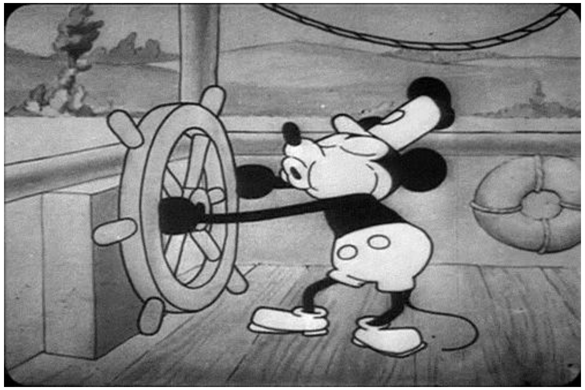
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



02 December 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

Lecture 21:
Unit 17: Bending of beams::
MV diagrams & MV general relationship

02 December 2025

General information

Instructor: Cosme Furlong
HL-152
(508) 831-5126
Email: cfurlong @ wpi.edu
http://www.wpi.edu/~cfurlong/es2502.html

Graduate Assistants:

→ Hamed Ghavami (TA)
Email: sghavami @ wpi.edu

→ Jay Patil (GA)
Email: jpatil1 @ wpi.edu

→ Mikayla Almeida (GA)
mpalmeida @ wpi.edu

The flexure formula

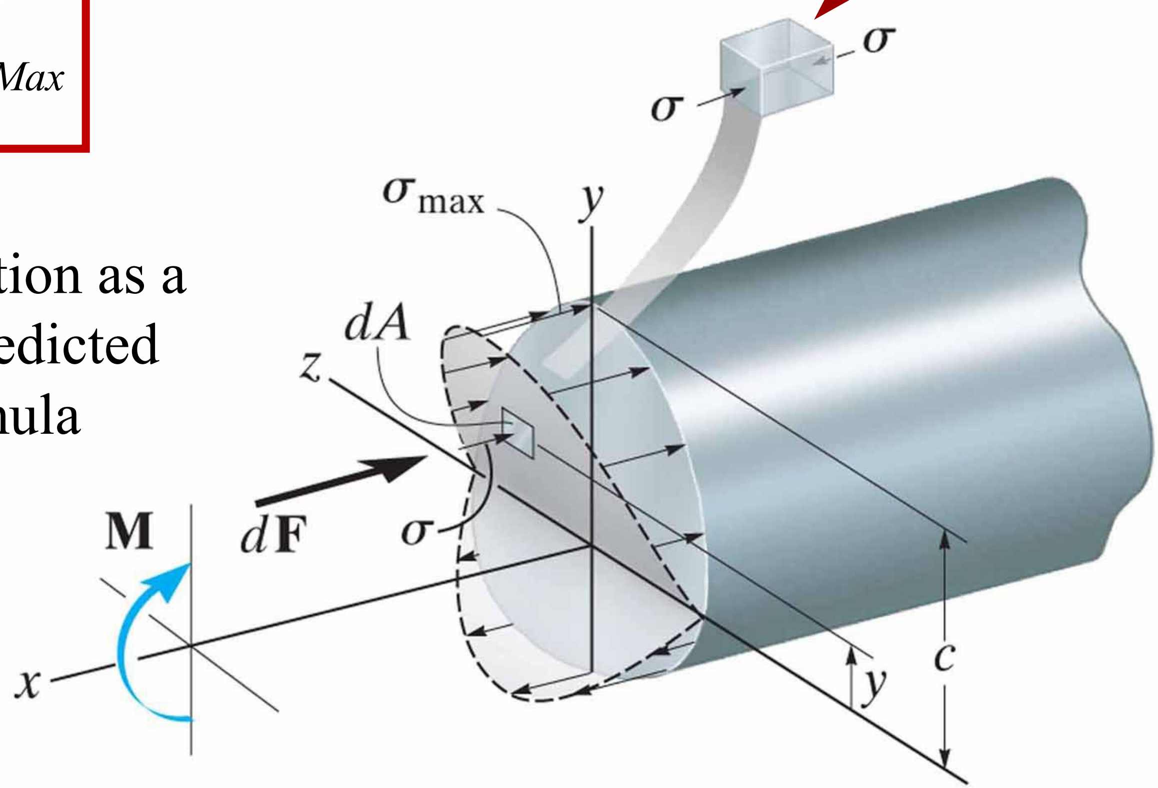
$$\sigma_x = \left(\frac{y}{c}\right) \cdot \sigma_{x\,Max}$$

Normal stress variation as a function of y as predicted by flexure formula

$$\sigma_x = -E\frac{y}{\rho}$$

$$\sigma_{x_{Max}} = -E\frac{c}{\rho}$$

Differential stress element (*stress cube*)



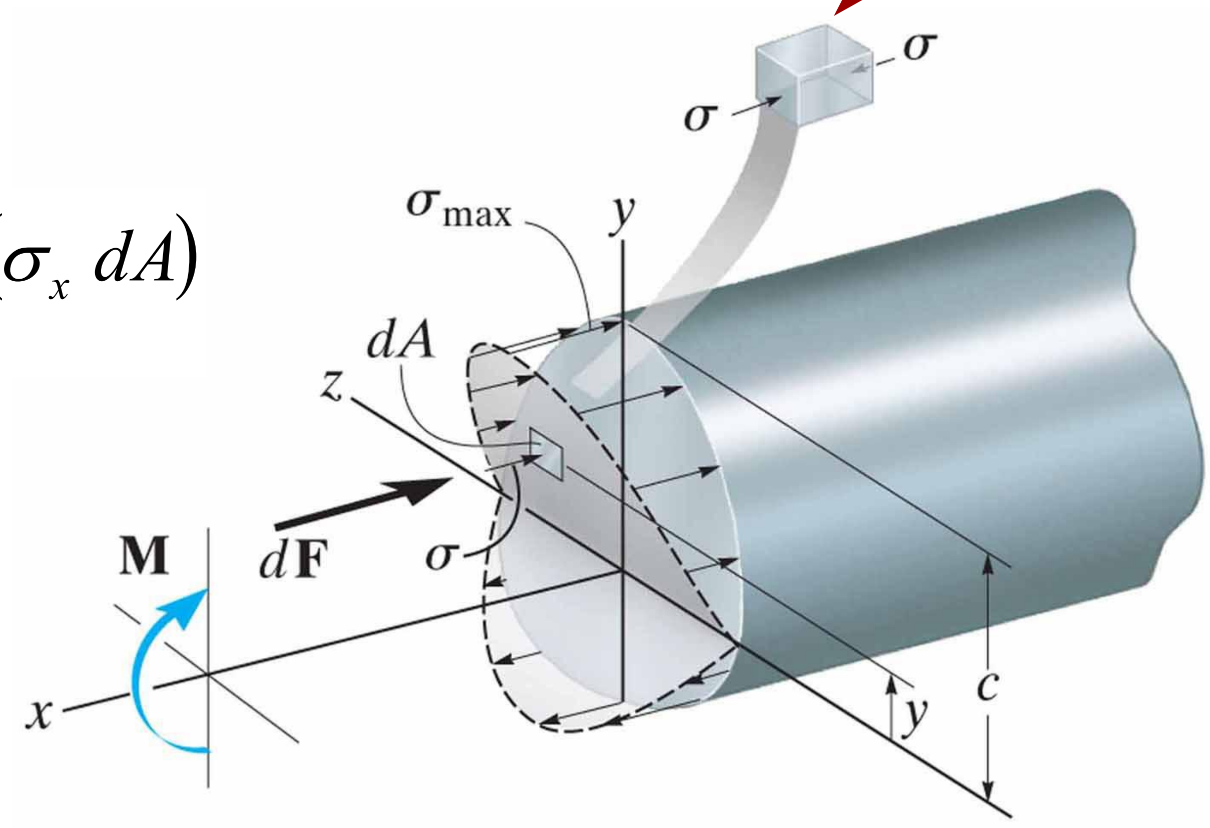
The flexure formula

Resultant internal moment:

$$M = \sum M_z$$

$$M = \int_A y\, dF = \int_A y\, (\sigma_x\, dA)$$

Differential stress element (*stress cube*)



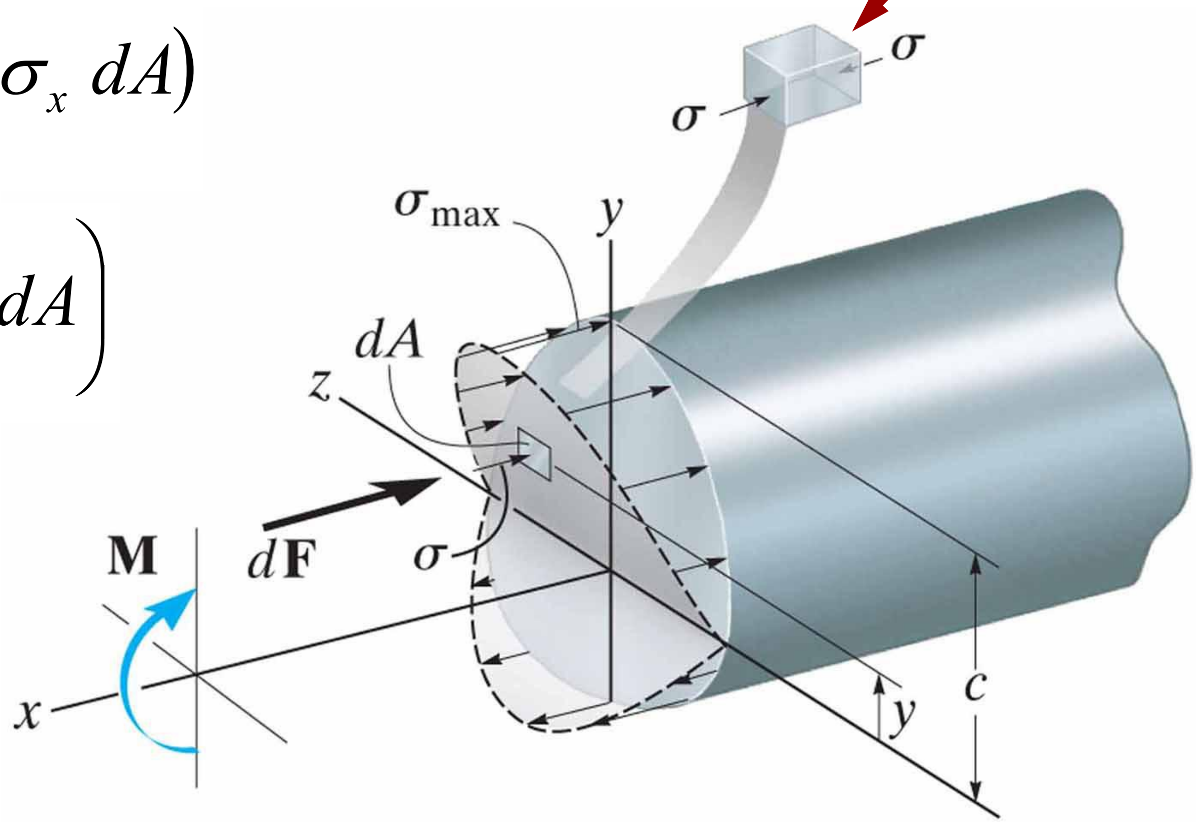
The flexure formula

Resultant internal moment:

$$M = \int_A y\, dF = \int_A y\left(\sigma_x\, dA\right)$$

$$= \int_A y\left(\frac{y}{c}\sigma_{x_{Max}}\, dA\right)$$

Differential stress element



The flexure formula

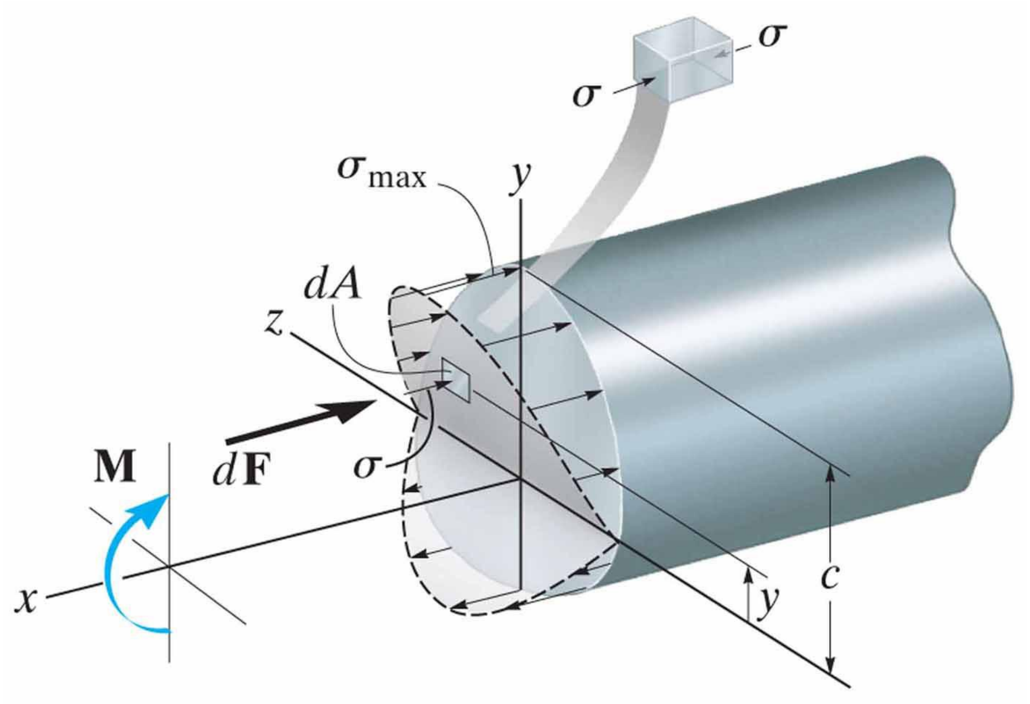
Resultant internal moment:

$$M = \frac{\sigma_{x_{Max}}}{c} \int_A y^2 \, dA$$

$$\sigma_{x_{Max}} = \frac{M\,c}{I_{zz}}$$

$$I_{zz} = \int_A y^2 \, dA$$

Area moment of inertia wrt to *z*-axis



The flexure formula

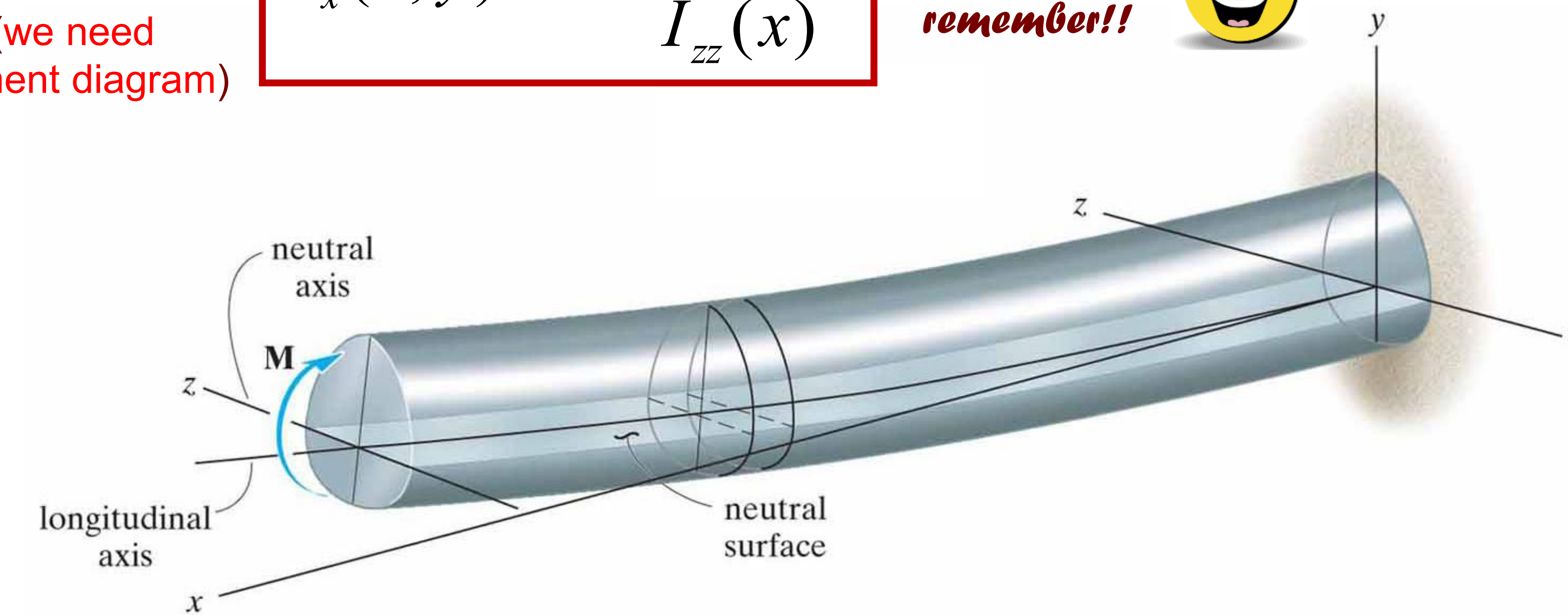
$$\sigma_{x\,Max} = \frac{M\ c}{I_{zz}}$$

$$\sigma_x = -\frac{M\ y}{I_{zz}}$$

Do note that:
(we need moment diagram)

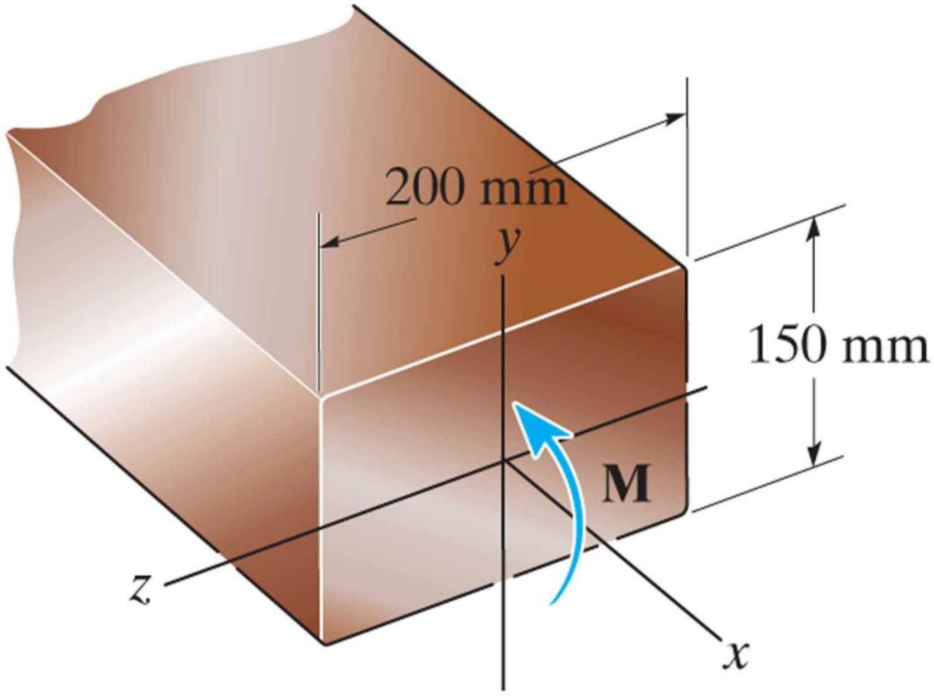
$$\sigma_x(x,y) = -\frac{M(x)\cdot y}{I_{zz}(x)}$$

Important to remember!!



Shear and bending diagrams: example C

A member having the dimensions shown is used to resist an internal bending moment of $M = 90\ \text{kN}\cdot\text{m}$. Determine the maximum stress in the member if the moment is applied (a) about the z-axis (as shown); and (b) about the y-axis. Sketch the stress distribution for each case.



Shear and bending diagrams: example C

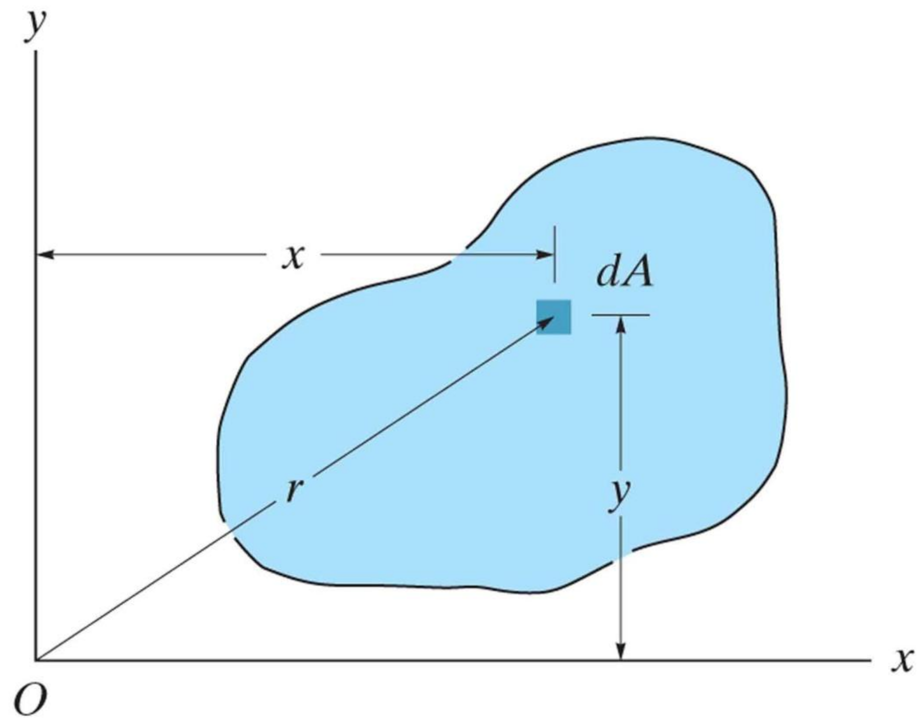
Area moment of inertia (**aka**, 2nd area moment of inertia):

$$I_{xx} = \int_A y^2 \, dA,$$

$$I_{yy} = \int_A x^2 \, dA$$

Polar area moment of inertia:

$$J_O = I_{xx} + I_{yy}$$



Shear and bending diagrams: example C

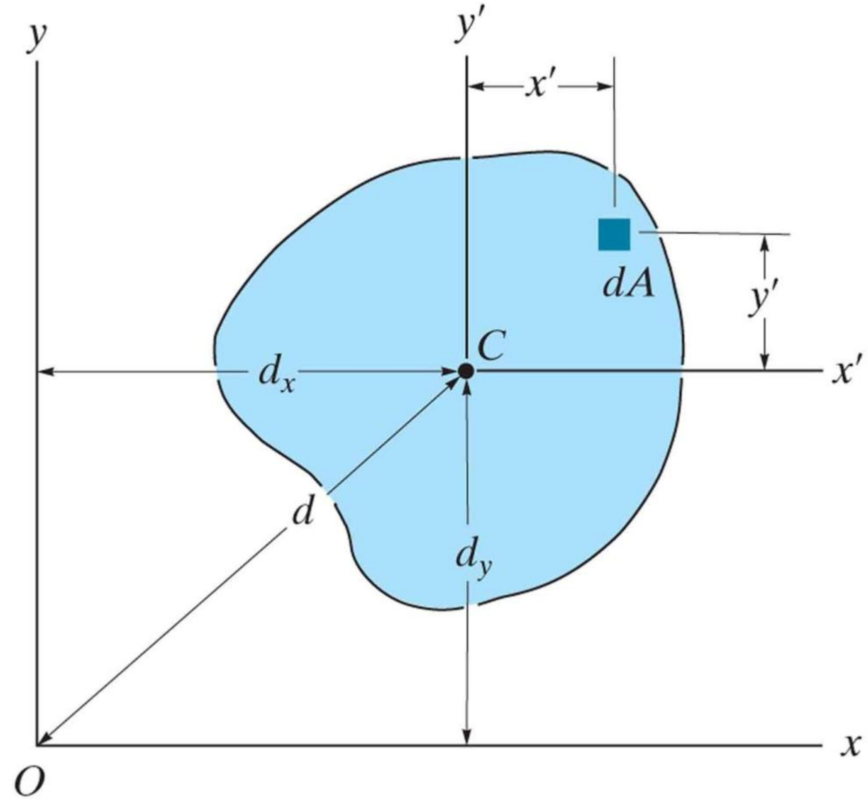
Area moment of inertia: *parallel-axis theorem*:

$$I_{xx} = \bar{I}_{x'x'} + A \cdot d_y^{\,2} \,,$$

$$I_{yy} = \bar{I}_{y'y'} + A \cdot d_x^{\,2}$$

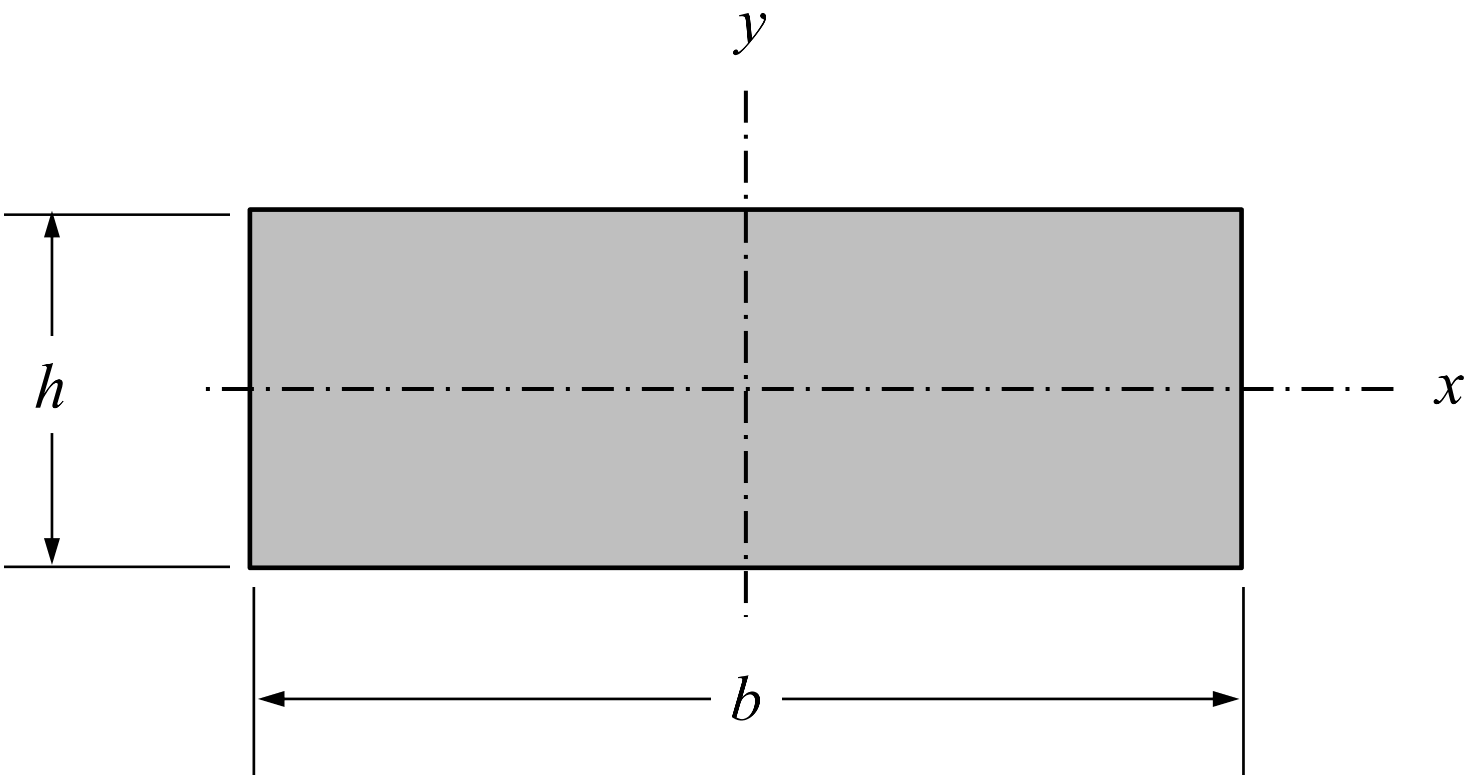
Polar area moment of inertia: *parallel-axis theorem*:

$$J_O = \bar{J}_C + A \cdot d^2$$



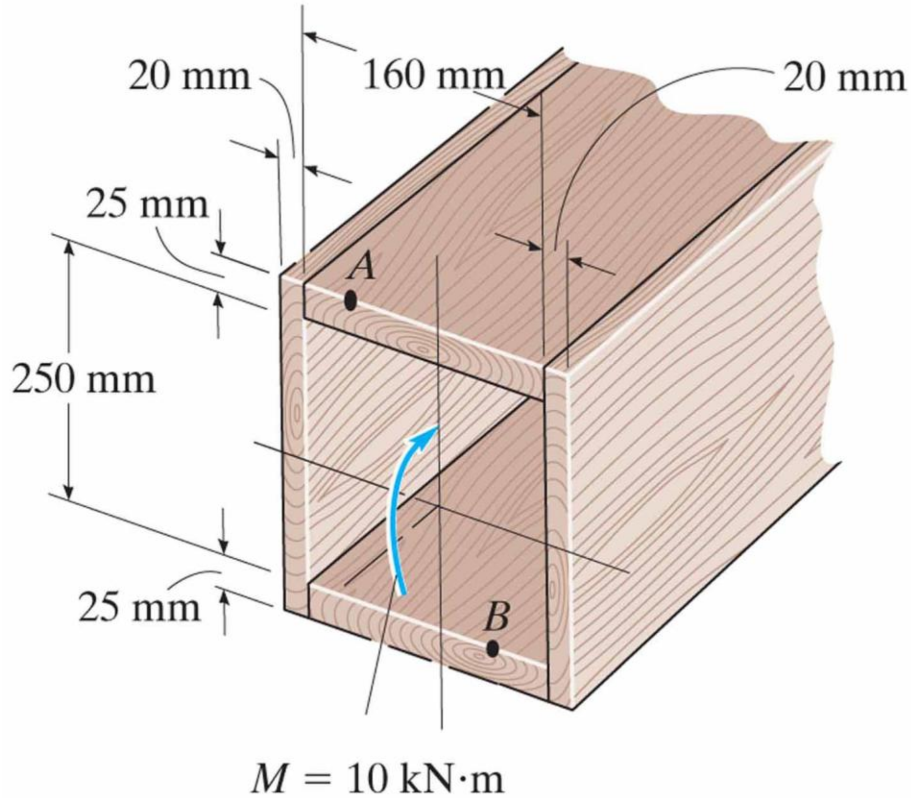
Shear and bending diagrams: example C

$$I_{xx} = \frac{1}{12} b \cdot h^3 \,, \quad I_{yy} = \frac{1}{12} h \cdot b^3$$



Shear and bending diagrams: example D

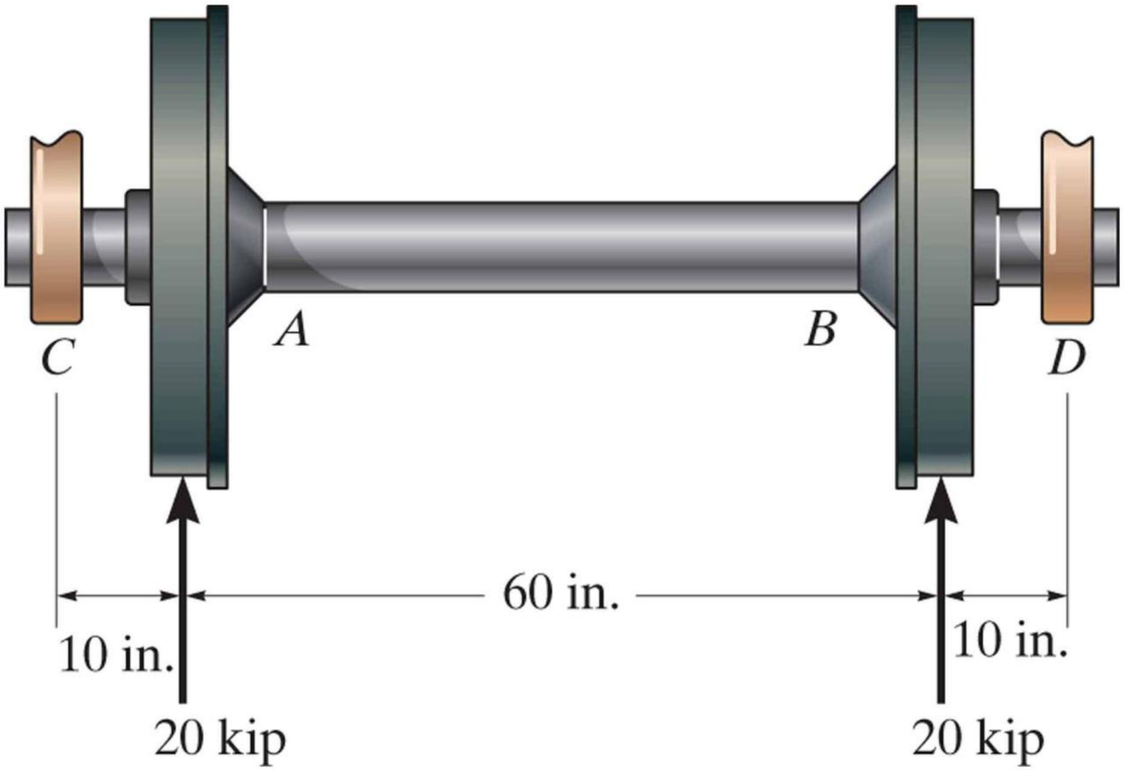
A box beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross-section is $10\ \mathrm{kN{\cdot}m}$, determine the stresses at points A and B and show the results acting on volume elements located at these points.



Note: state of stresses is *independent* of material (properties)

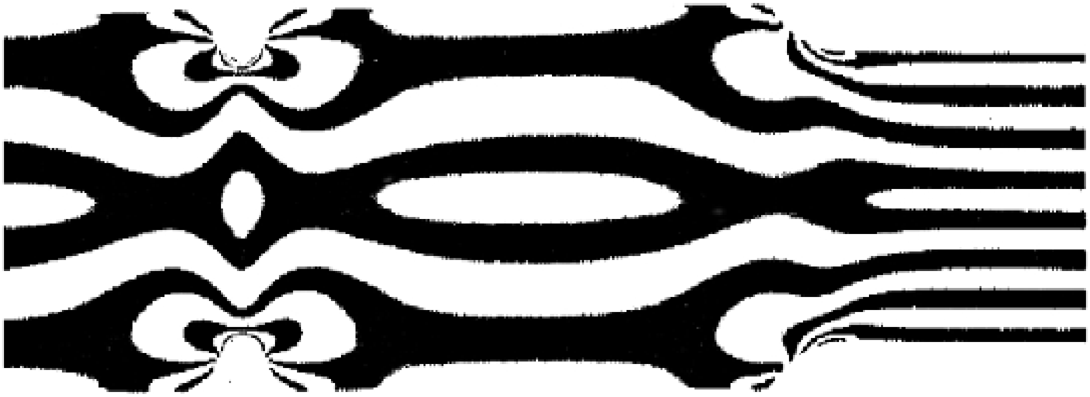
Shear and bending diagrams: example E

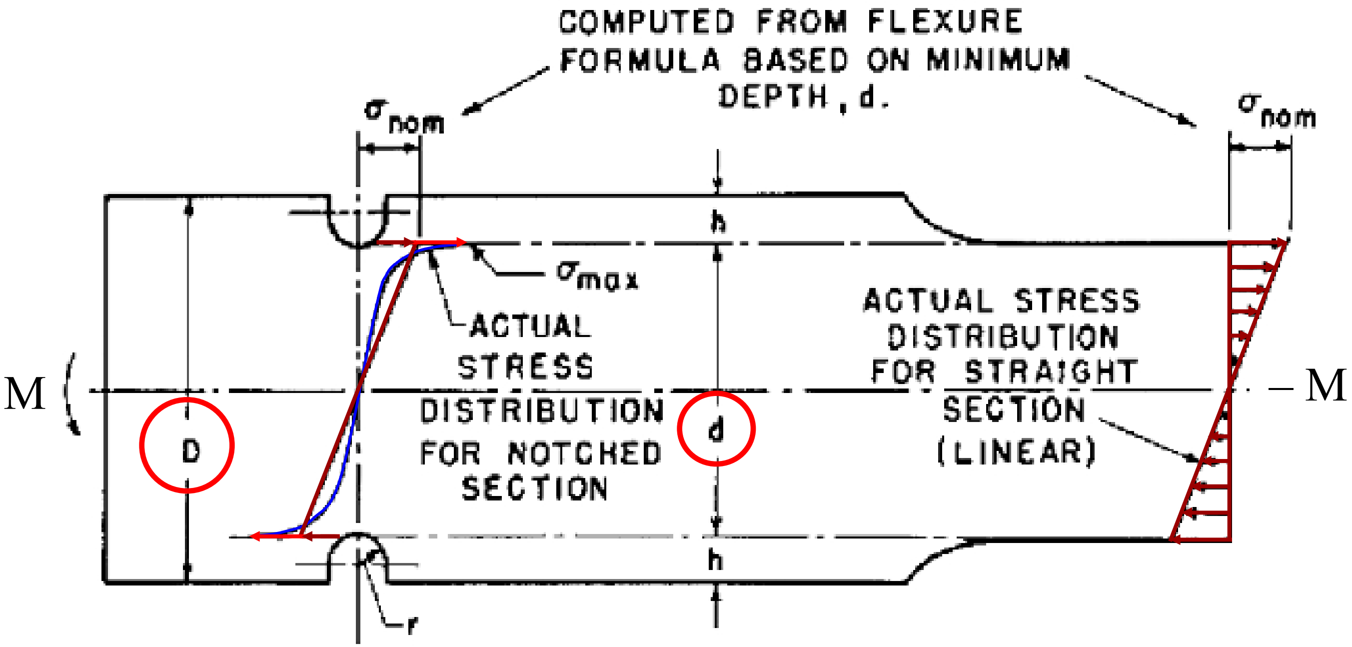
The axle of the freight car is subjected to wheel loadings of 20 kip. If it is supported by two journal bearings at *C* and *D*, determine the maximum bending stress developed at the center of the axle, where the diameter is 5.5 in. Apply two different methods



Stress concentrations: normal stresses: bending

Fringe pattern obtained with photoelasticity: pattern reveals distribution of internal stresses





Stress concentrations: bending: normal stresses + shear

Bending

Nominal bending stress:

$$\sigma_{nom} = \frac{Md}{I}$$

$$\sigma_{\max} = K\frac{Md}{I}$$

K is the stress concentration factor – normal stress

Transversal shear

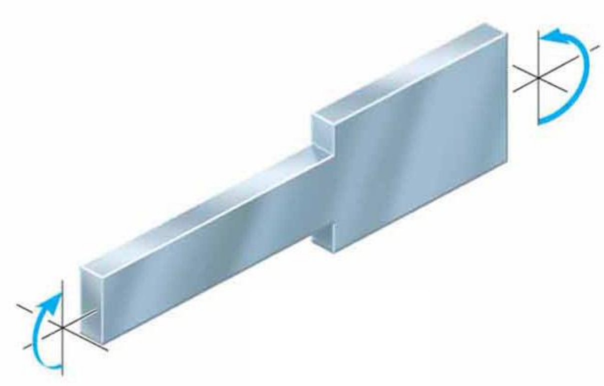
Nominal shear stress:

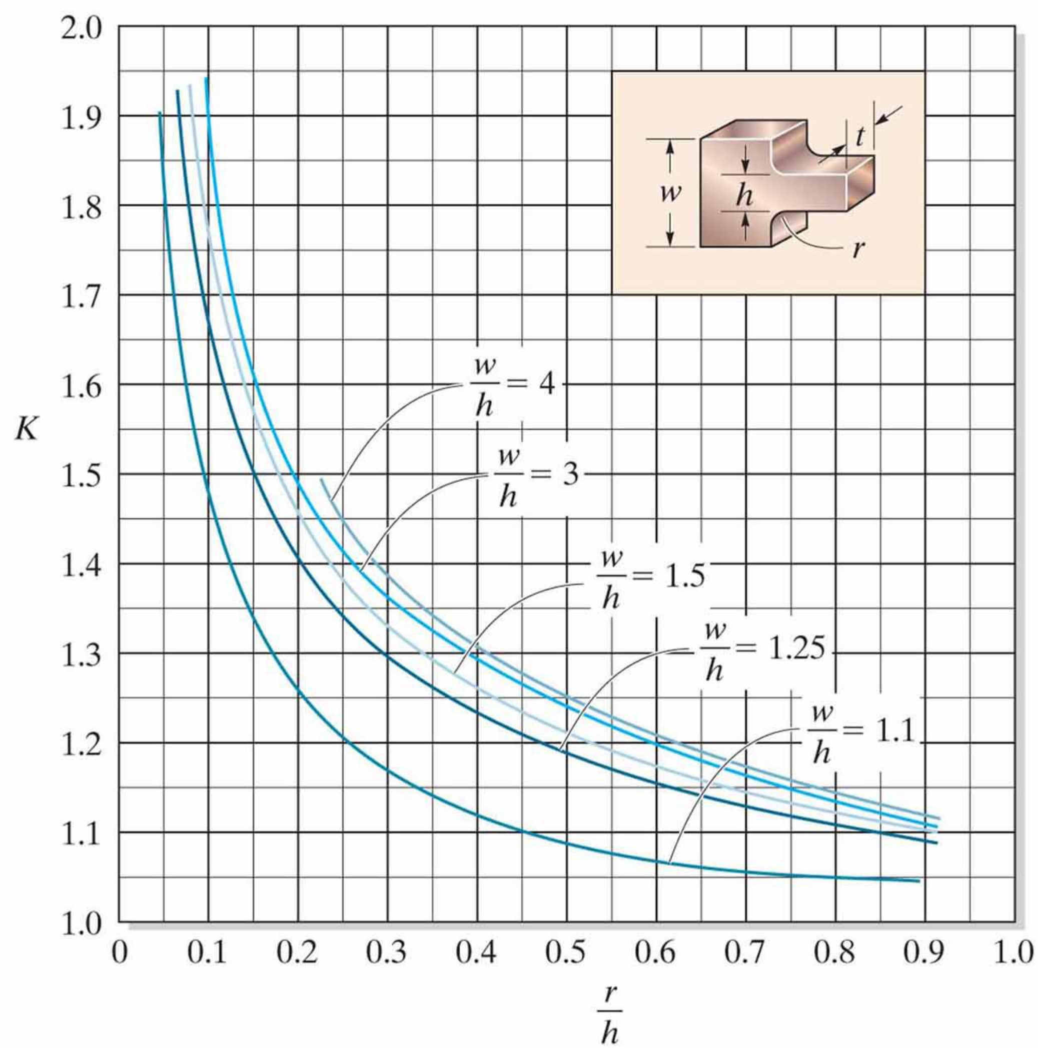
$$\tau_{nom}$$

$$\tau_{\max} = K_s\, \tau_{nom}$$

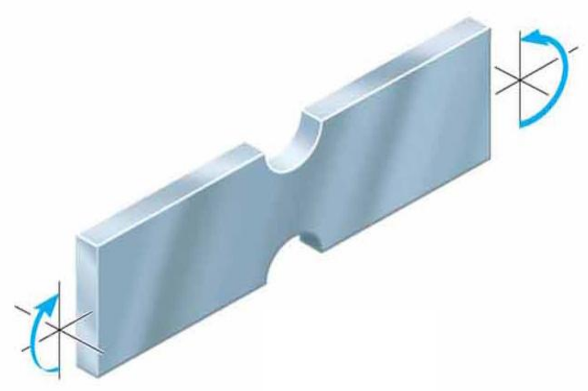
K_s is the stress concentration factor – transversal shear stress

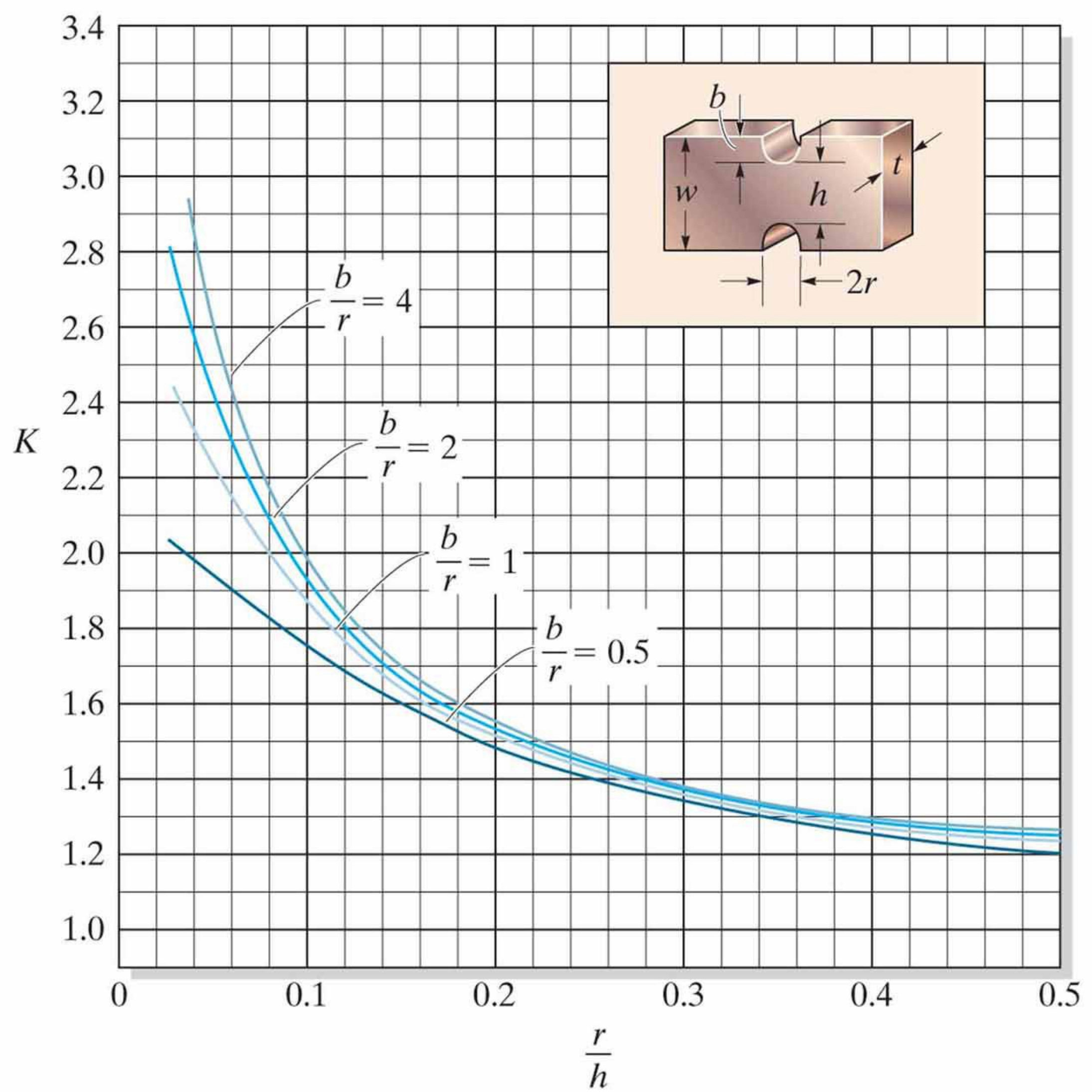
Stress concentrations: <u>normal stresses</u>: bending





Stress concentrations: normal stresses: bending





Reading assignment

- Chapter 6 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course