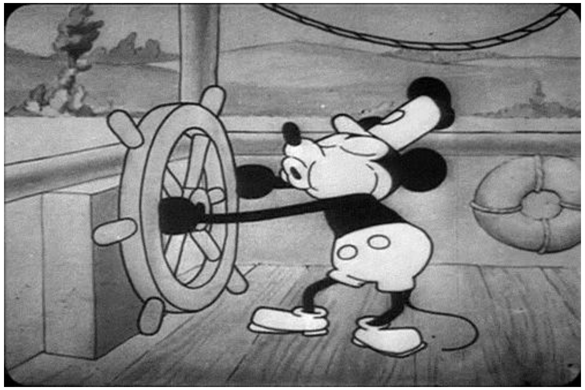
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



20/24 November 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

Lecture 19:
Unit 12, 13: Torsion of shafts:
circular cross-section:
statically indeterminate & stress concentrations

20/24 November 2025

General information

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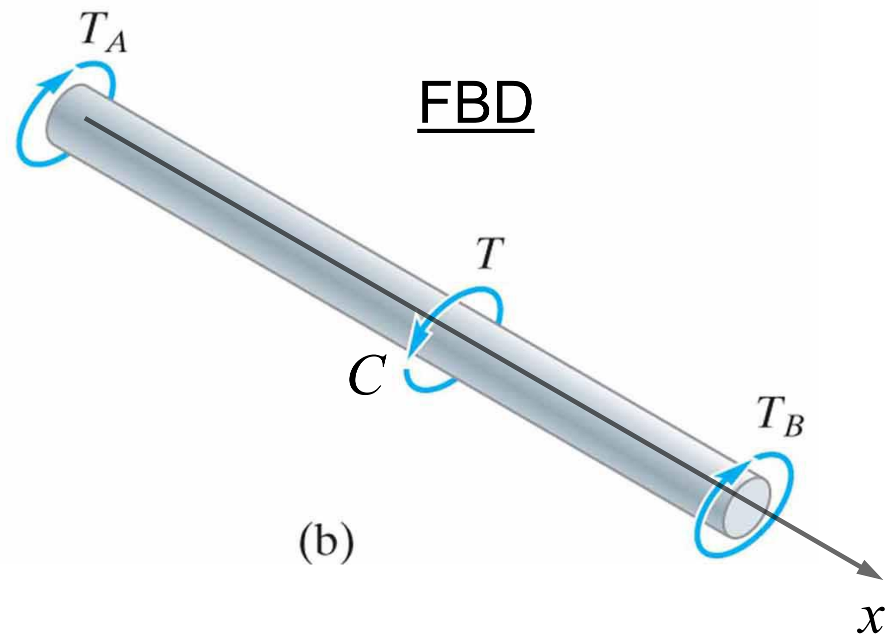
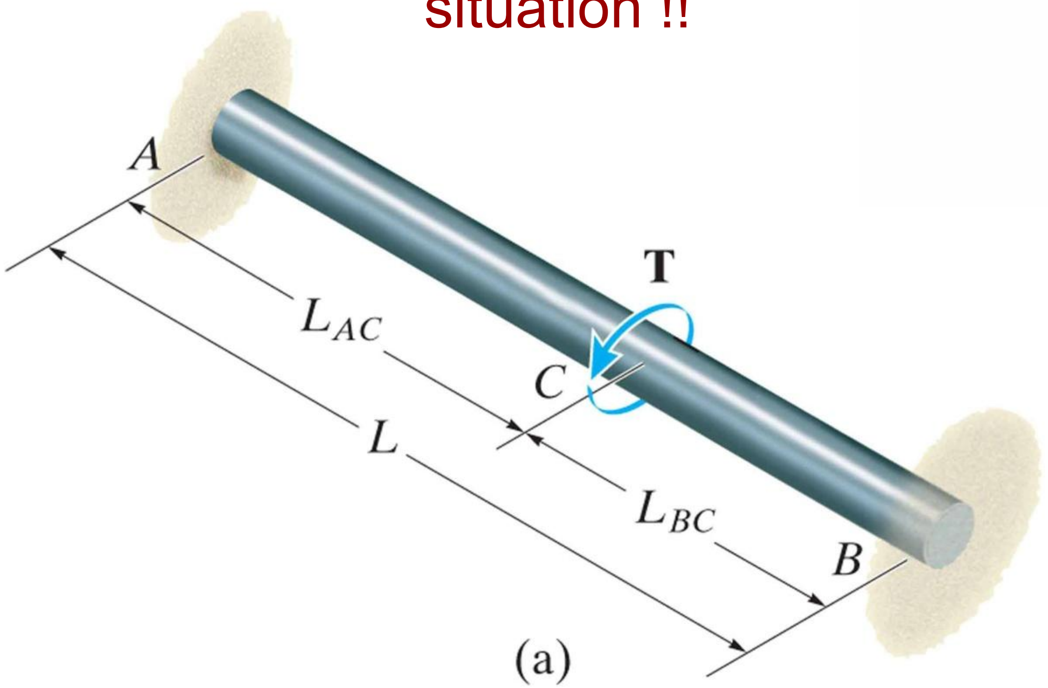
Statically indeterminate torque-loaded members

$$\sum M_x = 0$$

$$T - T_A - T_B = 0$$

Statically indeterminate situation !!

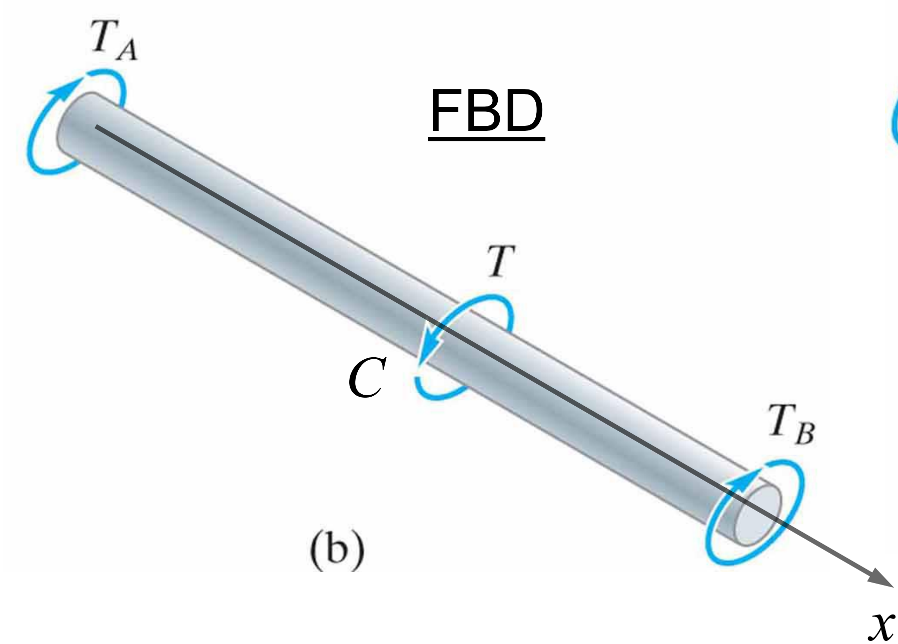


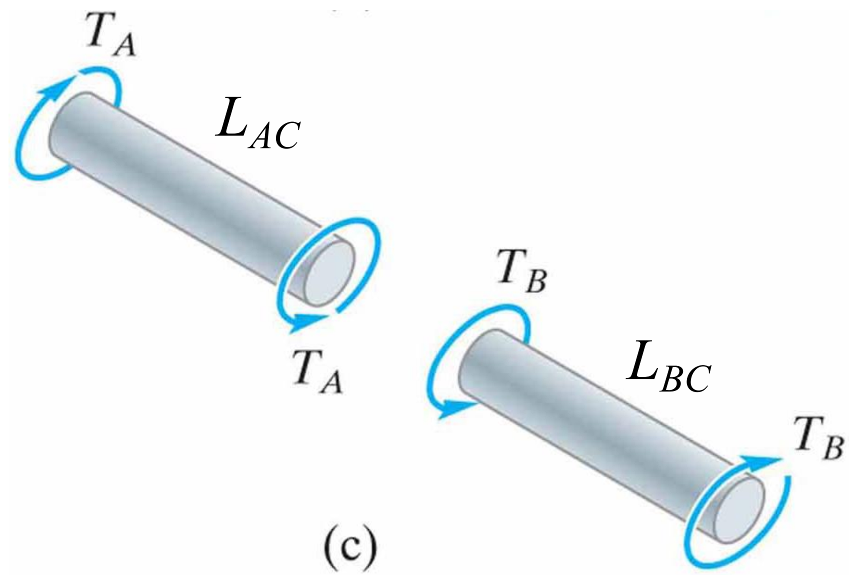


Statically indeterminate torque-loaded members

<u>Compatibility</u> equation:
(Same angle of twist)

$$\frac{T_A\, L_{AC}}{J\, G} = \frac{T_B\, L_{BC}}{J\, G}$$



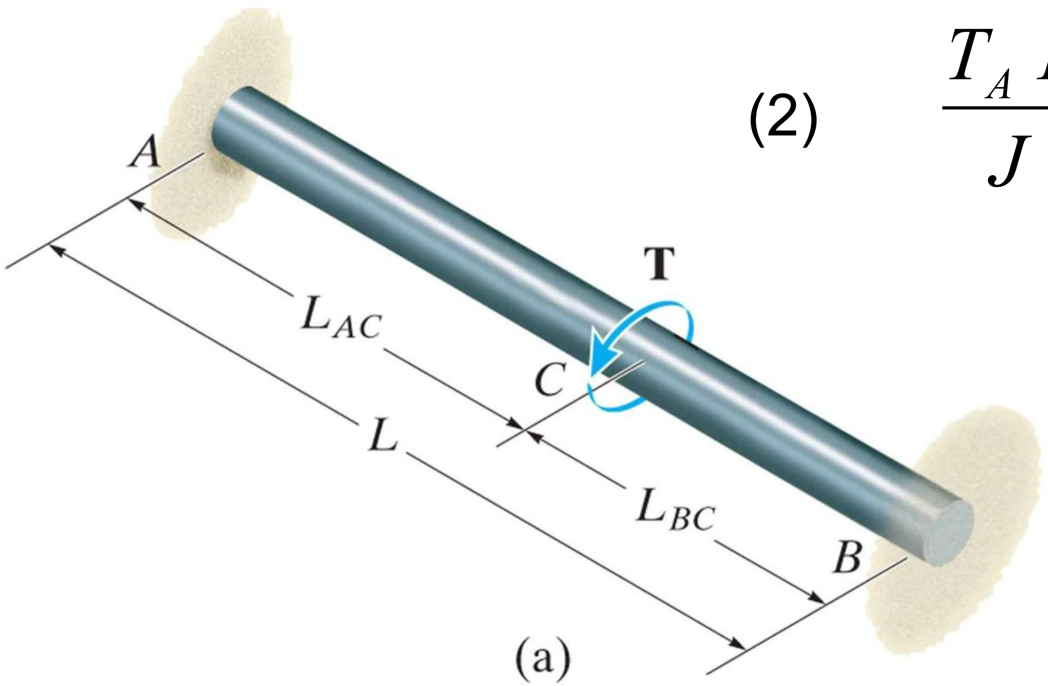


Statically indeterminate torque-loaded members

Solution with: two equations for two unknowns (T_A and T_B):

(1) $$T - T_A - T_B = 0$$

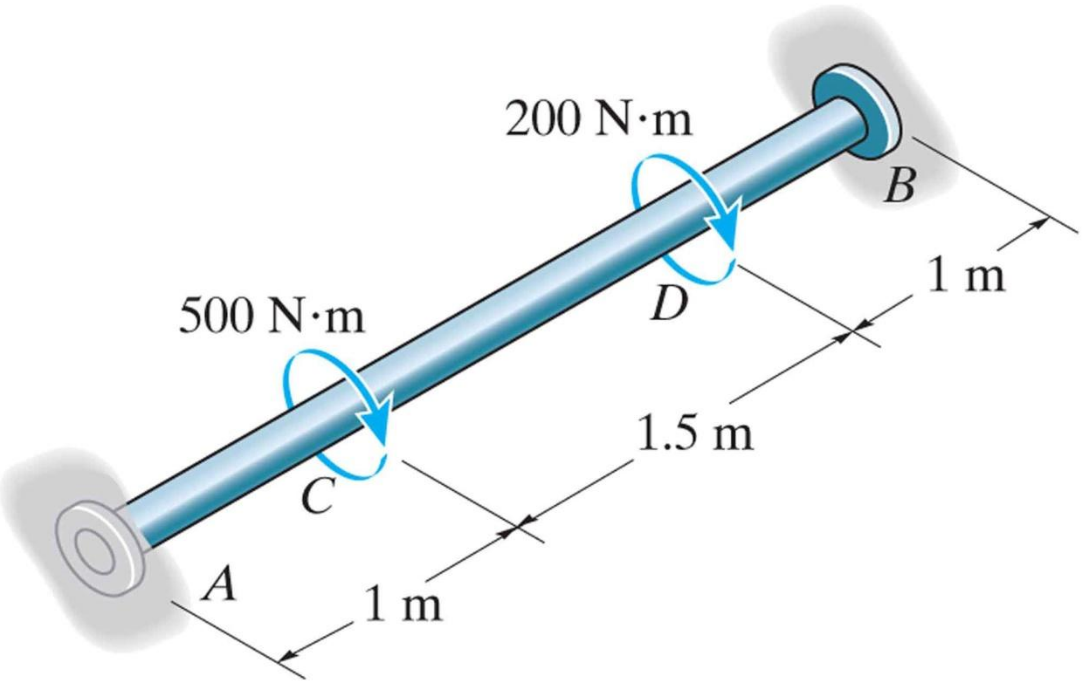
(2) $$\frac{T_A\, L_{AC}}{J\, G} - \frac{T_B\, L_{BC}}{J\, G} = 0$$



(a)

Statically indeterminate torque-loaded members: example A

The A-36 steel shaft has a diameter of $60\ \text{mm}$ and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft

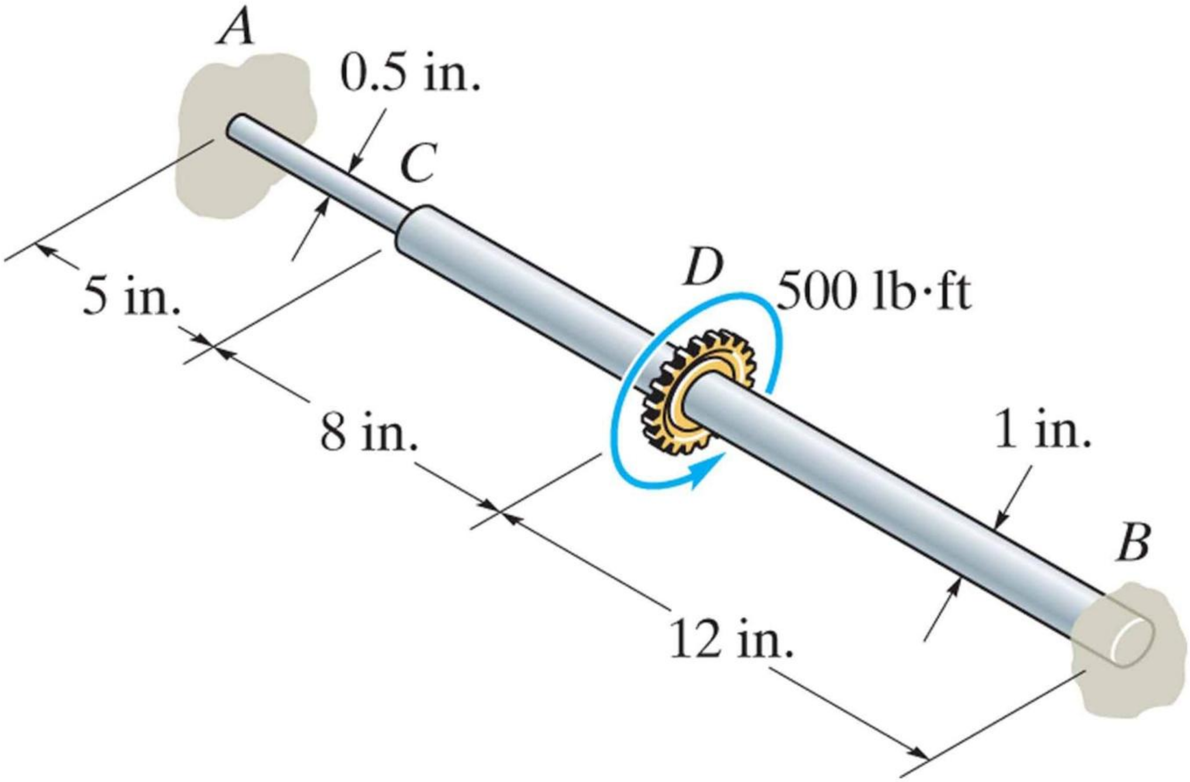


Approach:

1) Apply equilibrium equations
2) Apply compatibility equations (*two possible methods*)
3) Solve for stresses

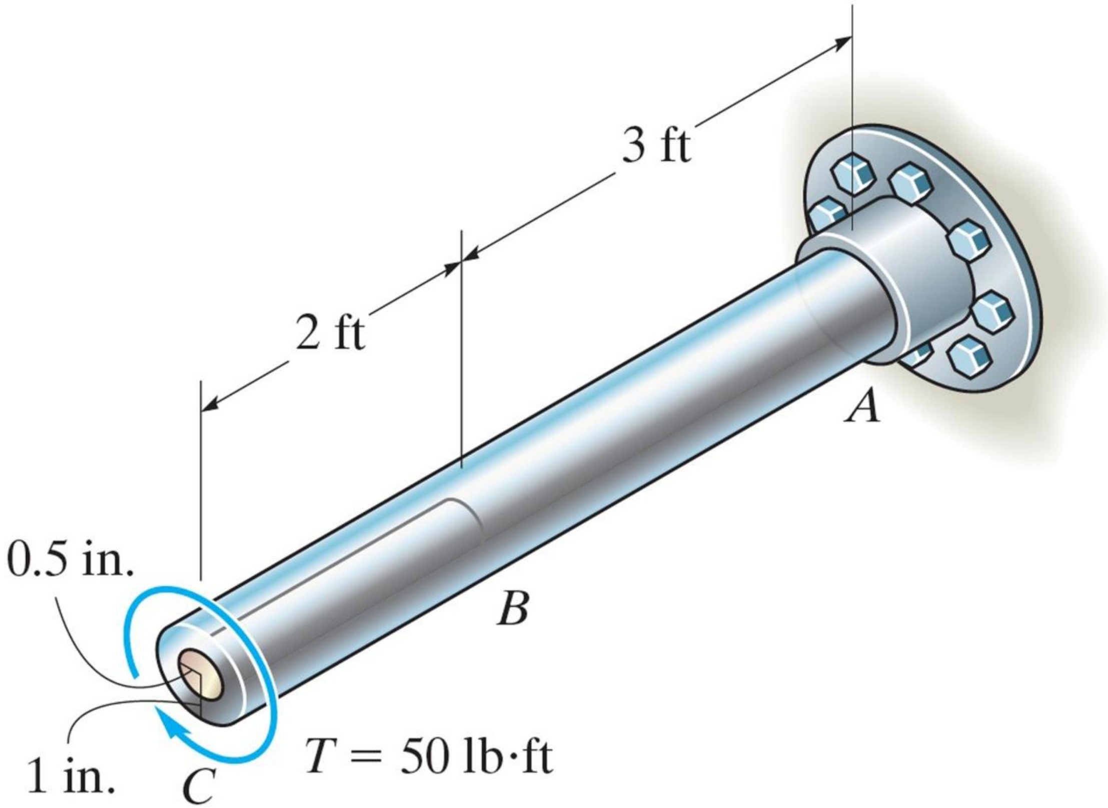
Statically indeterminate torque-loaded members: example B

The steel shaft is made from two segments: AC has a diameter of $0.5\ \text{in}$, and CB has a diameter of $1\ \text{in}$. If it is fixed at its ends A and B and subjected to a torque of $500\ \text{lb·in}$, determine the maximum shear stress in the shaft. Use $G_{\text{steel}} = 10.8\ \text{Mpsi}$.



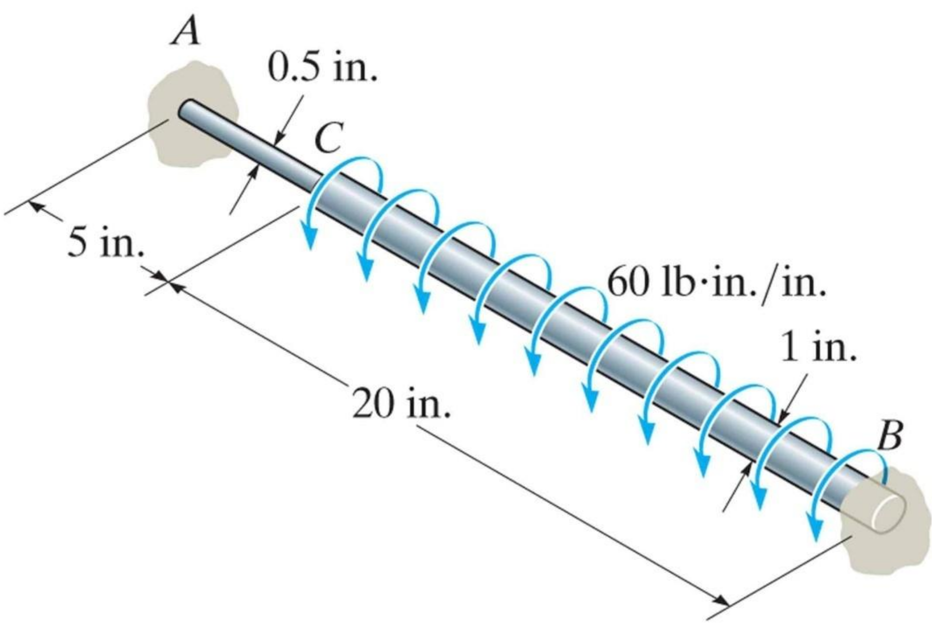
Statically indeterminate torque-loaded members: example C

The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50$ lb·ft is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Use $G_{steel} = 11.5$ Mpsi, $G_{brass} = 5.6$ Mpsi.



Statically indeterminate torque-loaded members: example D

The A-36 steel shaft is made from two segments: AC has a diameter of $0.5\ \text{in}$ and CB has a diameter of $1\ \text{in}$. If the shaft is fixed at its ends A and B and subjected to a uniform distributed torque of $60\ \text{lb}_f\cdot\text{in/in}$ along segment CB, determine the absolute maximum shear stress in the shaft

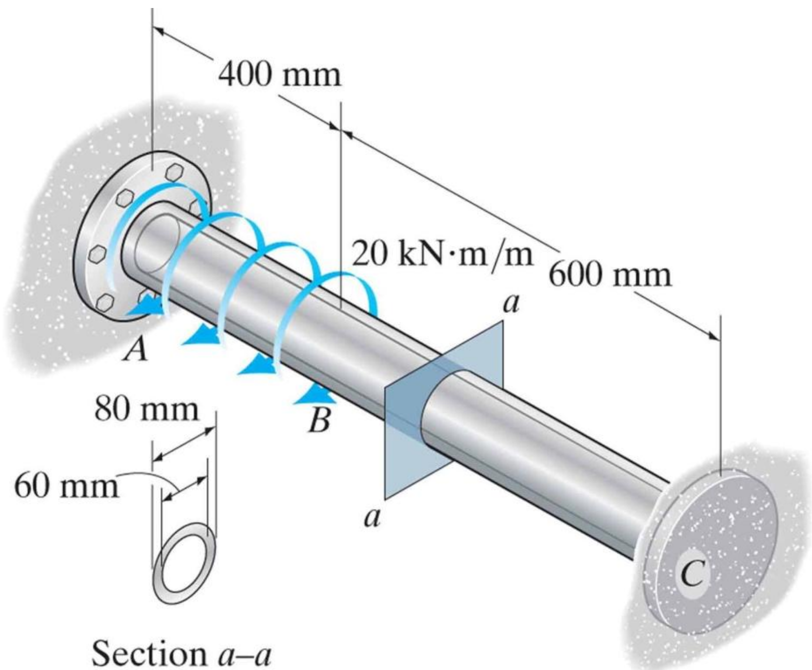


Approach:

1) Apply equilibrium equations
2) Apply compatibility equations
3) Solve for stresses

Statically indeterminate torque-loaded members: example E

If the shaft is subjected to a uniform distributed torque of $20\ \mathrm{kN{\cdot}m/m}$, determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is **fully** fixed at A and C.

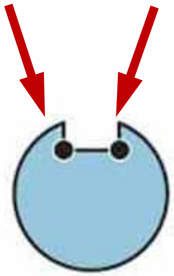


Approach:

1) Apply equilibrium equations
2) Apply compatibility equations
3) Solve for max. stress

Torque-loaded members: stress concentration

Locations producing stress concentrations



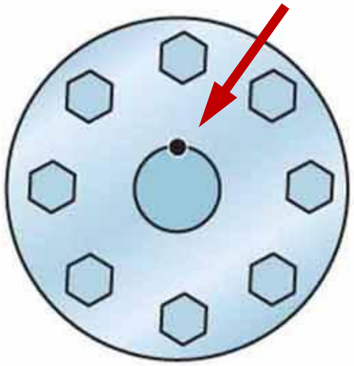
Keys and keyways

Failed component

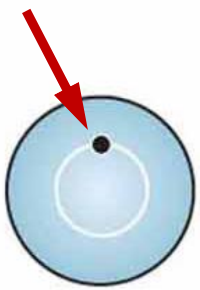
Torque-loaded members: stress concentration

Welded components

Locations with stress concentrations



Locations with stress concentrations



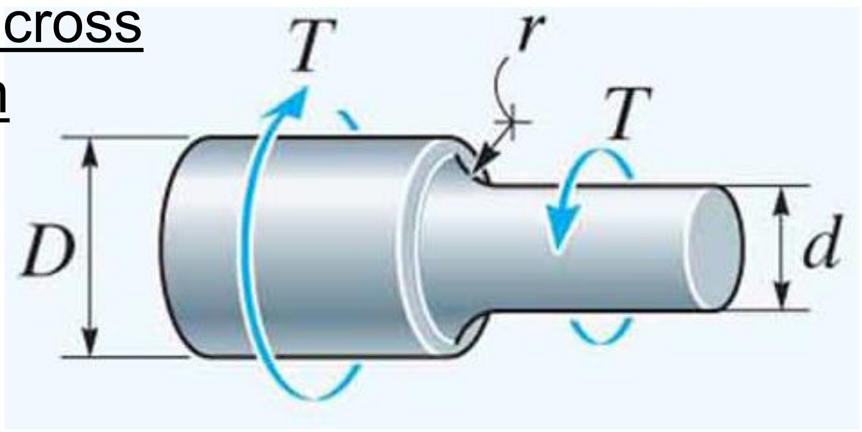
Changes in cross section

Torque-loaded members: stress concentration

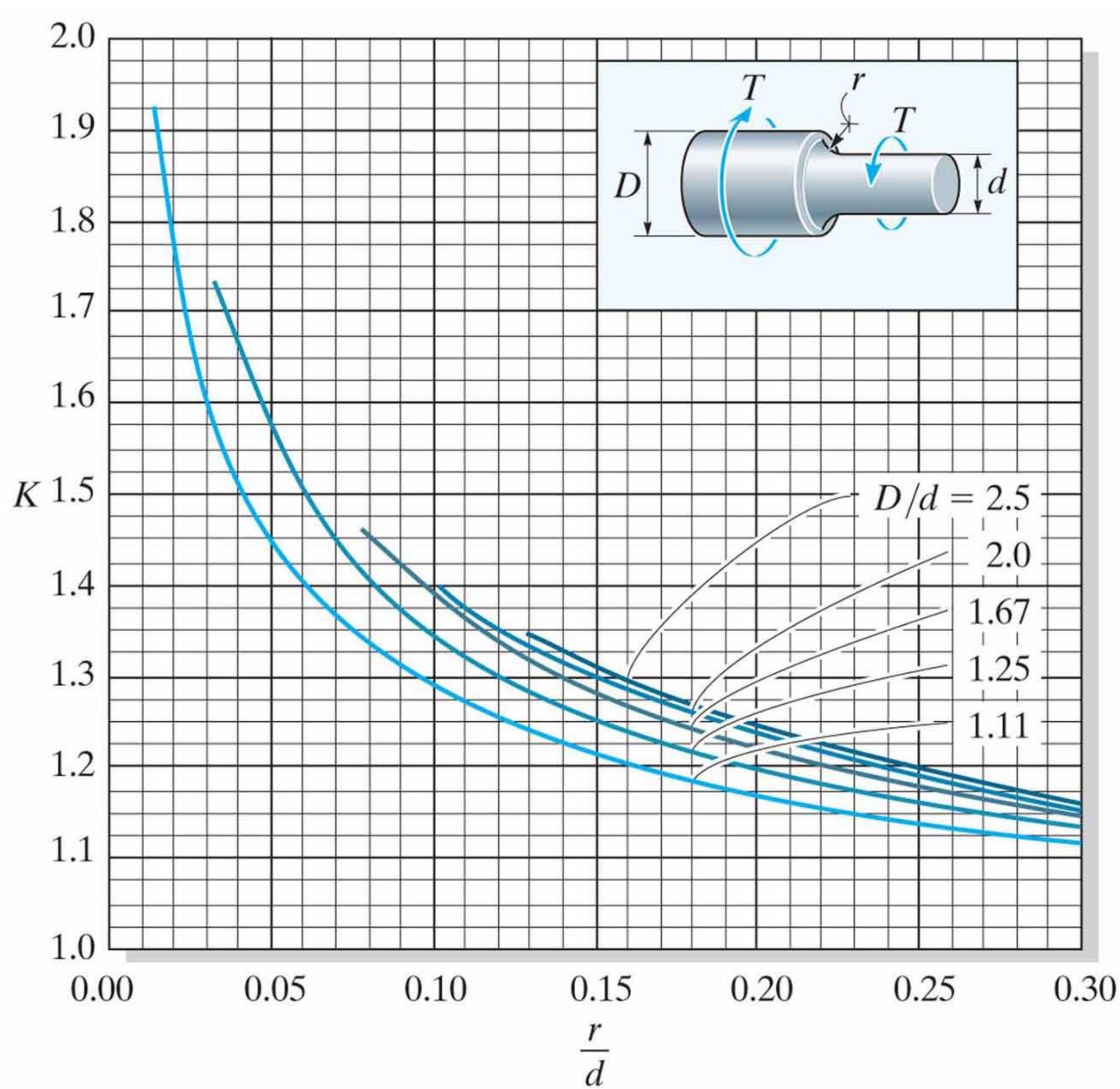
$$(\tau_{\max})_{amplified} = K\tau_{\max} = K\frac{Tc}{J}$$

Stress concentration factor: K
(amplification factor)

Changes in cross section

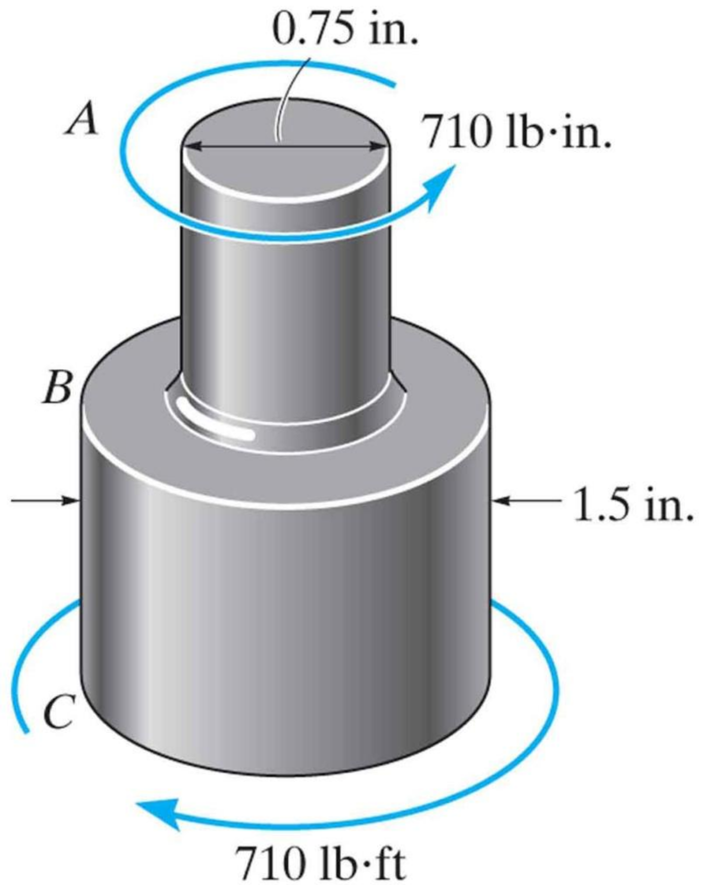


Torque-loaded members: stress concentration

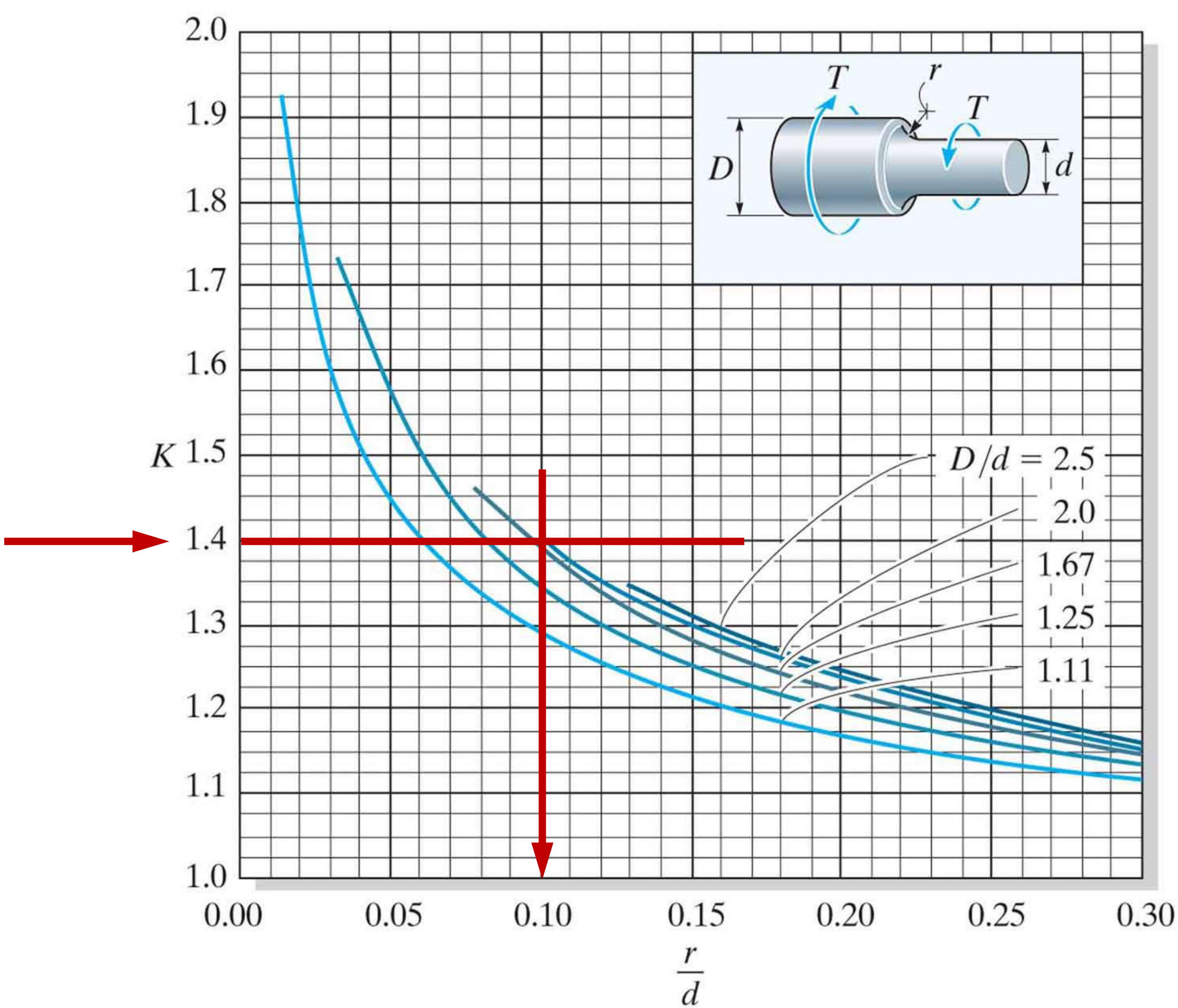


Stress concentrations, torsion: example A

The assembly is subjected to a torque of $710\ \mathrm{lb_f \cdot in}$. If the allowable shear stress for the material is $\tau_{allow} = 12\ \mathrm{ksi}$, determine the radius of the smallest size fillet that can be used to transmit the torque.



Stress concentrations, torsion: example A



Reading assignment

- Chapter 5 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course