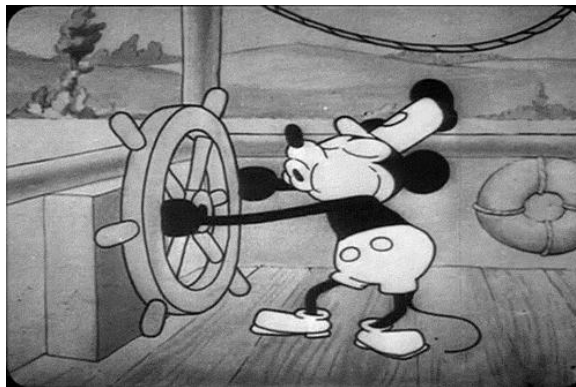


# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS ES-2502, B'2025

**We will get started soon...**



**25 November 2025**



# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

## STRESS ANALYSIS ES-2502, B'2025

Lecture 20:  
Unit 15, 16: Bending of beams::  
*MV diagrams & MV general relationship*

25 November 2025



# General information

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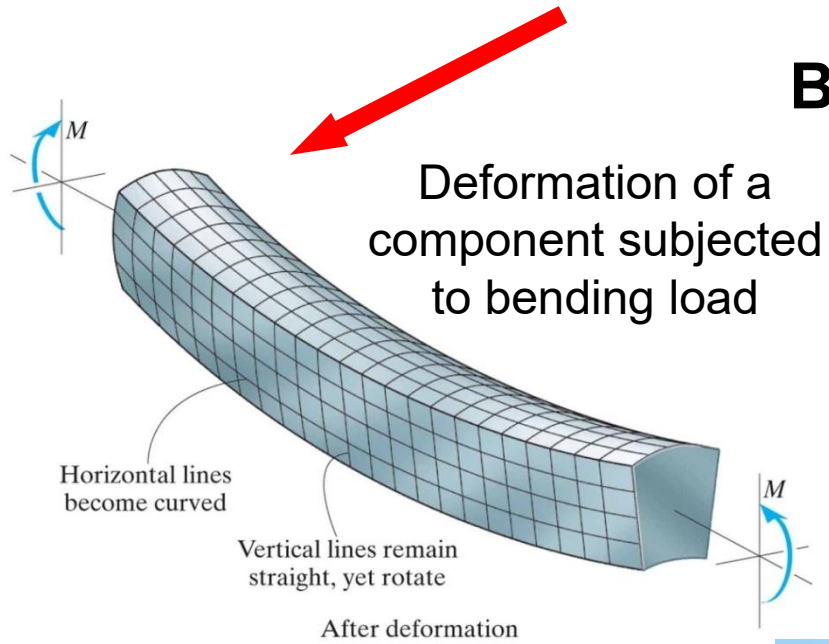
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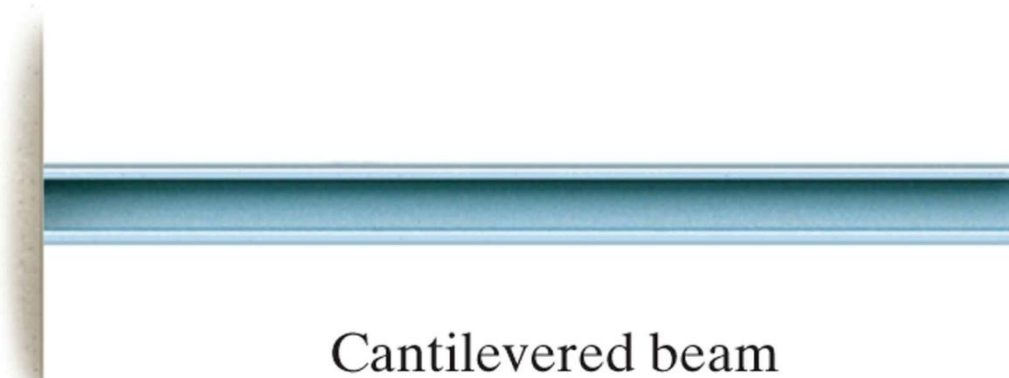
# Bending



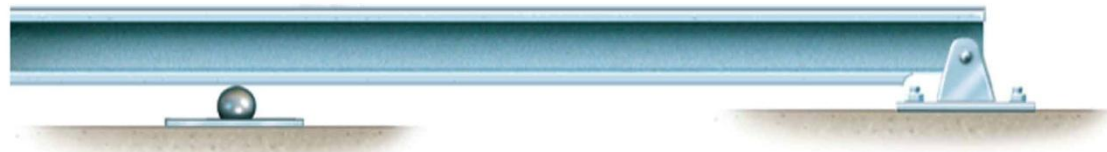
# Bending: many real components are modeled as “beams”



Simply supported beam



Cantilevered beam

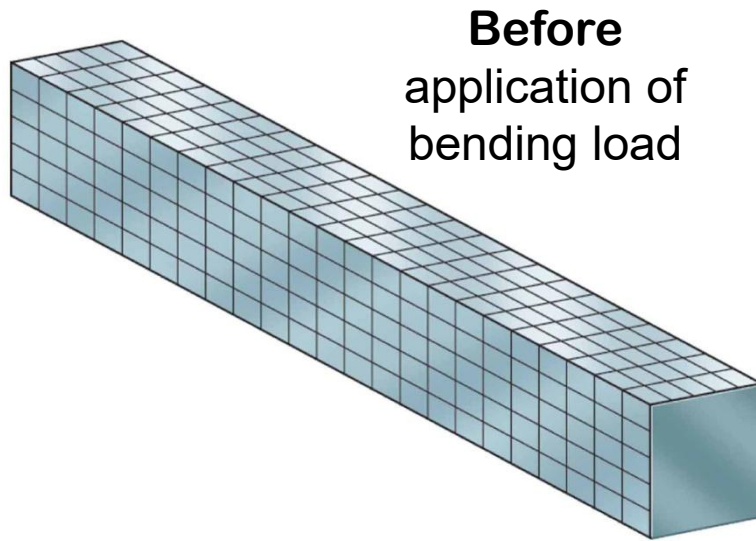


Overhanging beam



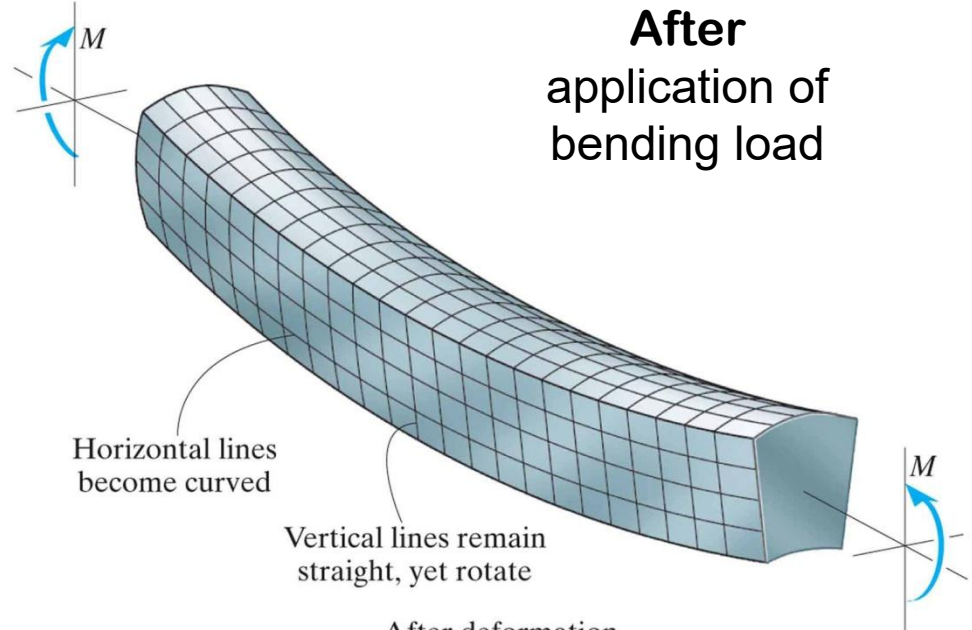


# Bending



**Before**  
application of  
bending load

Before deformation

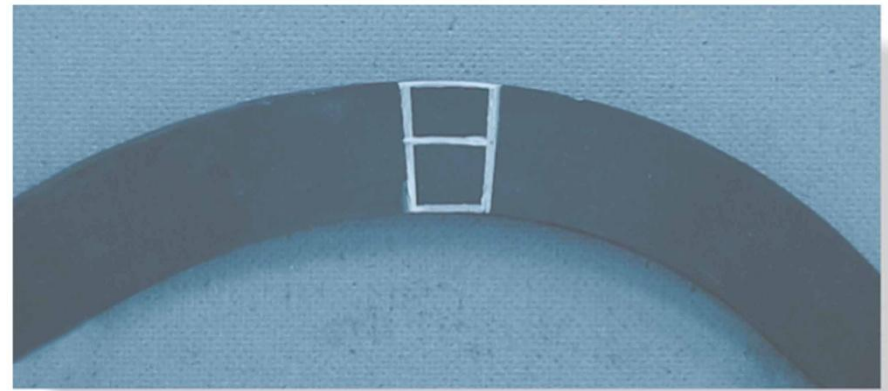
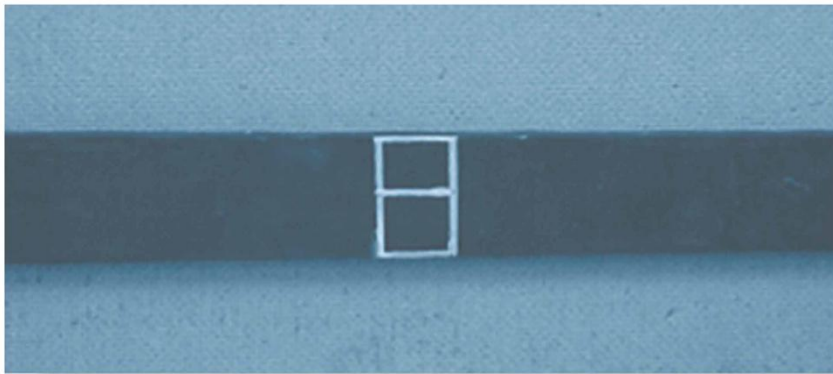


**After**  
application of  
bending load

Horizontal lines  
become curved

Vertical lines remain  
straight, yet rotate

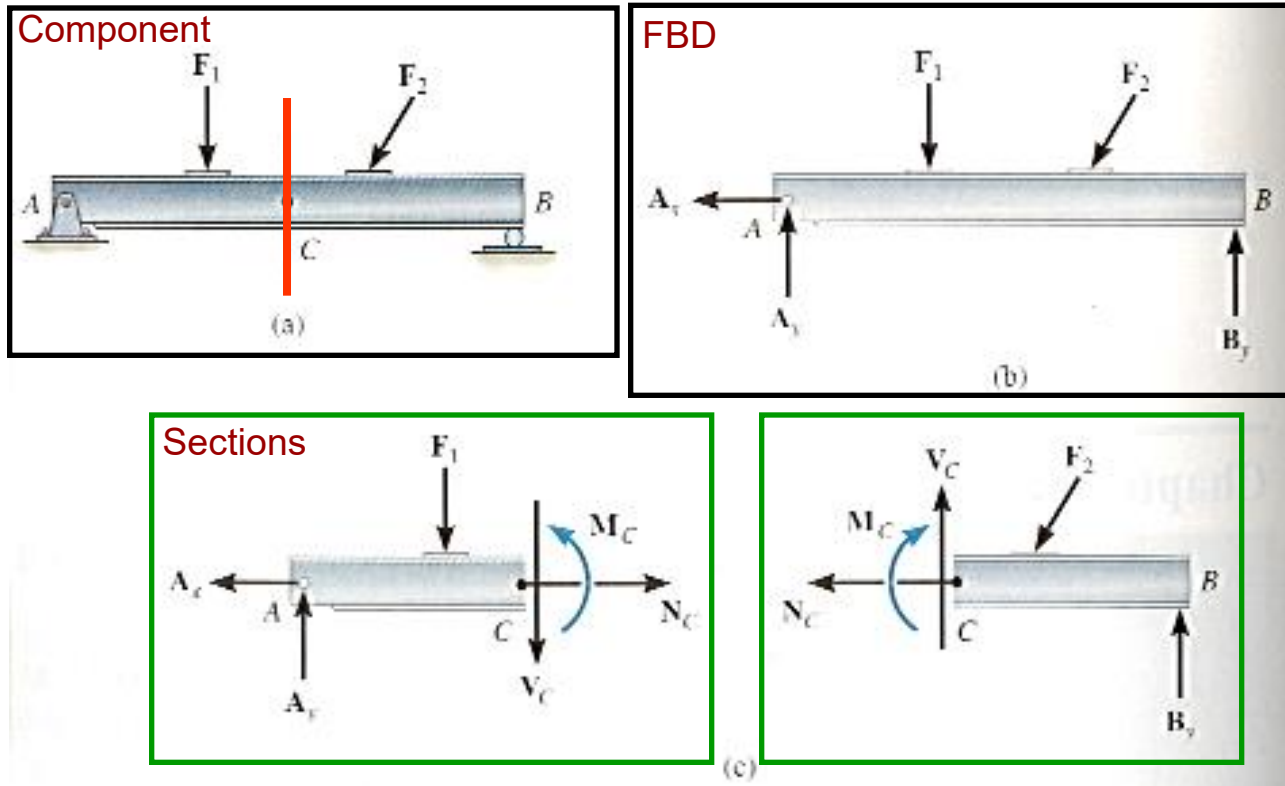
After deformation



# Internal forces and moments

## Shear and bending moments

### Internal forces (determination of shear and moment diagrams)

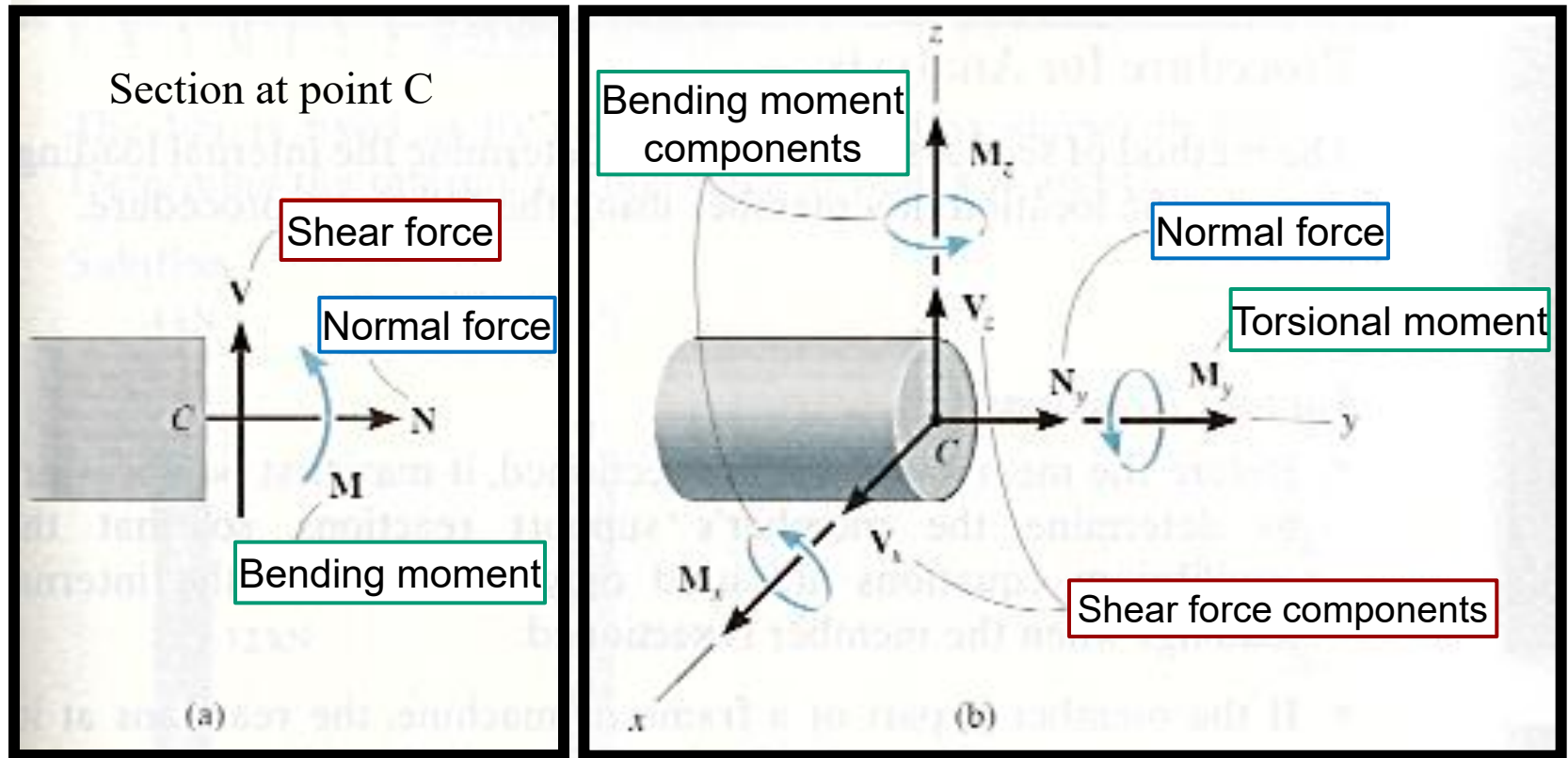


Internal: moments, shear, and normal forces at point C



# Internal forces and moments

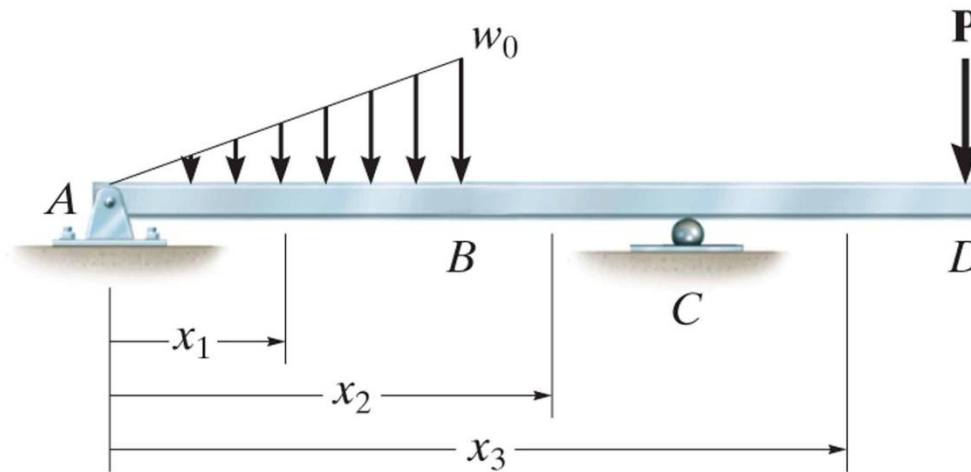
## Shear and bending moments



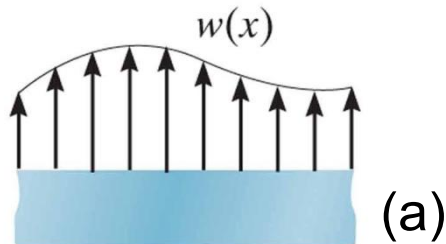


# Shear and bending diagrams

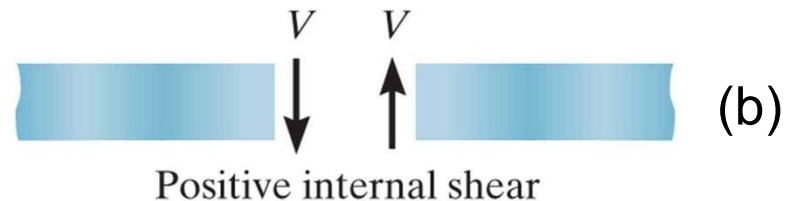
Diagrams are determined for *each region* of the beam *between* any two discontinuities of loading



## Beam sign convention



Positive external distributed load



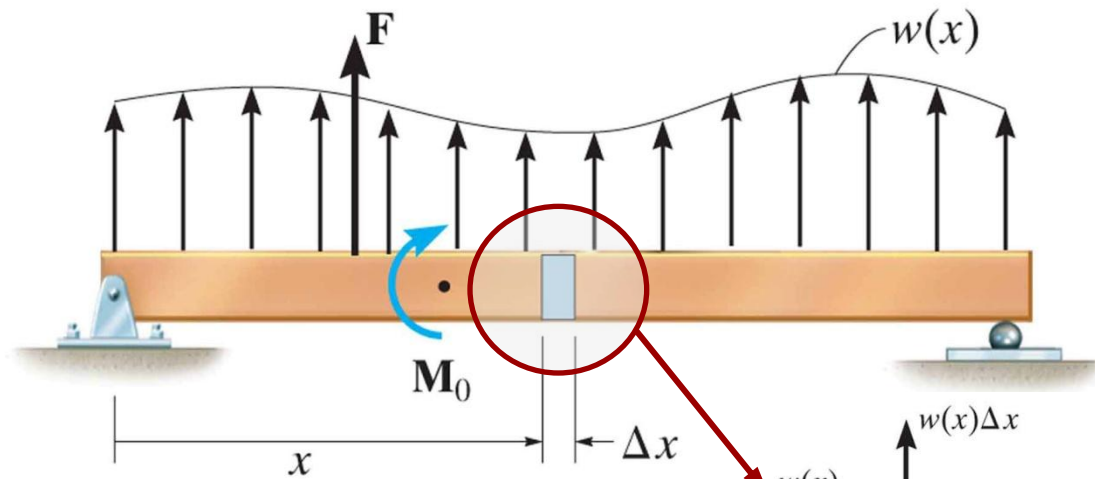
Positive internal shear



Positive internal moment



# Shear and bending diagrams: regions with distributed load



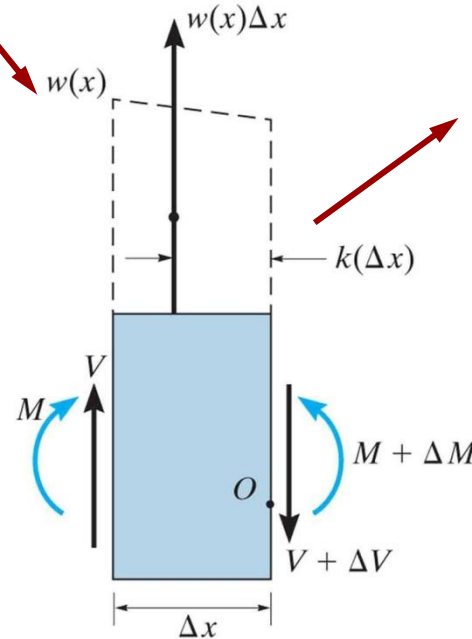
**Important to remember!!**



$$\frac{dV}{dx} = w(x);$$

$$\frac{dM}{dx} = V(x)$$

Free body diagram of element  $\Delta x$ :



Free-body diagram of segment  $\Delta x$



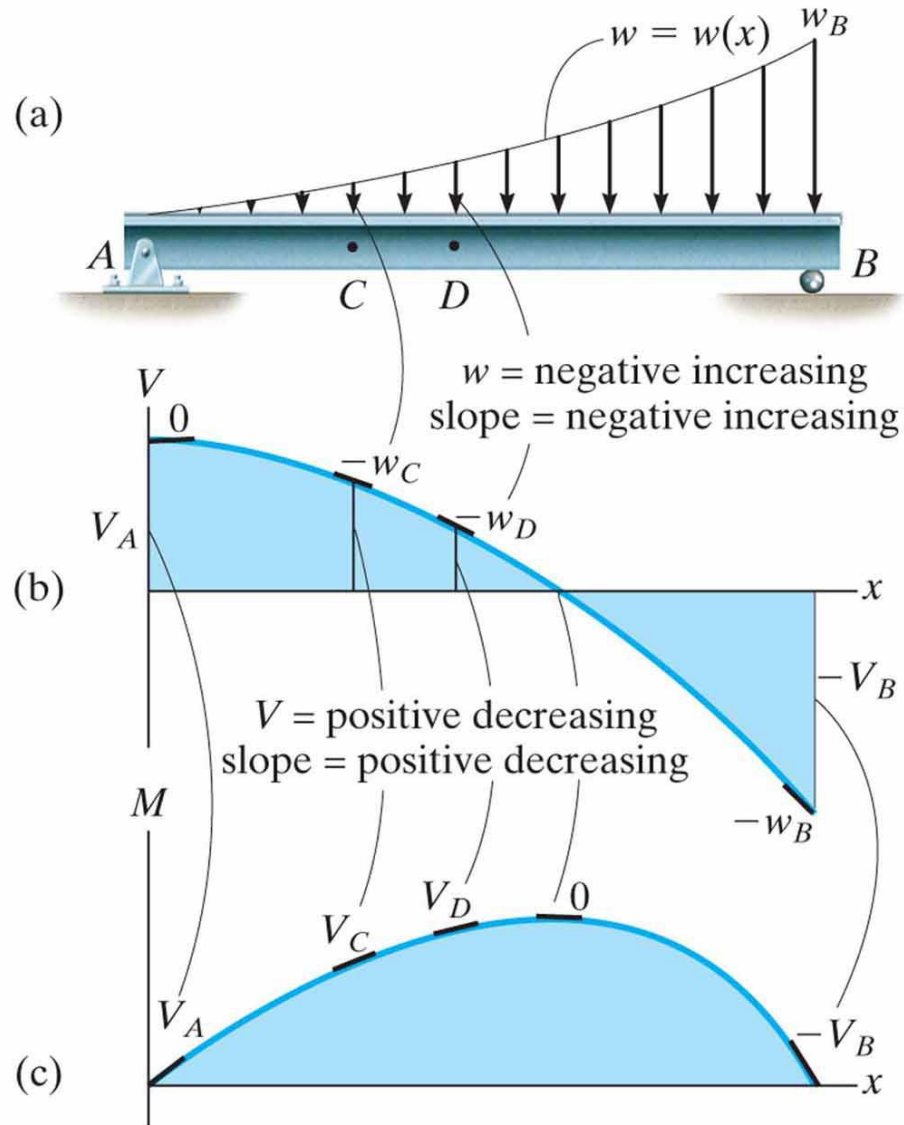
# Shear and bending diagrams: regions with distributed load

**Important to remember!!**

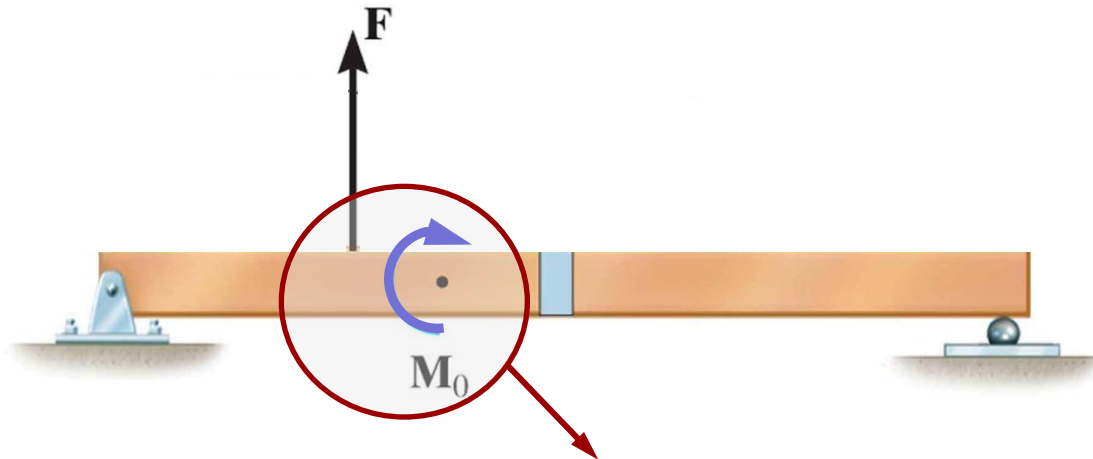


$$\frac{dV}{dx} = w(x);$$

$$\frac{dM}{dx} = V(x)$$



# Shear and bending diagrams: regions with concentrated force and moment

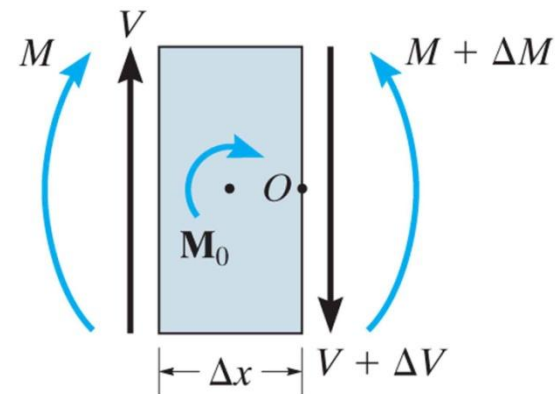
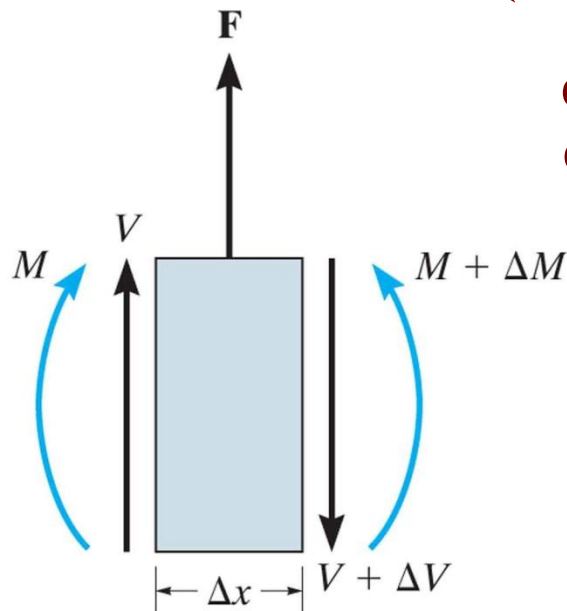


*Important to  
remember!!*



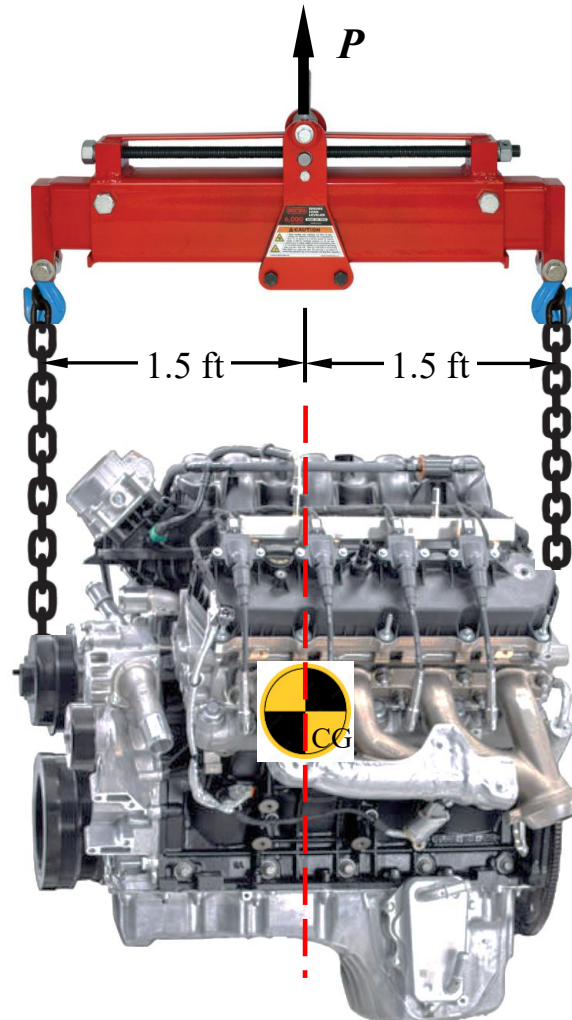
$$\Delta V = F ;$$
$$\Delta M = M_0$$

Free body  
diagrams of  
element  $\Delta x$ :



# Shear and bending diagrams: example A

A suspended bar supports a 600-lb engine. Plot the shear and moment diagrams for the bar





# Shear and bending-moment diagrams

## Method of sections: *plot using step functions + MathCad*

### S-M Diagrams – Bar: ES 2502

A suspended bar supports a 600-lb engine.  
Plot the shear and moment diagrams for the bar.

Input:

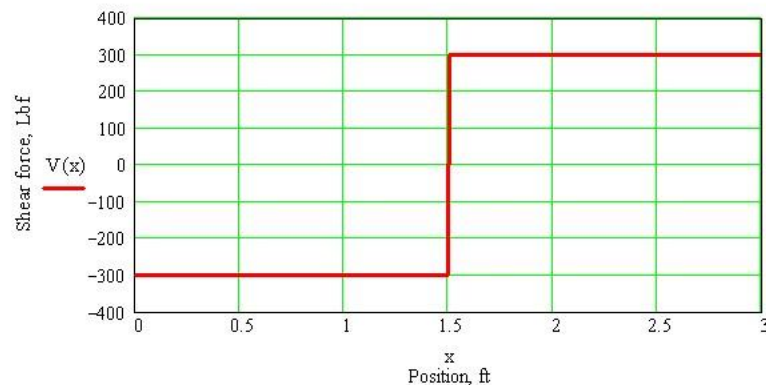
```
L := 3  
a := 1.5  
x := 0, 0.001..L
```

Define unit step function:

```
S(x,z) := if(x ≥ z, 1, 0)
```

Define shear function:

```
V(x) := -300·S(x, 0) + 600·S(x, 1.5)
```

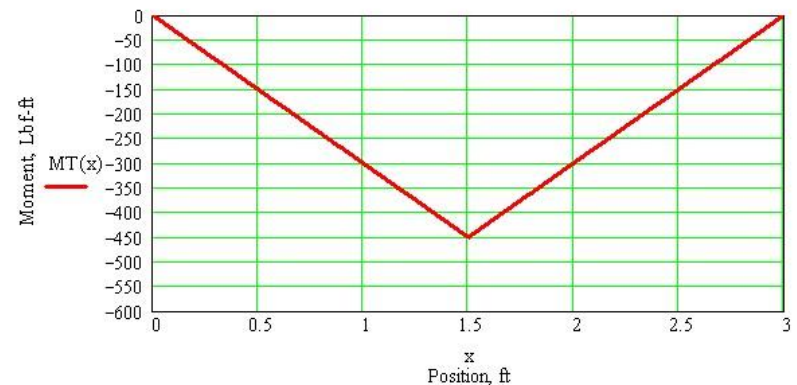


Define moments function:

```
M1(x) := -300·x
```

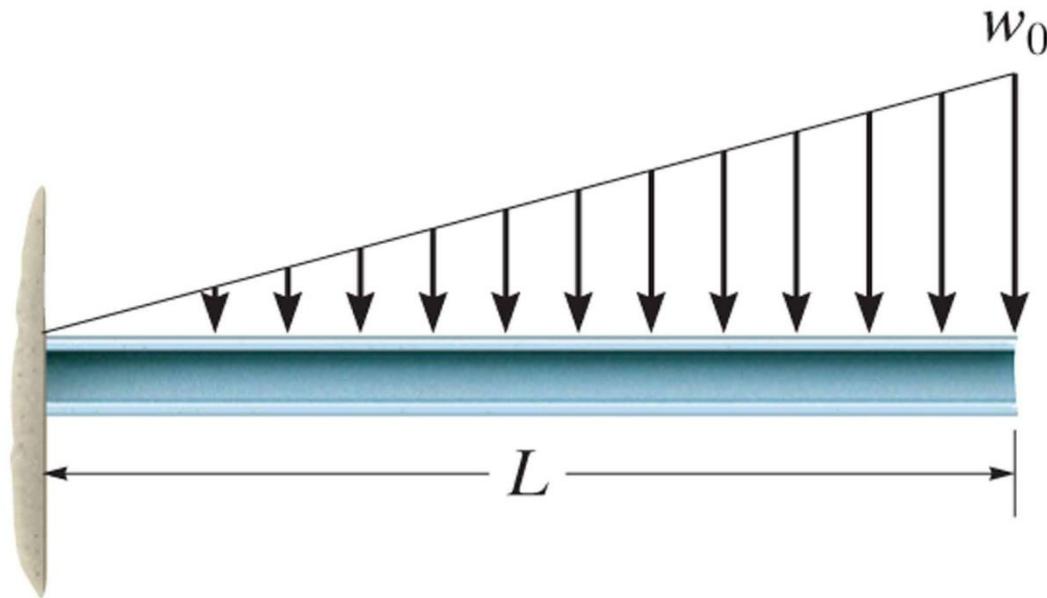
```
M2(x) := 300·x - 900
```

```
MT(x) := S(x, 0)·M1(x) - S(x, 1.5)·M1(x) + S(x, 1.5)·M2(x)
```



## Shear and bending diagrams: example B

Determine the shear and moment diagrams for the beam shown



# Reading assignment

- Chapter 6 of textbook
- Review notes and text: ES2001, ES2501



# Homework assignment

- As indicated on webpage of our course

