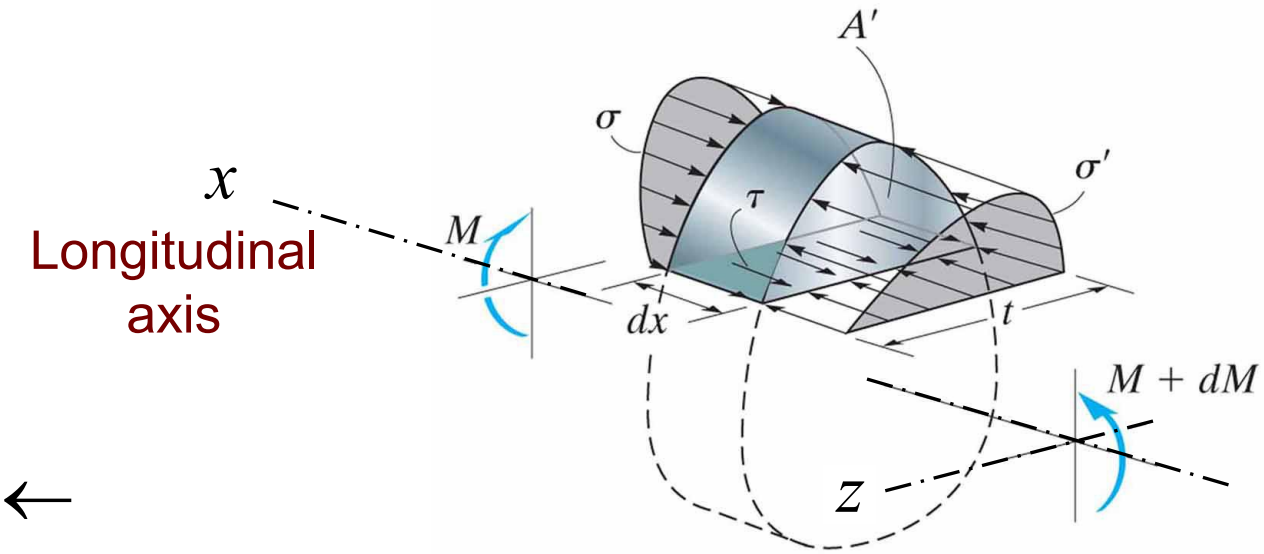

Three-dimensional view

$$\sum F_x = 0\,; \quad + \leftarrow$$

$$\int_{A'} \left( \frac{dM}{I_{zz}} \right) y \, dA' - \tau \, (t \cdot dx) = 0$$

# Transverse shear: *produced by bending*

Observed in components subjected to bending loads




Three-dimensional view

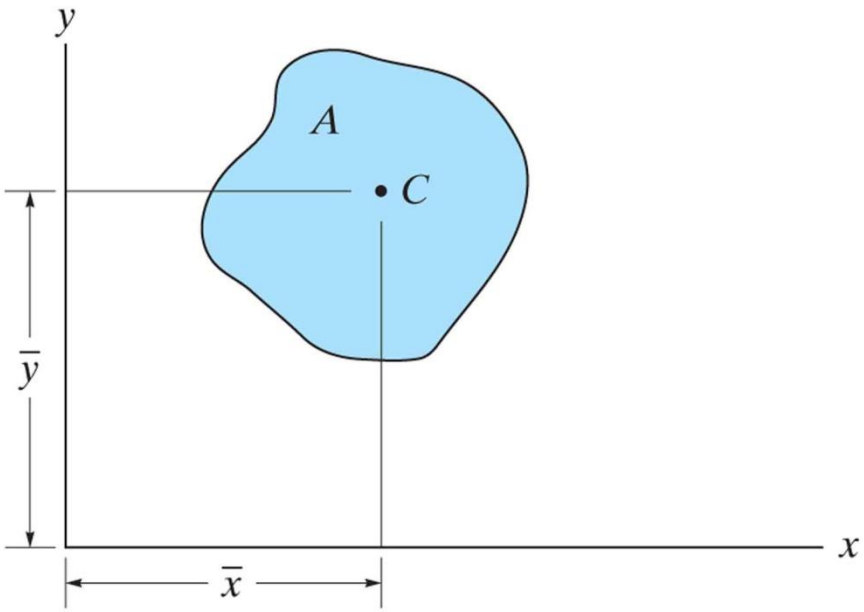
$$\sum F_x = 0\,; \quad + \leftarrow$$

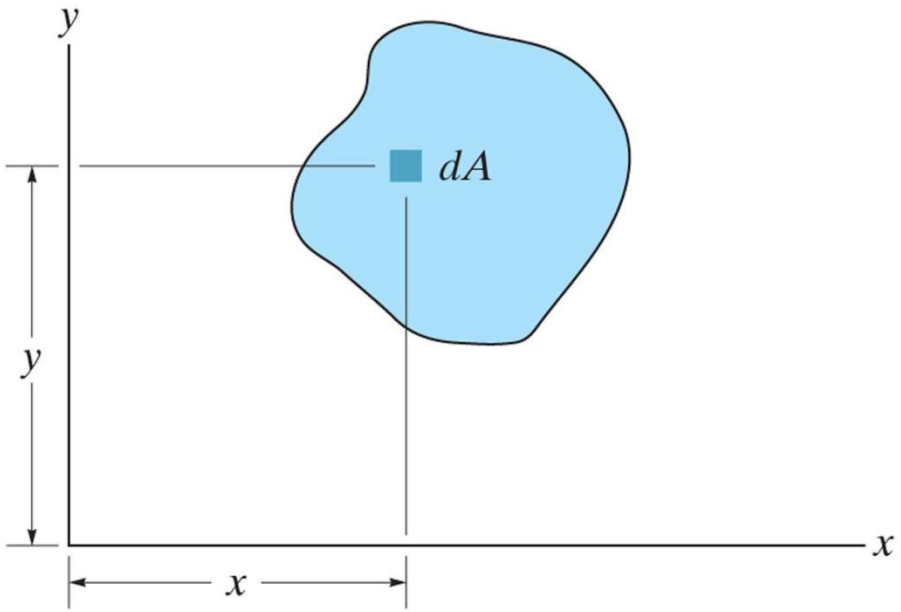
$$\tau = \frac{1}{I_{zz} \cdot t} \frac{dM}{dx} \int_{A'} y \, dA'$$

$$\frac{dM}{dx} = V$$

# Centroid of an area

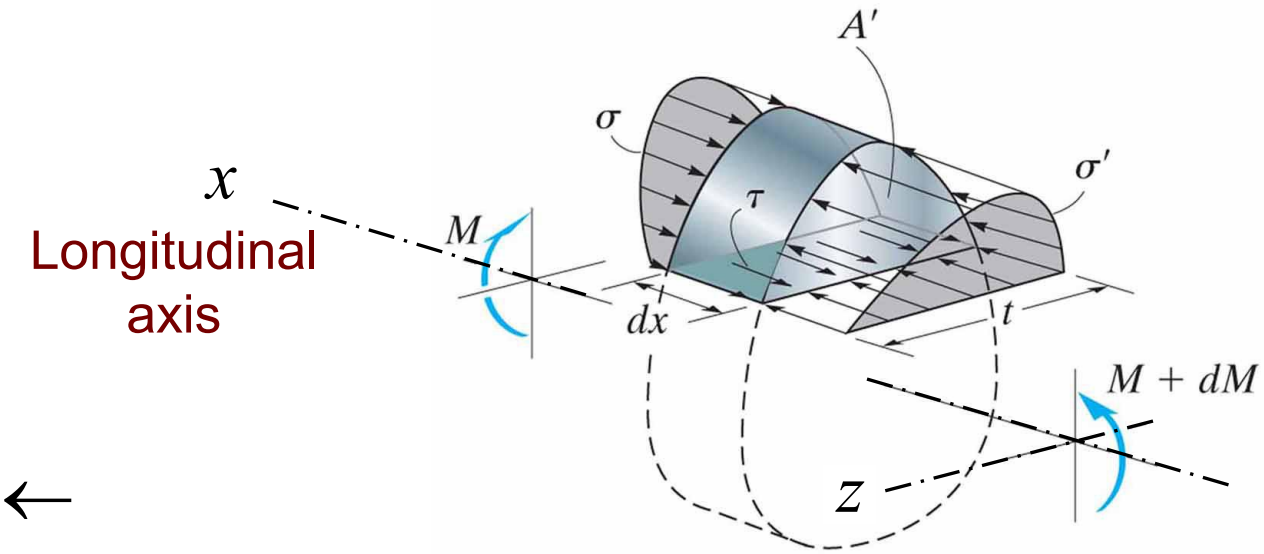
$$\bar{x} = \frac{\int_A x\, dA}{\int_A dA}; \qquad \bar{y} = \frac{\int_A y\, dA}{\int_A dA}$$




# Transverse shear: *produced by bending*

Observed in components subjected to bending loads




Three-dimensional view

$$\sum F_x = 0\,; \quad + \leftarrow$$

$$\tau = \frac{1}{I_{zz} \cdot t} \frac{dM}{dx} \int_{A'} y\, dA'$$

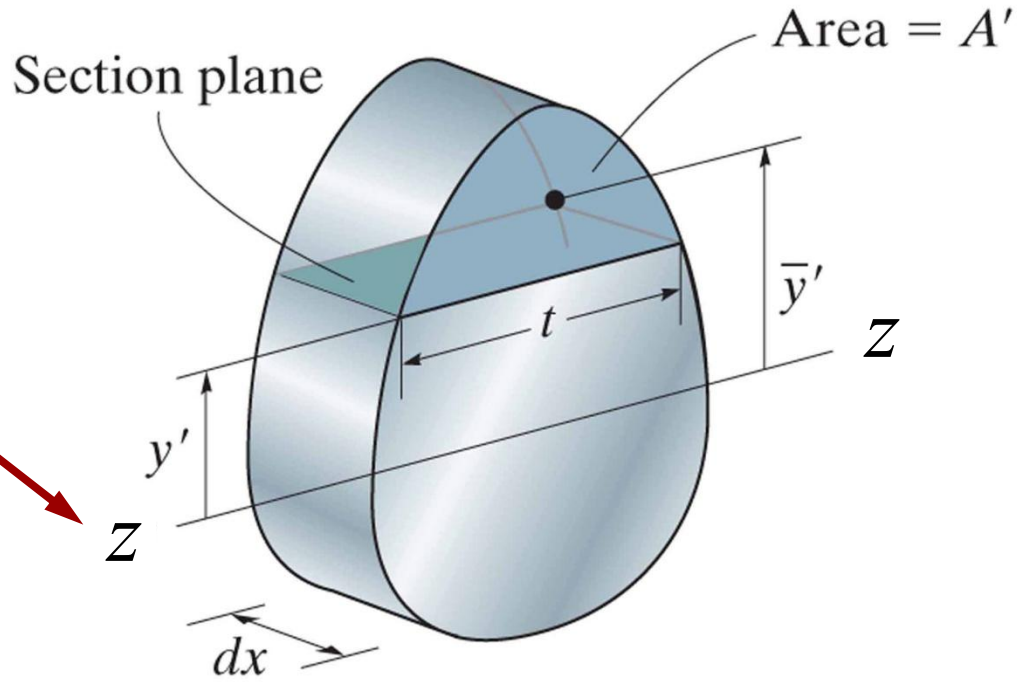
$$\int_{A'} y\, dA' = \bar{y}'\, A'$$

(related to centroid of $A'$)

# Transverse shear: *produced by bending*

Observed in components subjected to bending loads

$$\int_{A'} y\, dA' = \bar{y}'\, A' = Q$$

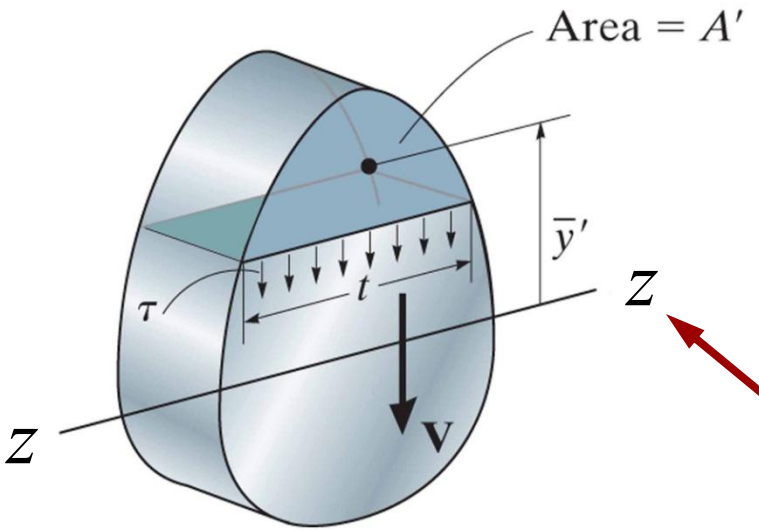



Passes through centroid of section

# **Transversal Shear** formula: ***produced by bending***

Observed in components subjected to bending loads

$$\sum F_x = 0\,; \quad + \leftarrow \qquad \tau = \frac{1}{I_{zz} \cdot t} \frac{dM}{dx} \int_{A'} y\, dA'$$




$$\tau = \frac{V \cdot Q}{I_{zz} \cdot t}$$

$$Q = \bar{y}'\, A'$$

Passes through centroid of section

*Important to remember!!*

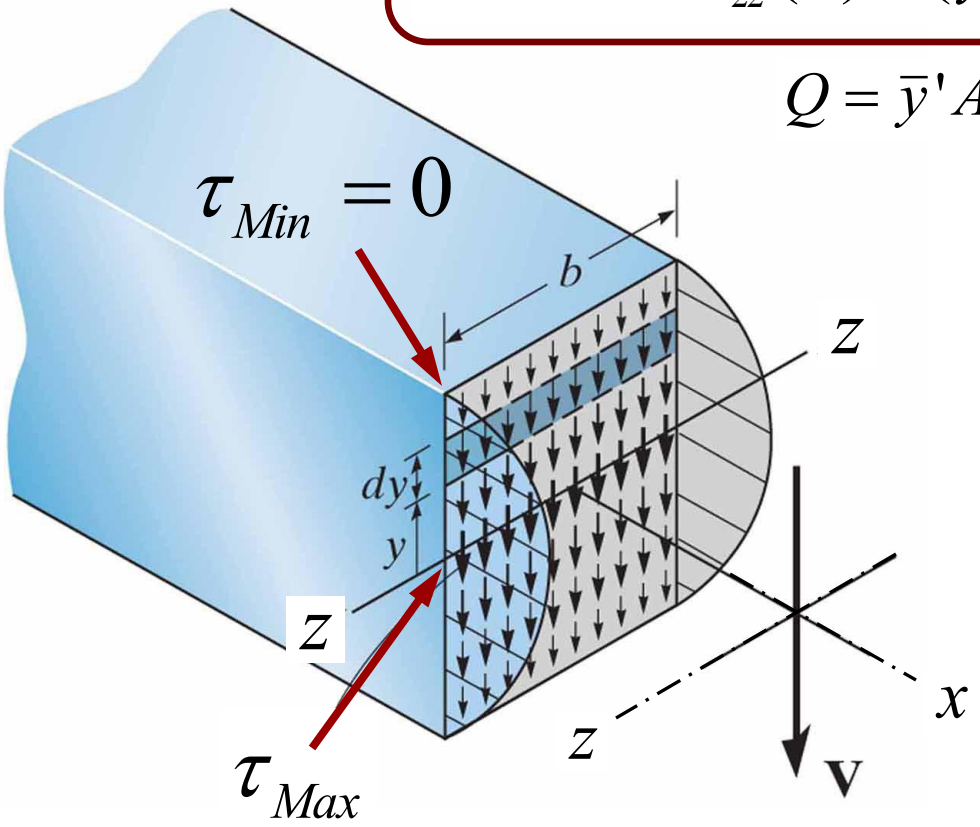
# **Transversal Shear** formula: ***produced by bending***

Observed in components subjected to bending loads

$$\tau(x,y) = \frac{V(x)\cdot Q(y)}{I_{zz}(x)\cdot t(y)}$$

*Important to remember!!*

$$Q = \bar{y}'A'$$

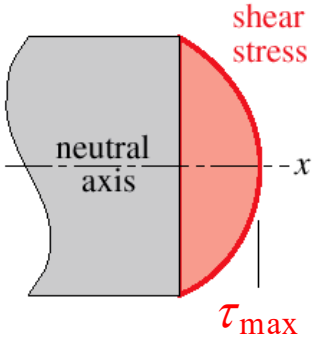



Internal distribution of shear stresses:

$$\tau_{xy} = \tau_{yx}$$

# **Transversal Shear** formula: ***produced by bending***

Observed in components subjected to bending loads

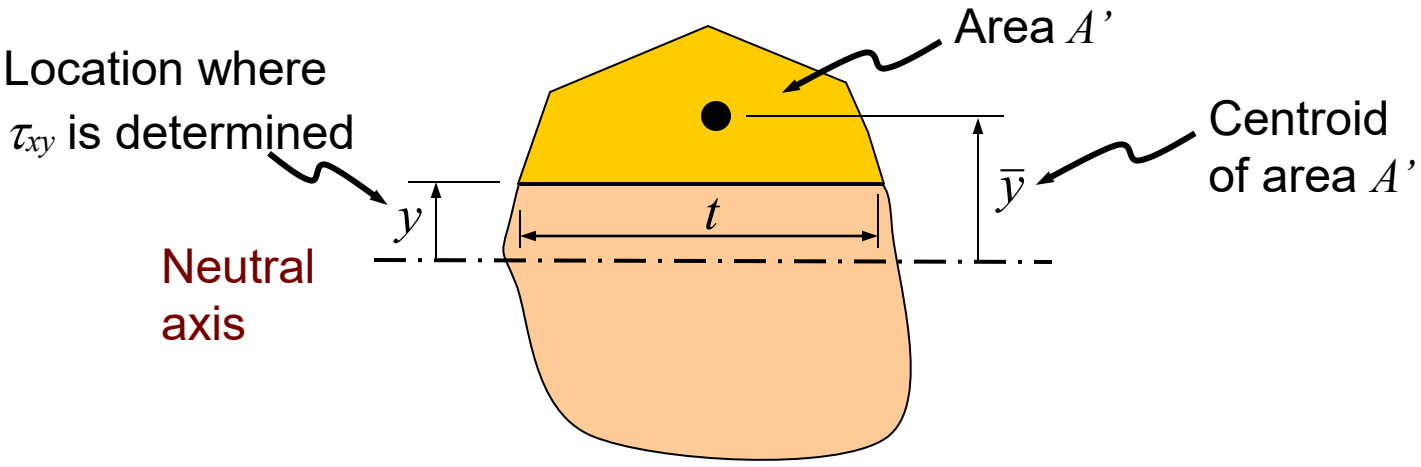



Transverse shear stress:

$$\tau_{xy} = \frac{V \cdot Q}{I \cdot t} \quad \text{with} \quad Q = \bar{y}\, A'$$

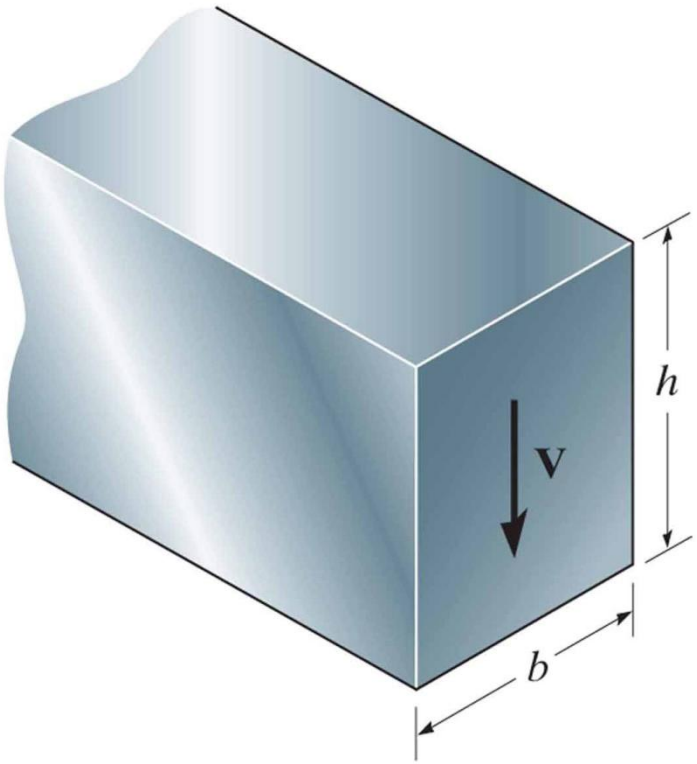
Recall that $V = V(x)$

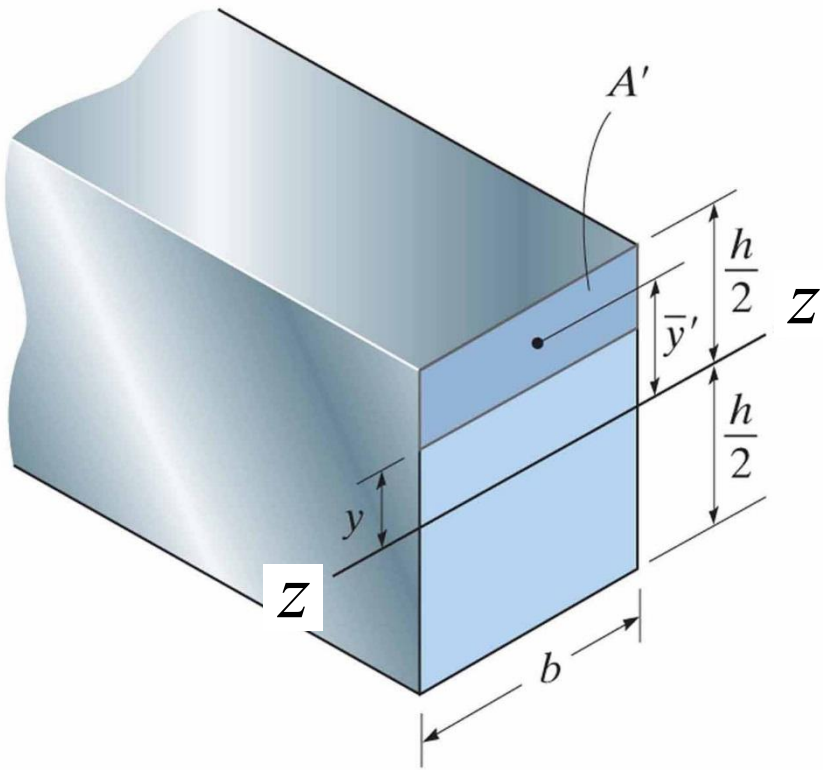
Generic cross-section:




# **Transversal shear** formula**: example A**

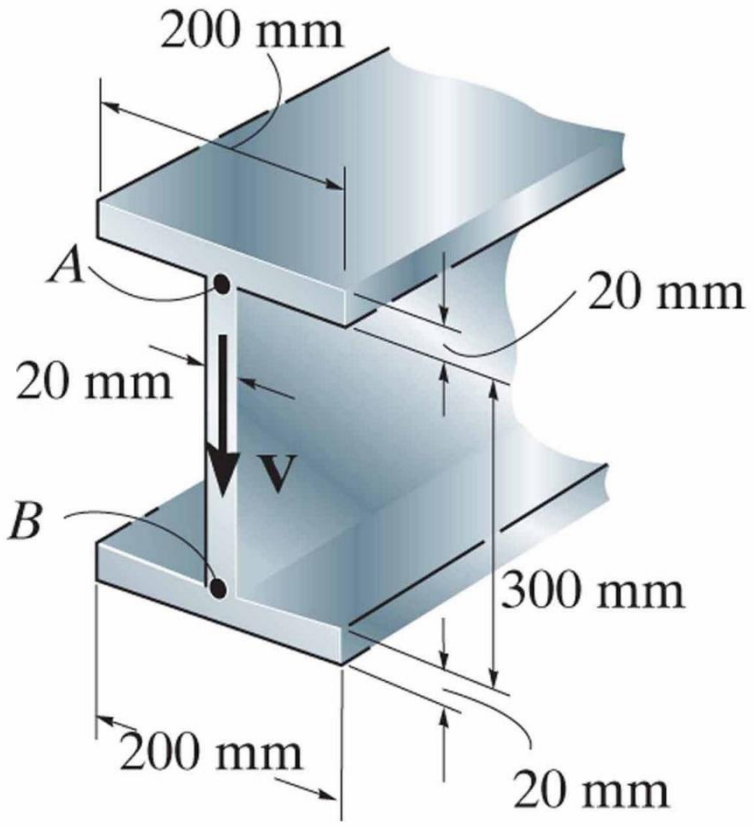
Determine the distribution of the transversal shear stresses over the cross section of the beam shown




# **Transversal shear** formula: **example B**

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear stress on the web at $A$. Indicate the shear-stress components on a volume element located at this point. Determine the distribution of the transversal shear stresses over the cross section of the beam shown




Note that $V$ is given:

It is obtained from $V(x)$ function evaluated at location $x$ of interest.

# Reading assignment

- Chapter 7 of textbook
- Review notes and text: ES2001, ES2501

# Homework assignment

- As indicated on webpage of our course