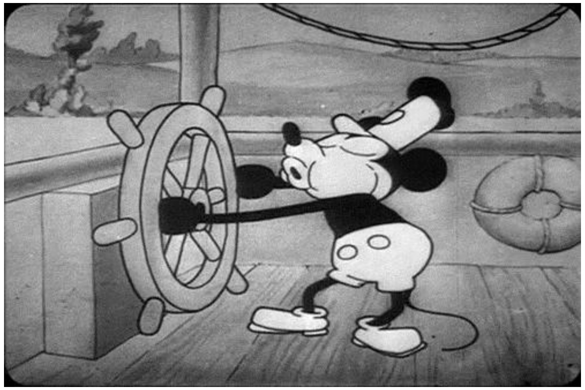
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



01 December 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

Lecture 21:
Unit 17: Bending of beams::
MV diagrams & MV general relationship

01 December 2025

General information

Instructor: Cosme Furlong
HL-152
(508) 831-5126
Email: cfurlong @ wpi.edu
http://www.wpi.edu/~cfurlong/es2502.html

Graduate Assistants:

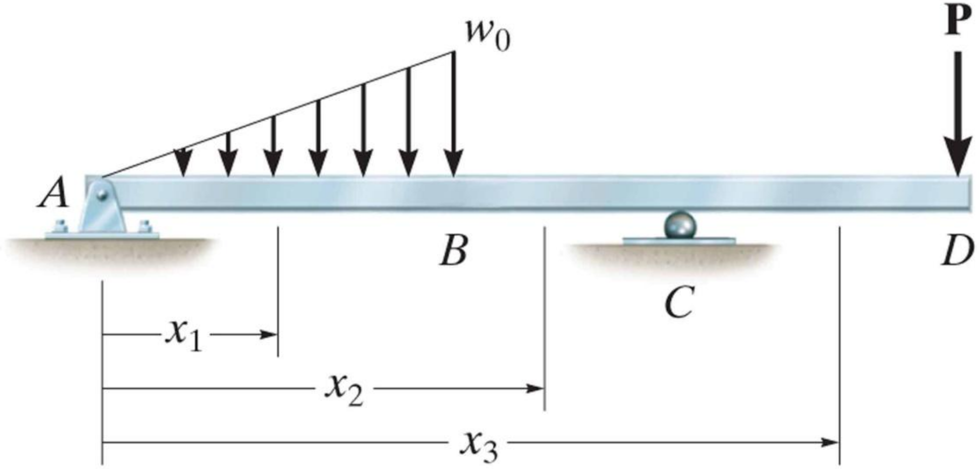
→ Hamed Ghavami (TA)
Email: sghavami @ wpi.edu

→ Jay Patil (GA)
Email: jpatil1 @ wpi.edu

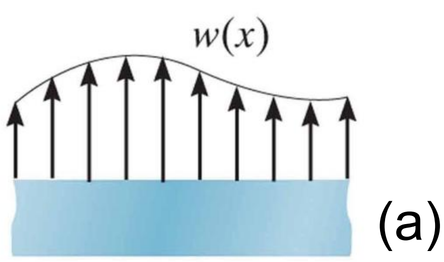
→ Mikayla Almeida (GA)
mpalmeida @ wpi.edu

Shear and bending diagrams

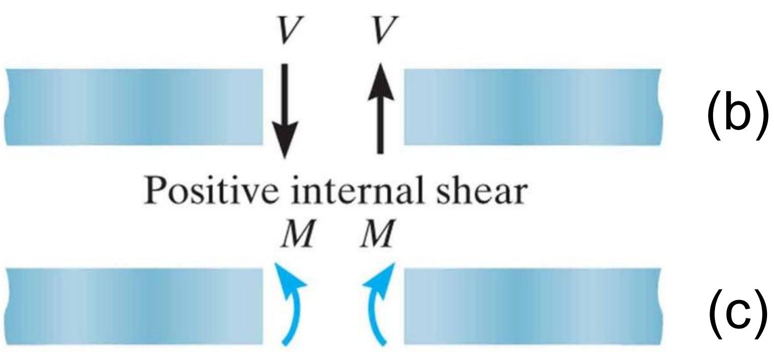
Diagrams are determined for *each region* of the beam *between* any two discontinuities of loading



Beam sign convention

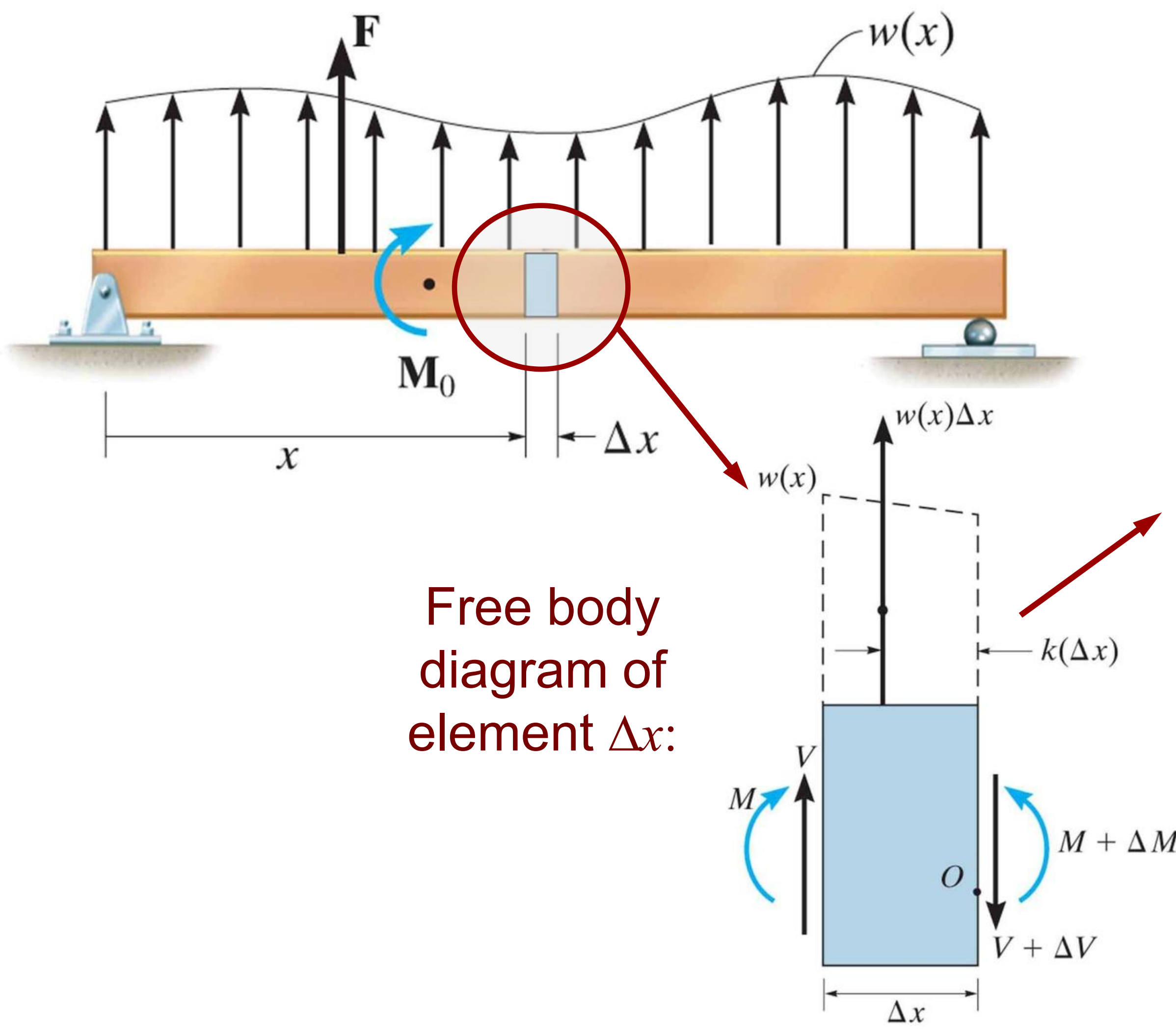


Positive external distributed load



Positive internal moment

Shear and bending diagrams: regions with distributed load



Free-body diagram of segment Δx

Free body diagram of element Δx:

Important to remember!!

$$\frac{dV}{dx} = w(x)\,;$$

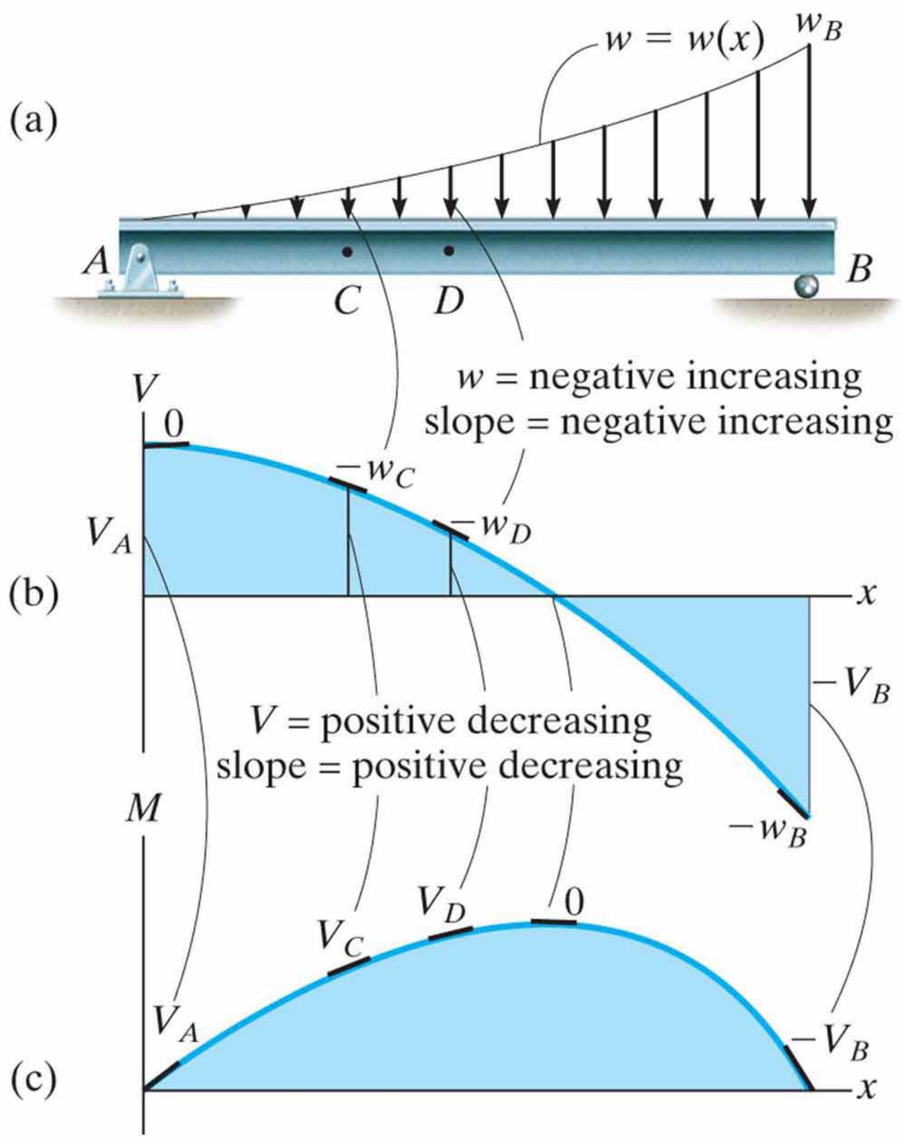
$$\frac{dM}{dx} = V(x)$$

Shear and bending diagrams: regions with distributed load

Important to remember!!

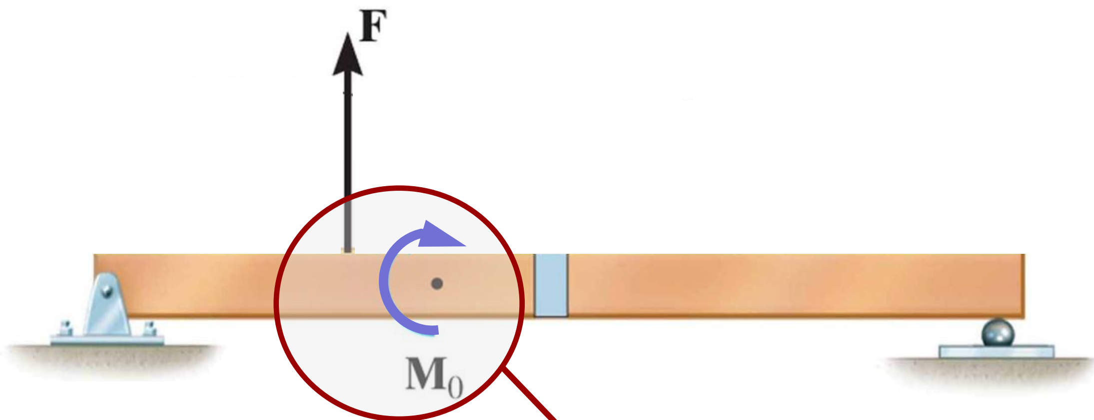
$$\frac{dV}{dx} = w(x);$$

$$\frac{dM}{dx} = V(x)$$



Shear and bending diagrams:

regions with concentrated force and moment

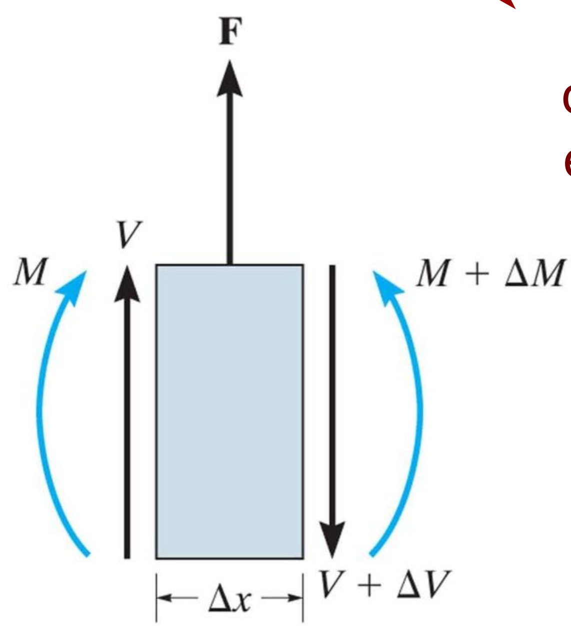


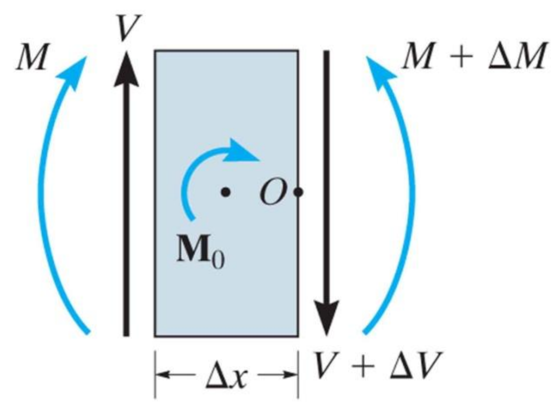
Important to remember!!

$$\Delta V = F\ ;$$

$$\Delta M = M_0$$

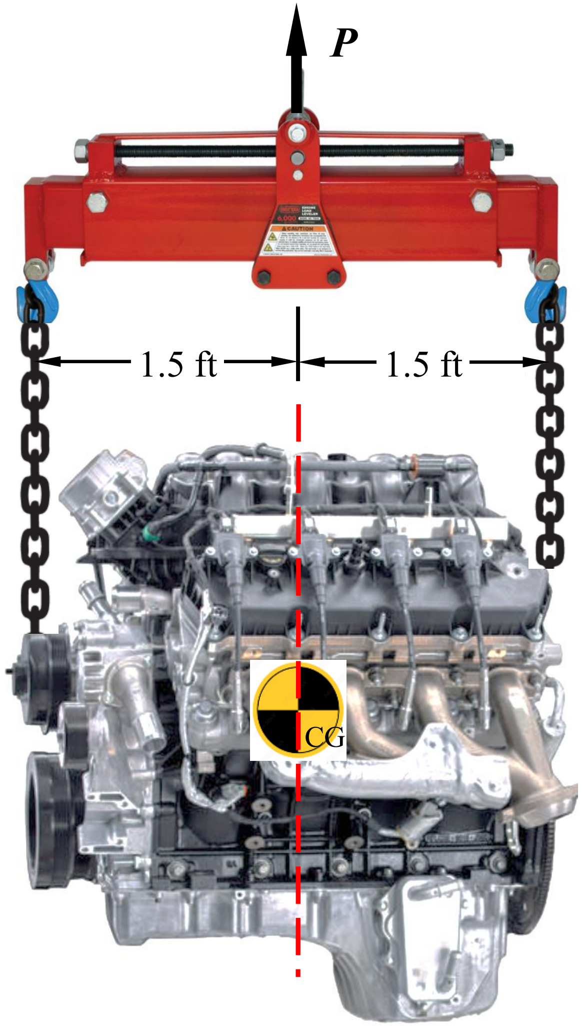
Free body diagrams of element Δx:





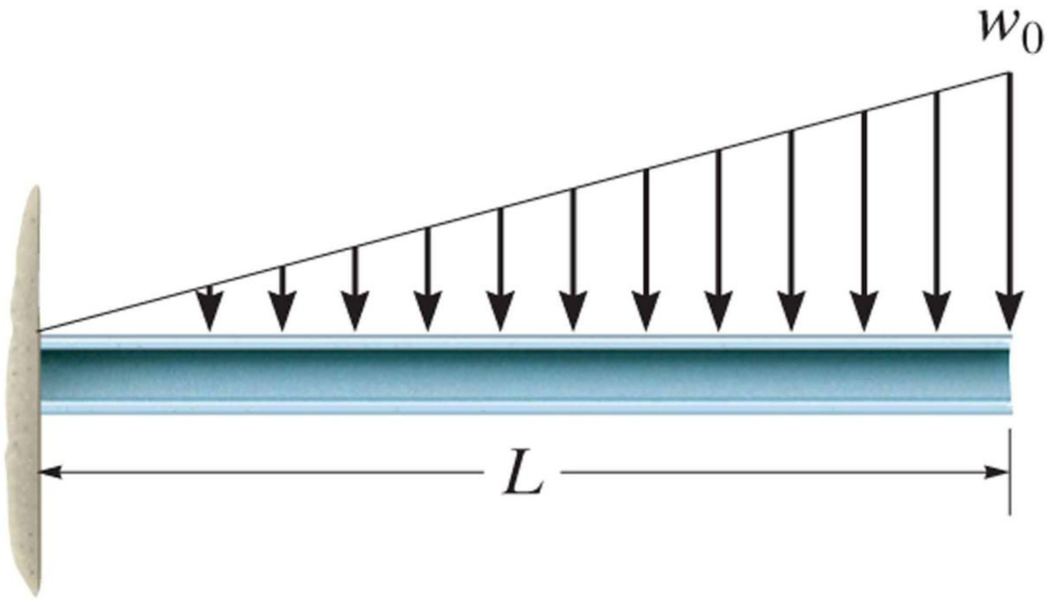
Shear and bending diagrams: example A

A suspended bar supports a 600-lb engine. Plot the shear and moment diagrams for the bar

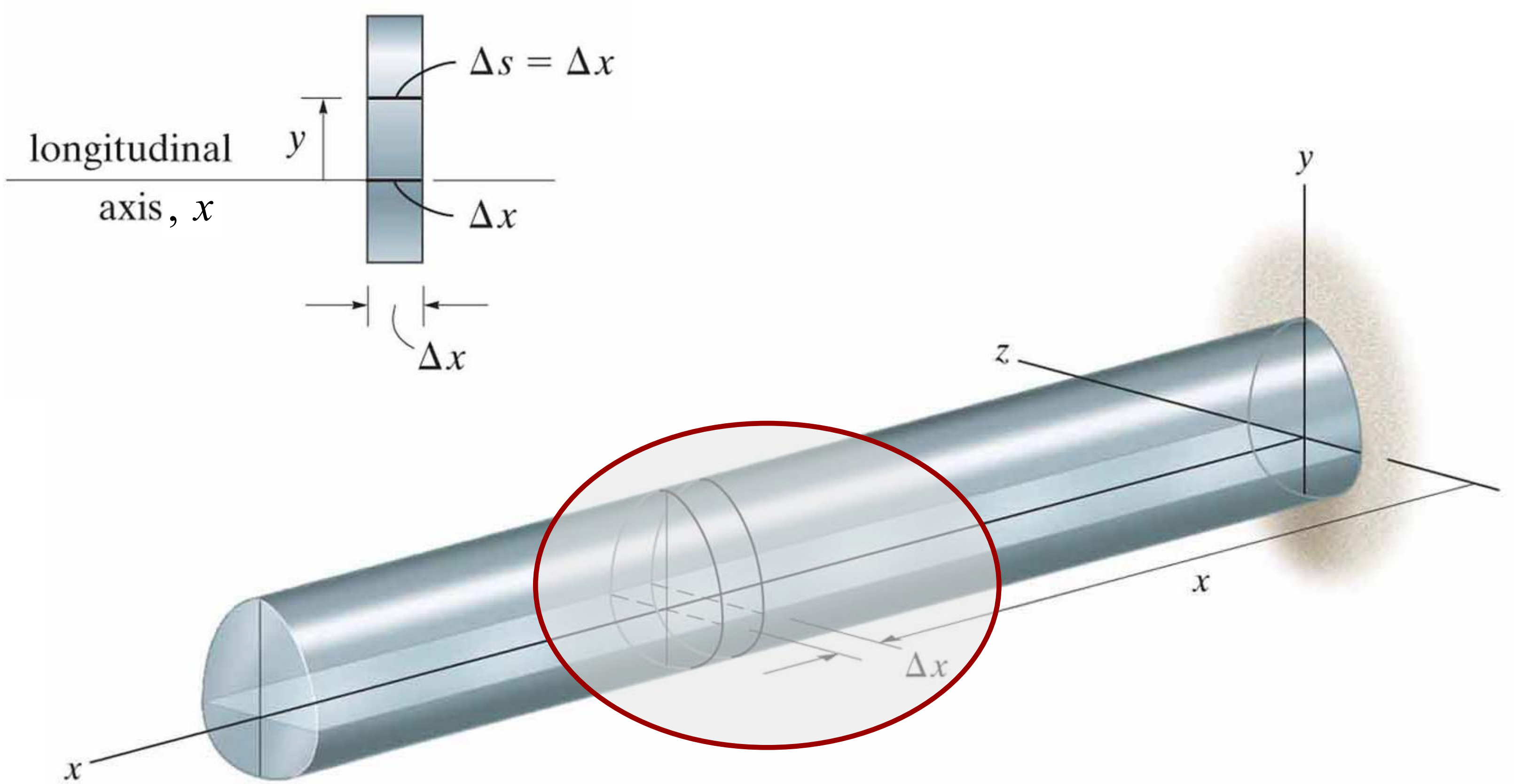


Shear and bending diagrams: example B

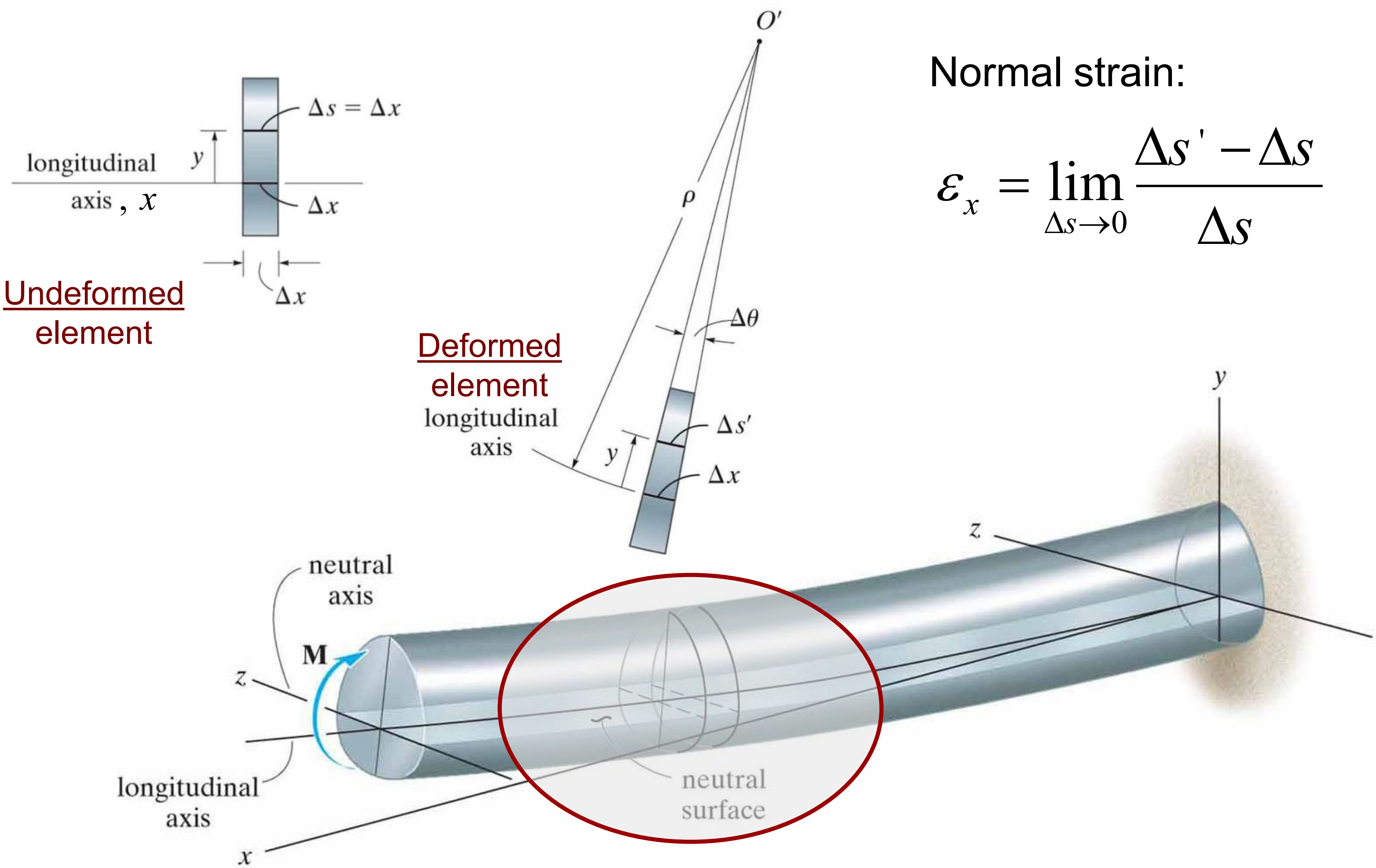
Determine the shear and moment diagrams for the beam shown



Bending deformation of straight beams



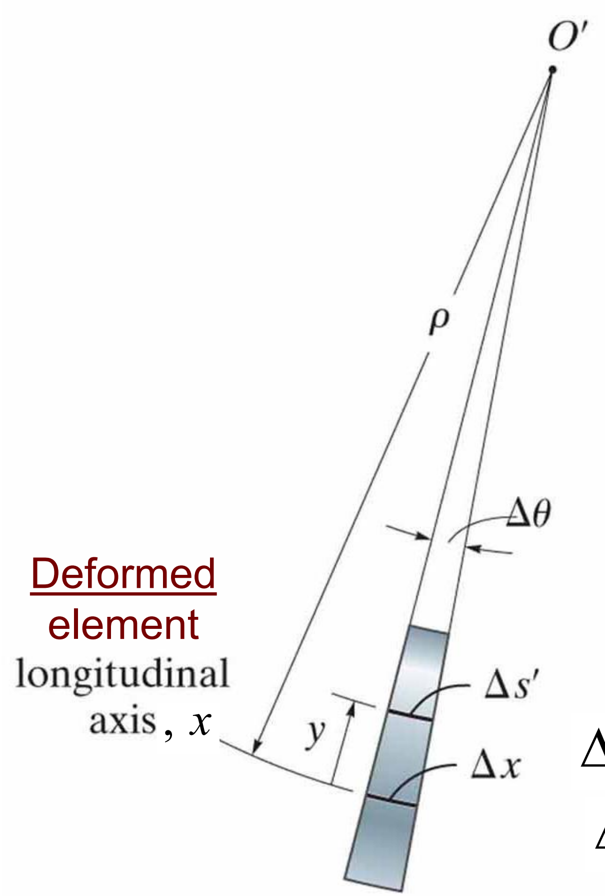
Bending deformation of straight beams



Normal strain:

$$\varepsilon_x = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

Bending deformation of straight beams



Normal strain:

$$\varepsilon_x = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

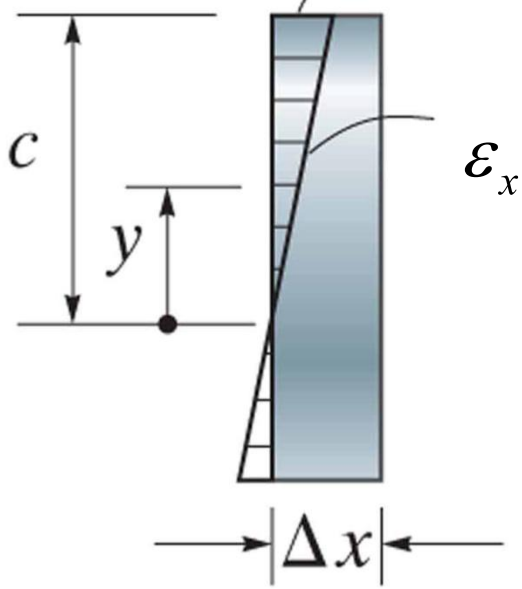
$$\varepsilon_x = \lim_{\Delta \theta \to 0} \frac{(\rho - y)\cdot\Delta\theta - \rho\cdot\Delta\theta}{\rho\cdot\Delta\theta}$$

$$\varepsilon_x = -\frac{y}{\rho}$$

$$\Delta s' = (\rho - y)\cdot\Delta\theta$$

$$\Delta x = \Delta s = \rho\,\Delta\theta$$

Bending deformation of straight beams



$$\varepsilon_{x\,Max} = -\frac{c}{\rho}$$

$$\varepsilon_{x} = -\frac{y}{\rho}$$

$$\frac{\varepsilon_x}{\varepsilon_{x\,Max}} = \frac{y/\rho}{c/\rho}$$

$$\varepsilon_x = \left(\frac{y}{c}\right)\cdot \varepsilon_{x\,Max}$$

The flexure formula

Hook's Law:

$$\sigma_x = E \cdot \varepsilon_x$$

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_{x\,Max} = \frac{\sigma_{x\,Max}}{E}$$

$$\frac{\varepsilon_x}{\varepsilon_{x\,Max}} = \frac{\sigma_x}{\sigma_{x\,Max}} \rightarrow \sigma_x = \left(\frac{y}{c}\right) \cdot \sigma_{x\,Max}$$

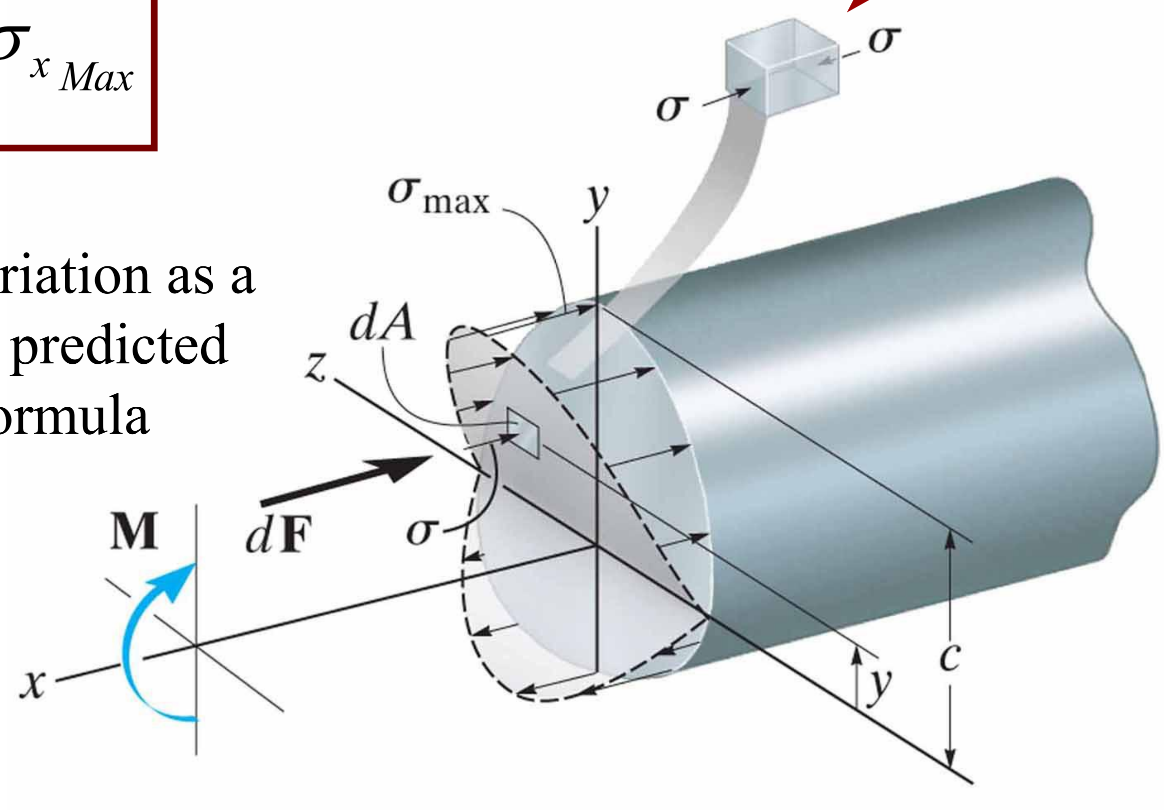
$$\varepsilon_x = -\frac{y}{\rho} \rightarrow \sigma_x = -E\frac{y}{\rho}, \quad \sigma_{x_{Max}} = -E\frac{c}{\rho}$$

The flexure formula

$$\sigma_x = \left(\frac{y}{c}\right) \cdot \sigma_{x\,Max}$$

Normal stress variation as a function of *y* as predicted by flexure formula

Differential stress element



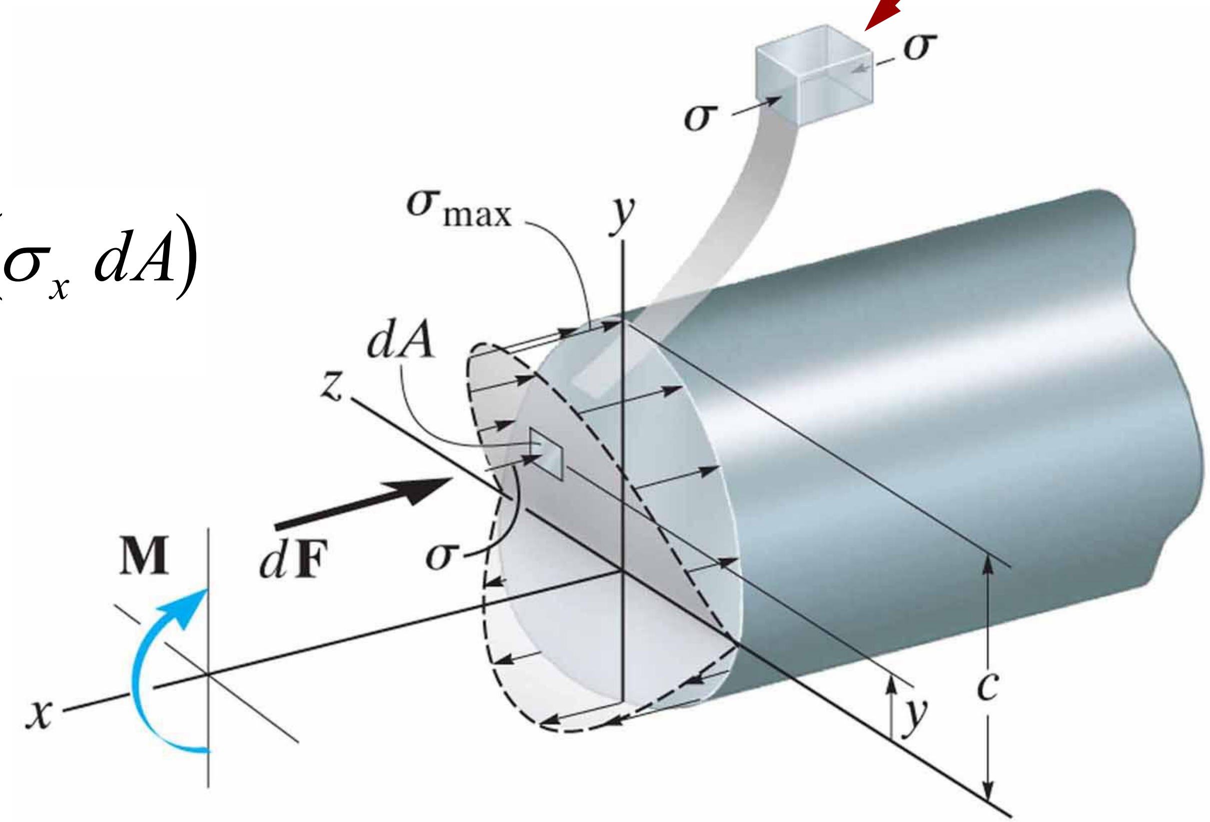
The flexure formula

Resultant internal moment:

$$M = \sum M_z$$

$$M = \int_A y\, dF = \int_A y\left(\sigma_x\, dA\right)$$

Differential stress element



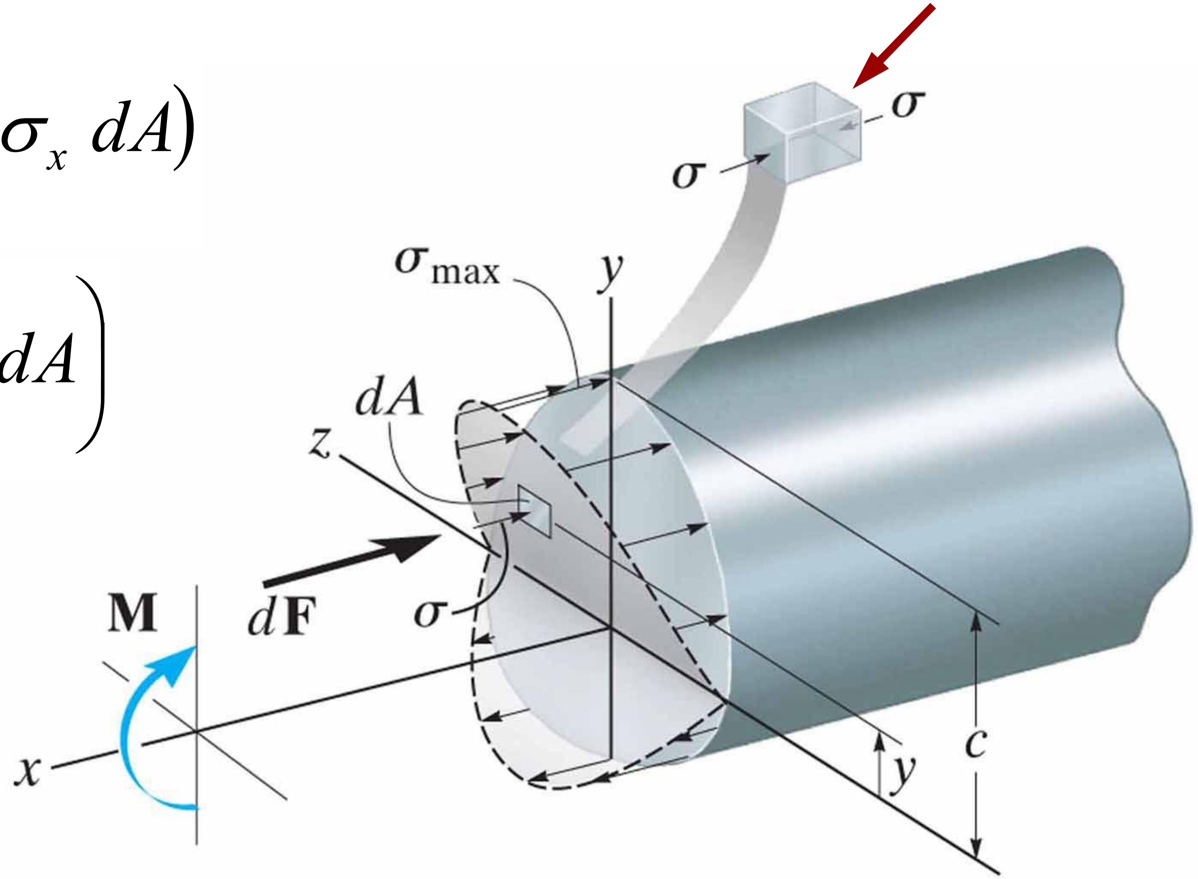
The flexure formula

Resultant internal moment:

$$M = \int_A y\, dF = \int_A y\,(\sigma_x\, dA)$$

$$= \int_A y\left(\frac{y}{c}\sigma_{x_{Max}}\, dA\right)$$

Differential stress element



The flexure formula

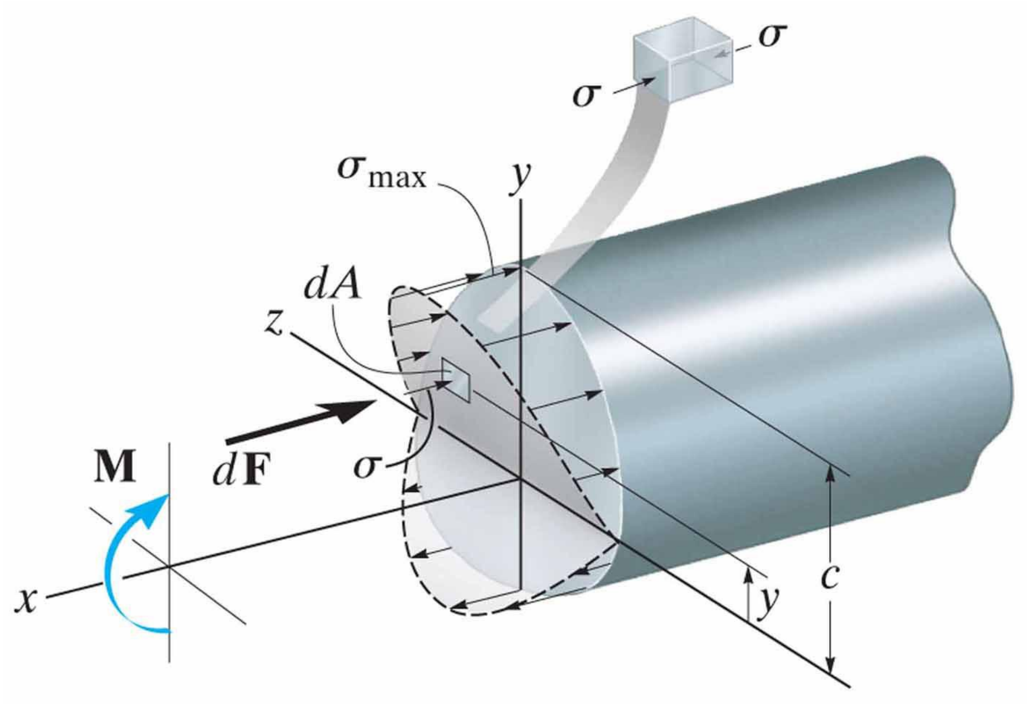
Resultant internal moment:

$$M = \frac{\sigma_{x_{Max}}}{c} \int_A y^2 \, dA$$

$$\boxed{\sigma_{x_{Max}} = \frac{M\,c}{I_{zz}}}$$

$$I_{zz} = \int_A y^2 \, dA$$

Area moment of inertia *wrt* to *z*-axis



The flexure formula

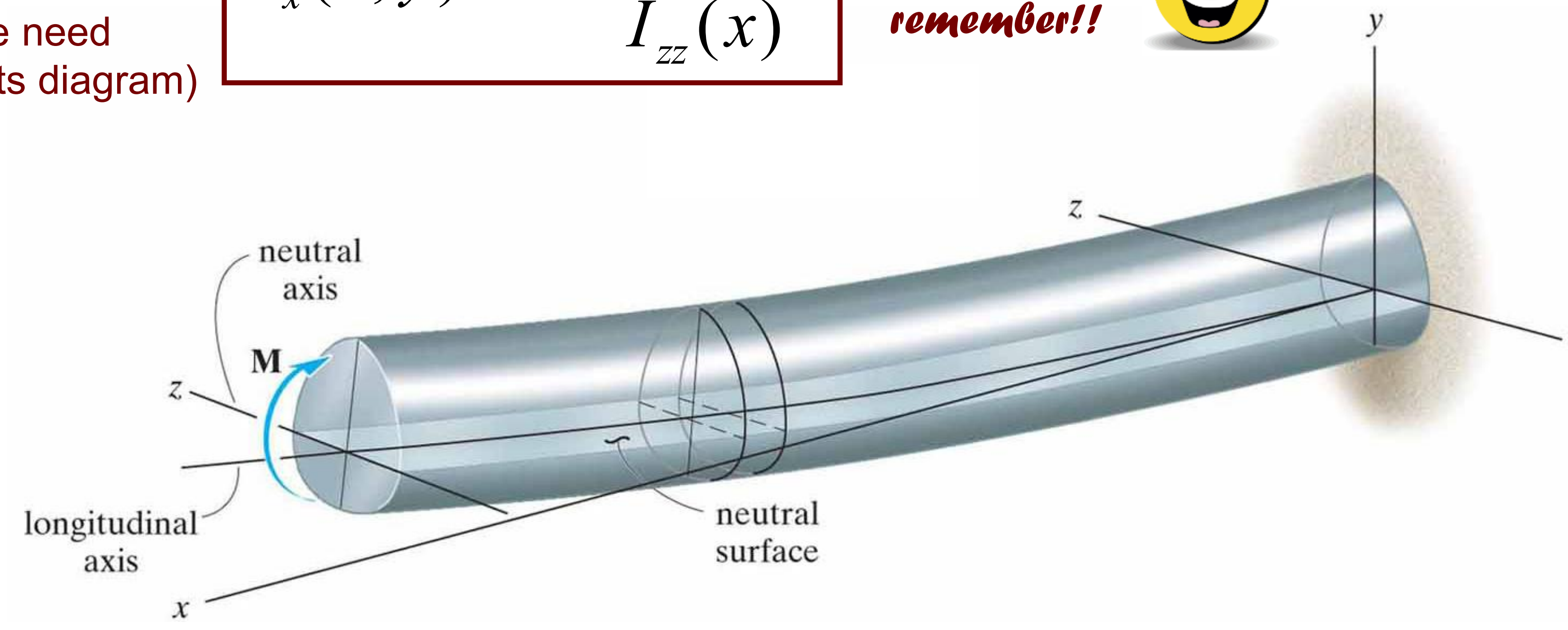
$$\sigma_{x\,Max} = \frac{M\ c}{I_{zz}}$$

$$\sigma_x = -\frac{M\ y}{I_{zz}}$$

Do note that:
(we need moments diagram)

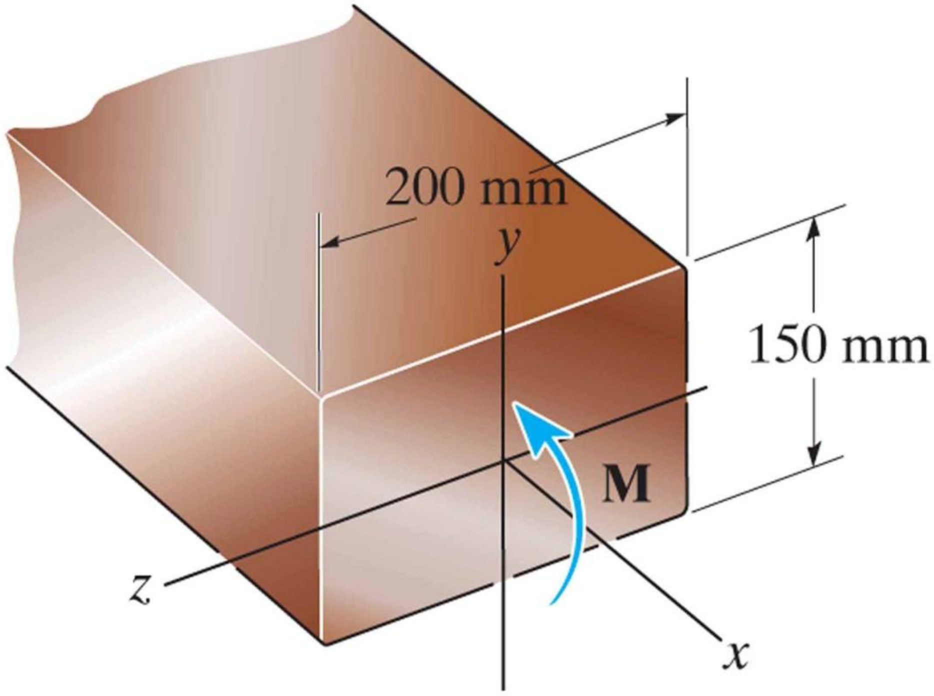
$$\sigma_x(x, y) = -\frac{M(x) \cdot y}{I_{zz}(x)}$$

Important to remember!!



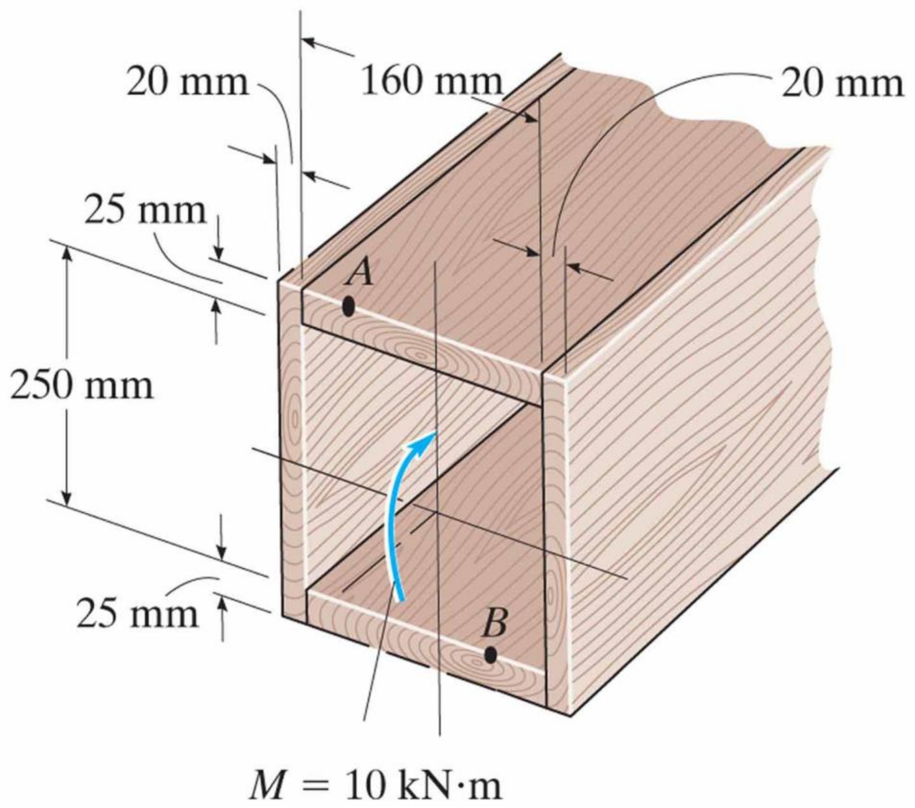
Shear and bending diagrams: example C

A member having the dimensions shown is used to resist an internal bending moment of $M = 90\ \mathrm{kN{\cdot}m}$. Determine the maximum stress in the member if the moment is applied (a) about the z-axis (as shown); and (b) about the y-axis. Sketch the stress distribution for each case.



Shear and bending diagrams: example D

A box beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross-section is 10 kN·m, determine the stress at points A and B and show the results acting on volume elements located at these points.



Reading assignment

- Chapter 6 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course