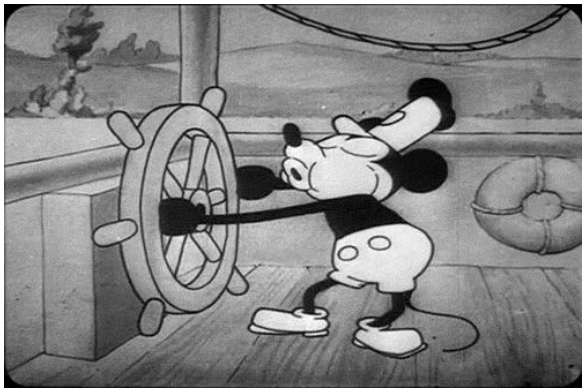
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



19 November 2025

WORCESTER POLYTECHNIC INSTITUTE
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STRESS ANALYSIS
ES-2502, B'2025

Lecture 18:
Unit 12: Torsion of shafts:
circular cross-section: *angle of twist & statically indeterminate*

19 November 2025

General information

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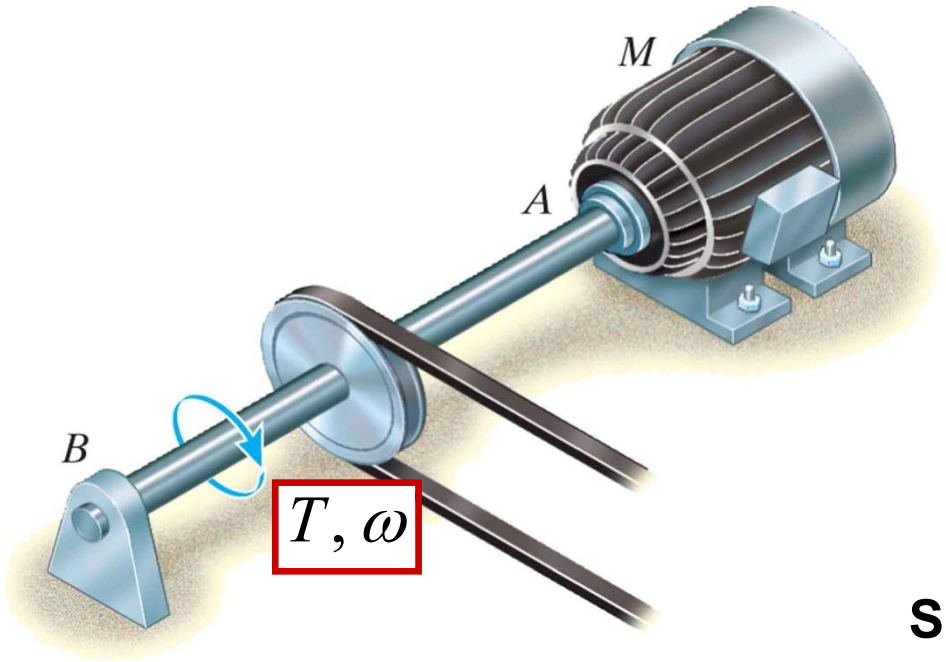
Power transmission



$$P = T\,\omega$$

with:

$$\omega = 2\pi \cdot f$$

$$\omega \left[\frac{rad}{sec}\right]$$

$$f\ [Hz]$$

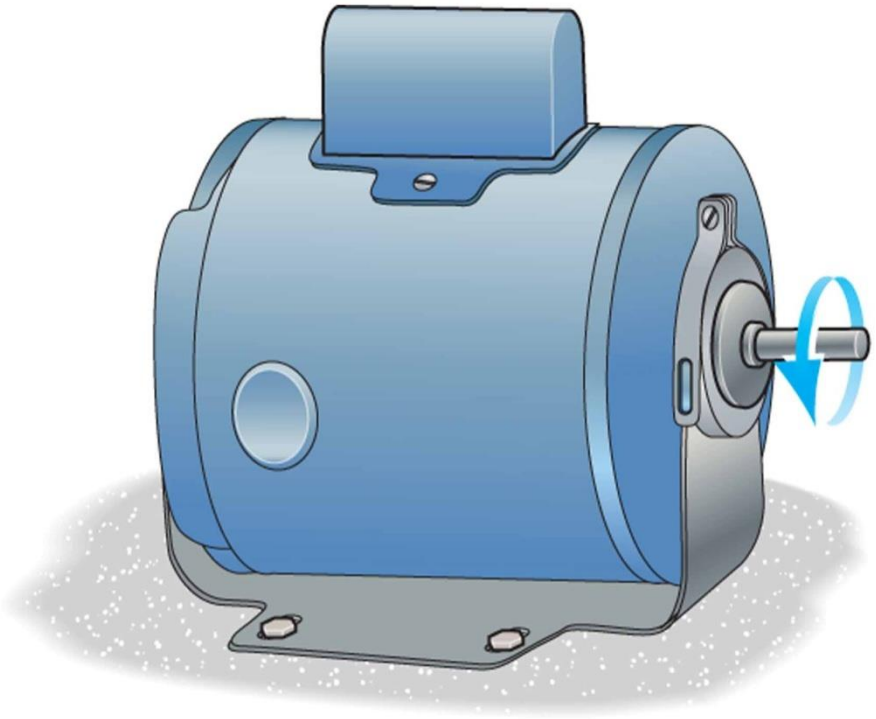
SI: $1W = 1N \cdot \frac{m}{s}$

(units)

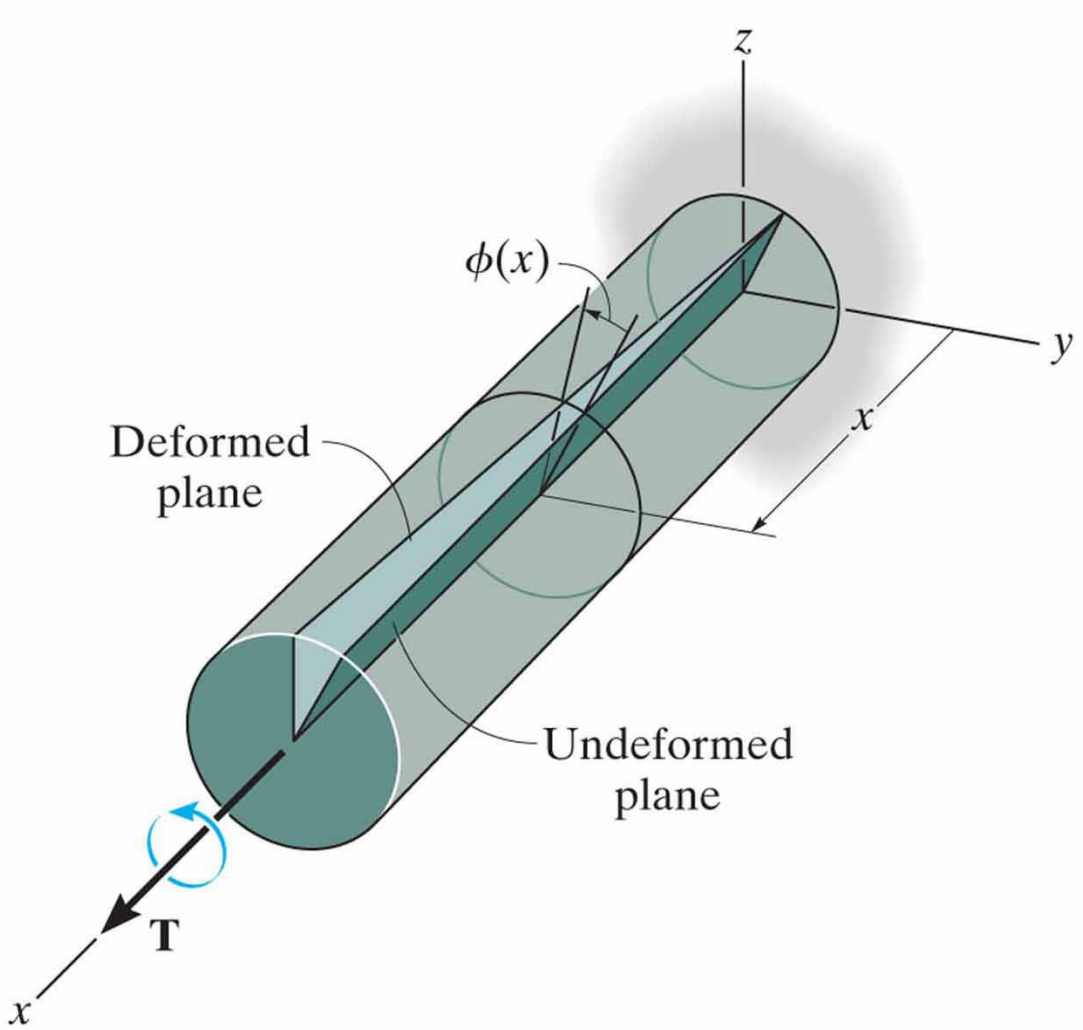
FPS: $1hp = 550\ ft \cdot \frac{lb}{s}$

Torsion: example E

The $25\ \text{mm}$ diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{\text{allow}} = 75\ \text{MPa}$. If the motor is operating at its maximum power of $5\ \text{kW}$, determine the minimum allowable rotation of the shaft.

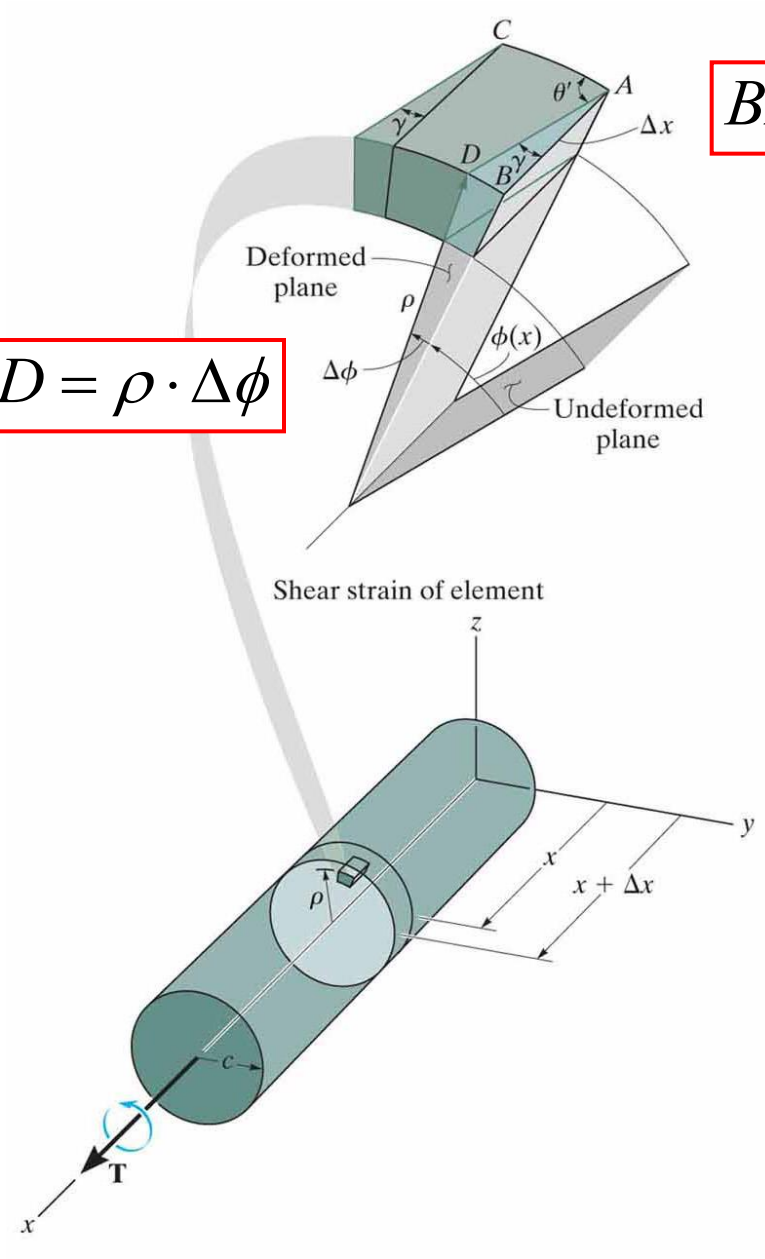


Torsion: angle of twist



The angle of twist $\phi(x)$ increases as x increases.

Torsion: angle of twist ϕ



$$BD = \gamma \cdot \Delta x$$

$$BD = \rho \cdot \Delta\phi$$

Shear strain: $\gamma = \rho \dfrac{d\phi}{dx}$

Therefore: $d\phi = \dfrac{\gamma}{\rho} dx$

Torsion: angle of twist ϕ

From before: $$d\phi = \frac{\gamma}{\rho} dx$$

By Hook's law: $$\gamma = \frac{\tau}{G} = \frac{1}{G} \frac{T\ \rho}{J}$$

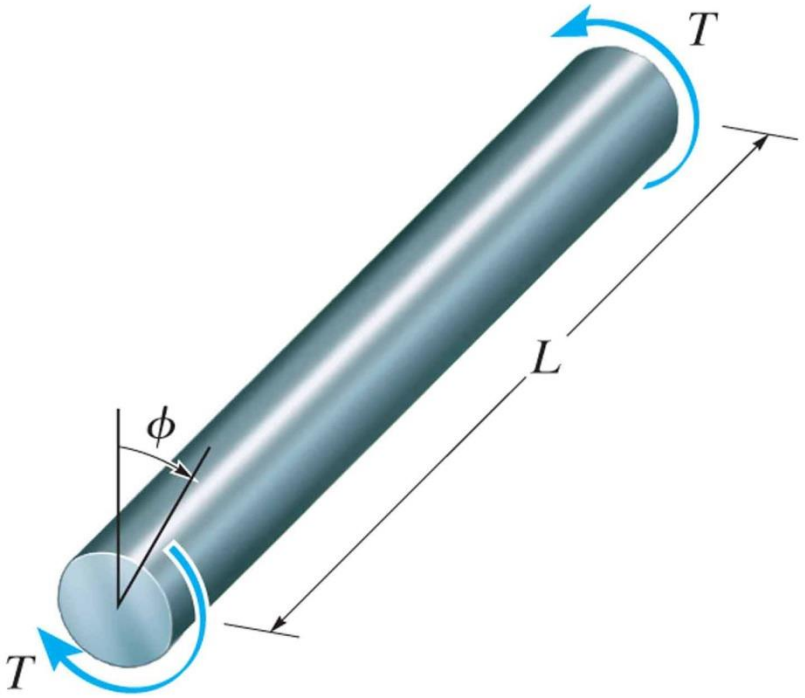
$$\gamma(x,\rho) = \frac{1}{G} \frac{T(x)}{J(x)} \rho$$

Angle of twist: $$\phi(x) = \int_0^L \frac{1}{G} \frac{T(x)}{J(x)} dx$$

Torsion: angle of twist ϕ

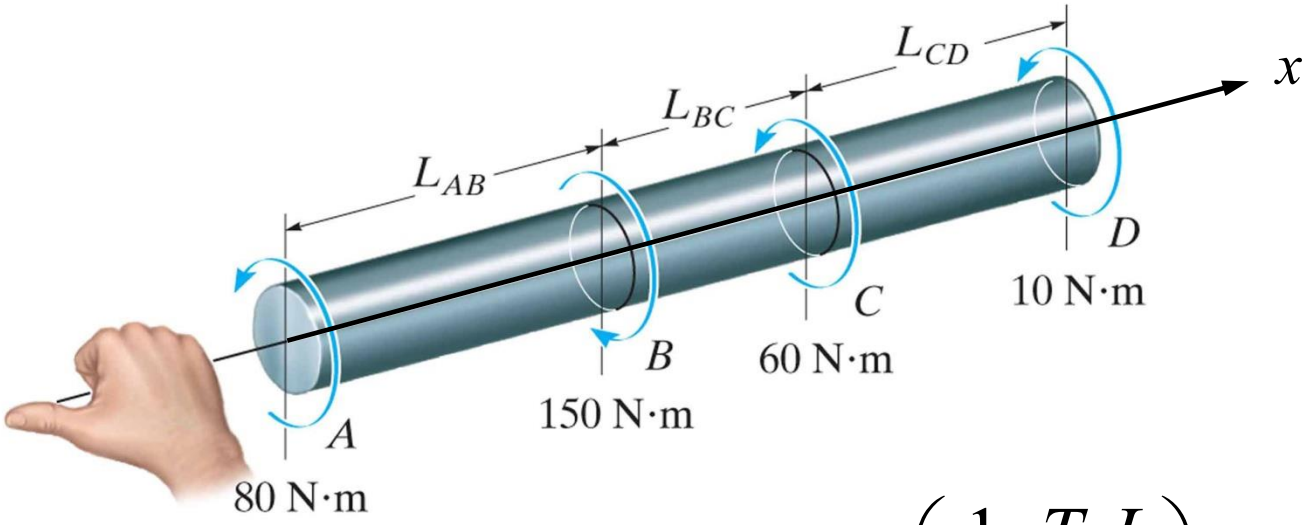
Constant torque and cross sectional area

Angle of twist: $$\phi(x = L) = \frac{1}{G}\frac{T\,L}{J}$$

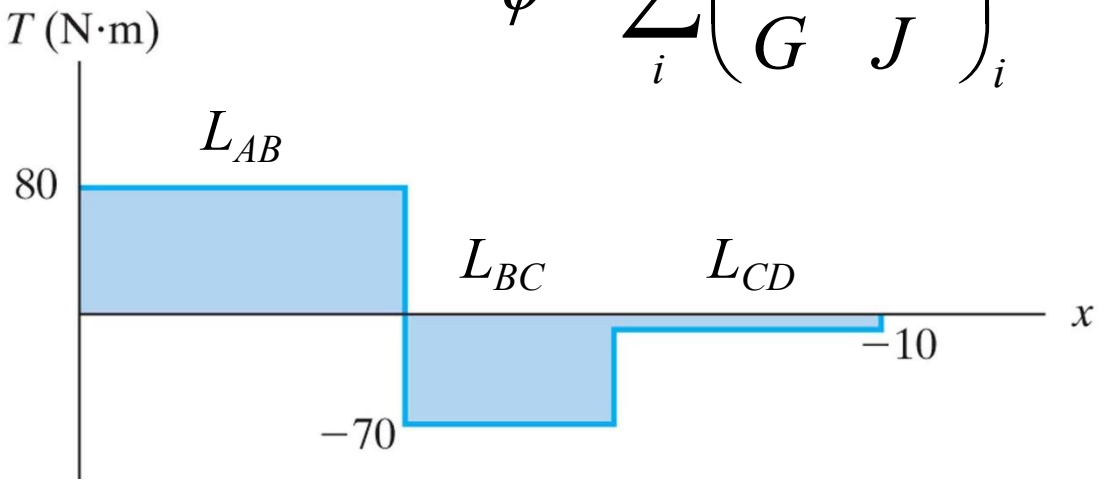


Torsion: angle of twist ϕ

Multiple torques

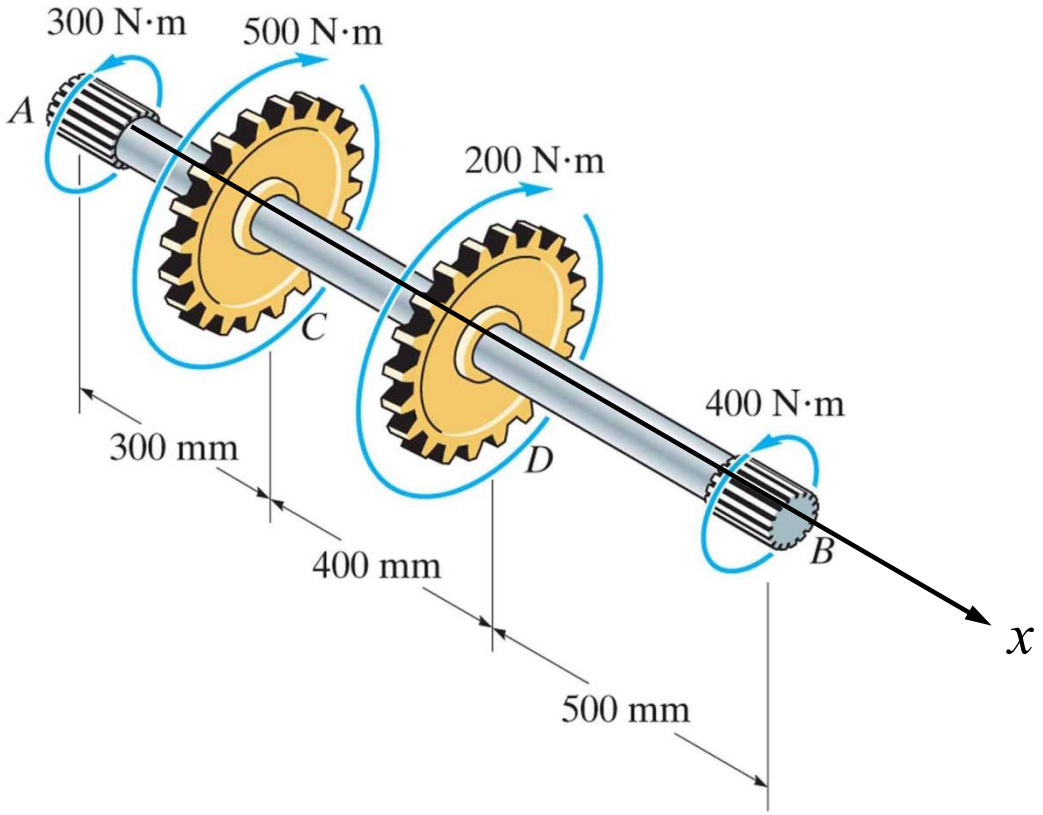


$$\phi = \sum_i \left(\frac{1}{G} \frac{T\,L}{J} \right)_i$$



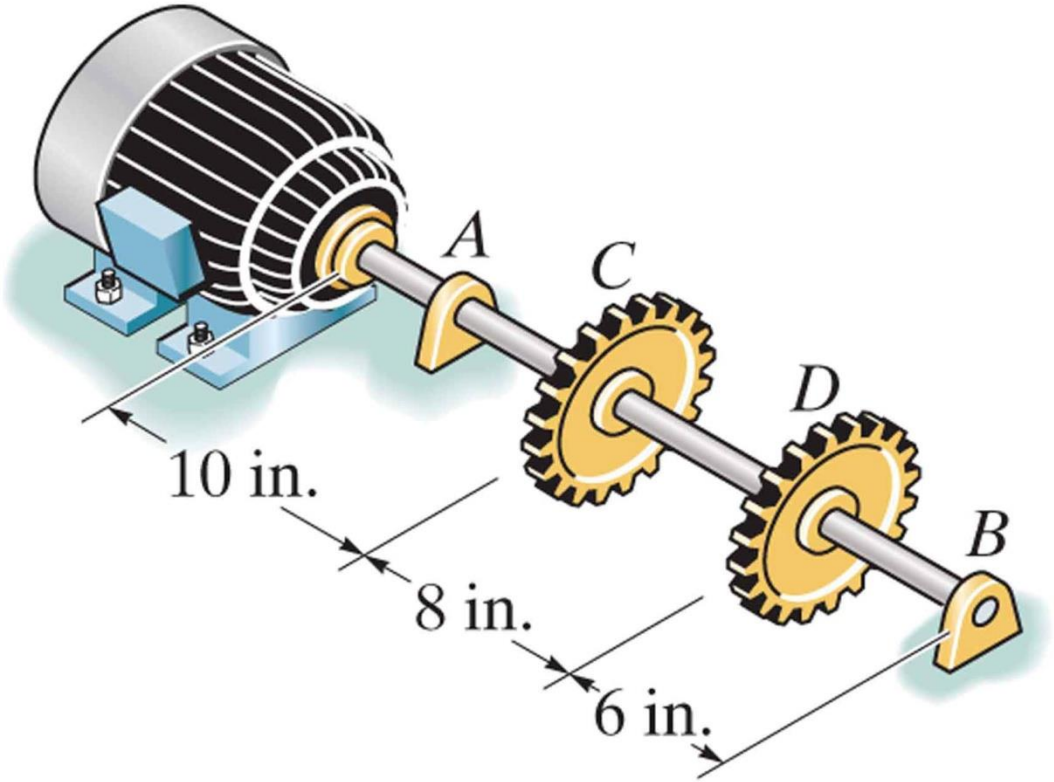
Torsion: example F

The splined ends and gears attached to the A-36vsteel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of $40\ \text{mm}$.



Torsion: example G

The motor delivers 40 hp to the 304 stainless steel shaft while it rotates at 20 Hz. The shaft is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the diameter of the shaft to the nearest $1/8$ in. if the allowable shear stress is $\tau_{\text{allow}} = 8$ ksi and the allowable angle of twist of C with respect to D is 0.20°.

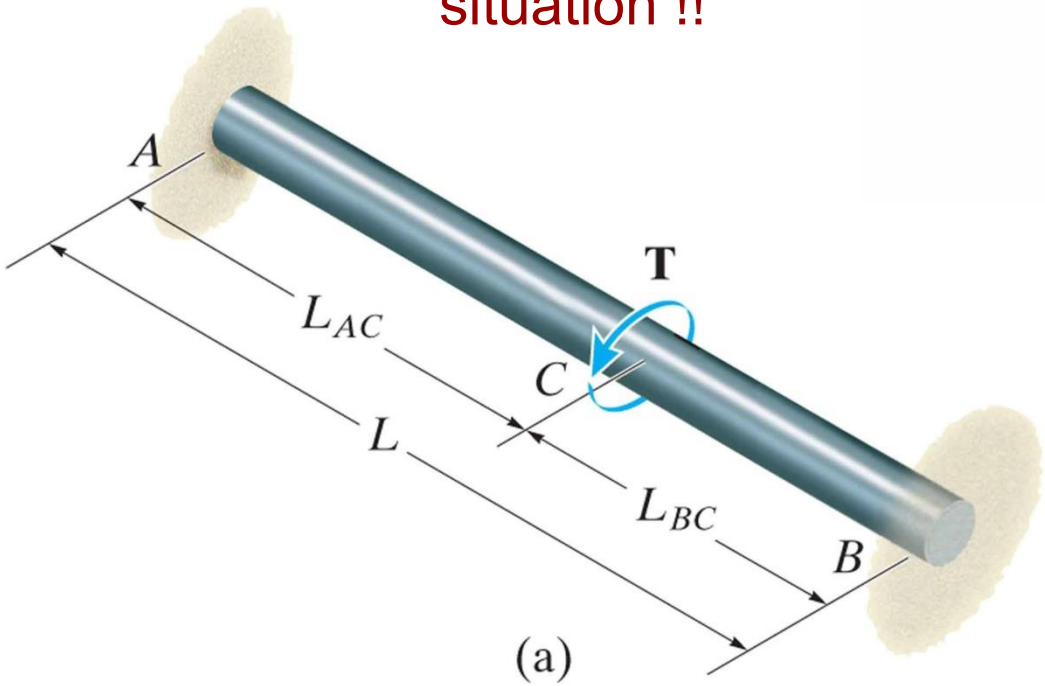


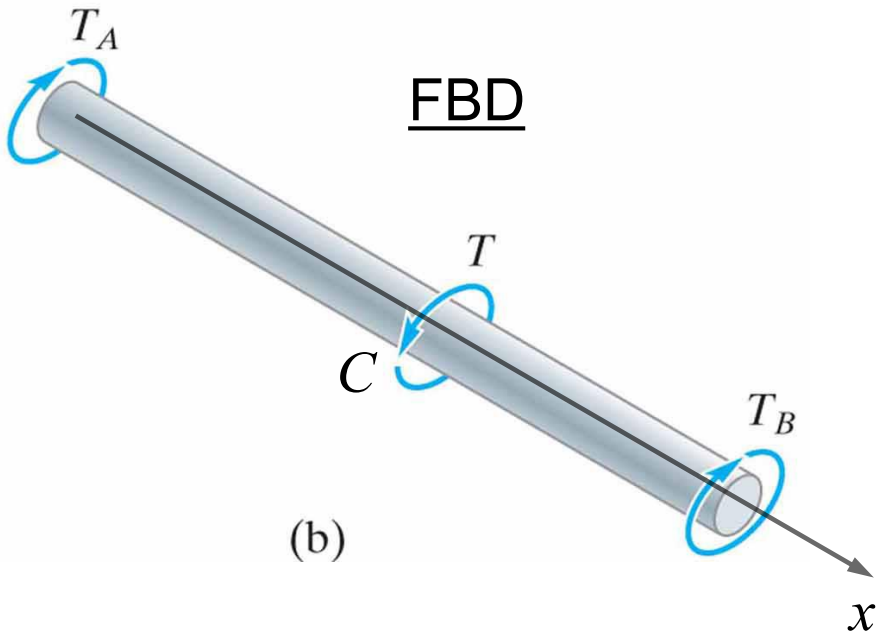
Statically indeterminate torque-loaded members

$$\sum M_x = 0$$

$$T - T_A - T_B = 0$$

Statically indeterminate situation !!

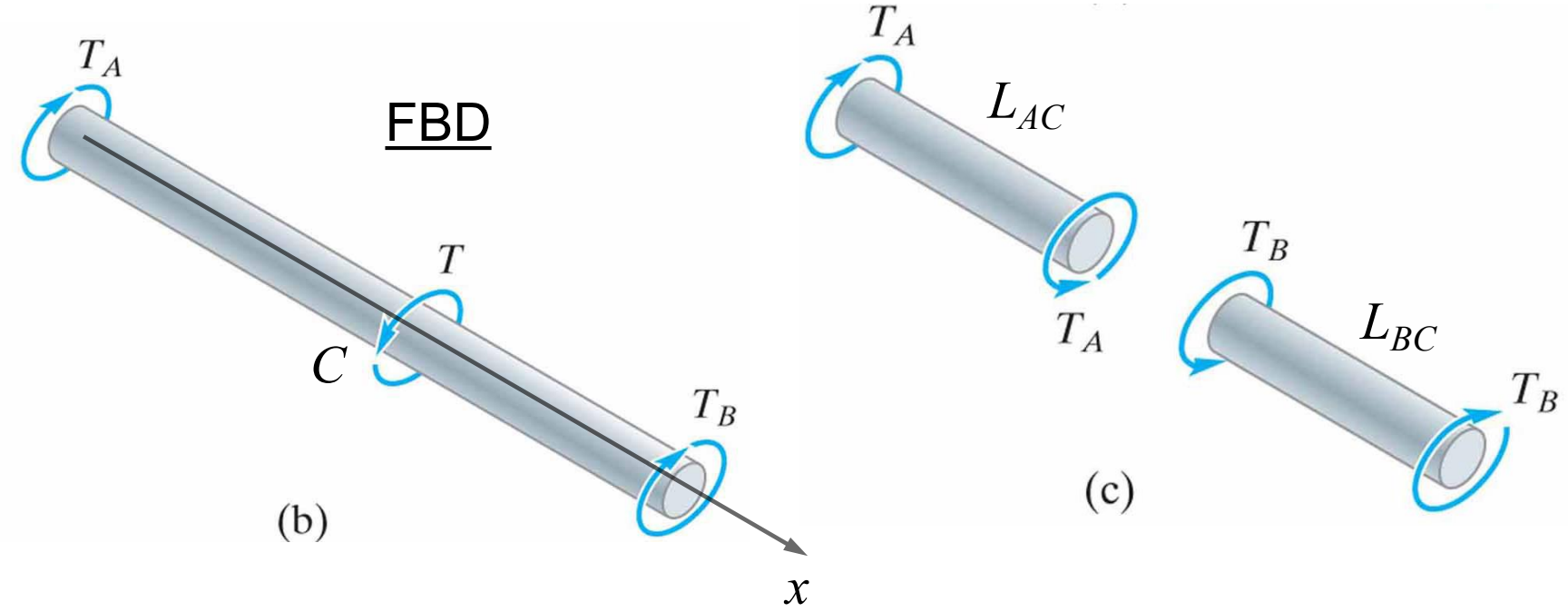




Statically indeterminate torque-loaded members

Compatibility equation:
(Same angle of twist)

$$\frac{T_A L_{AC}}{J G} = \frac{T_B L_{BC}}{J G}$$

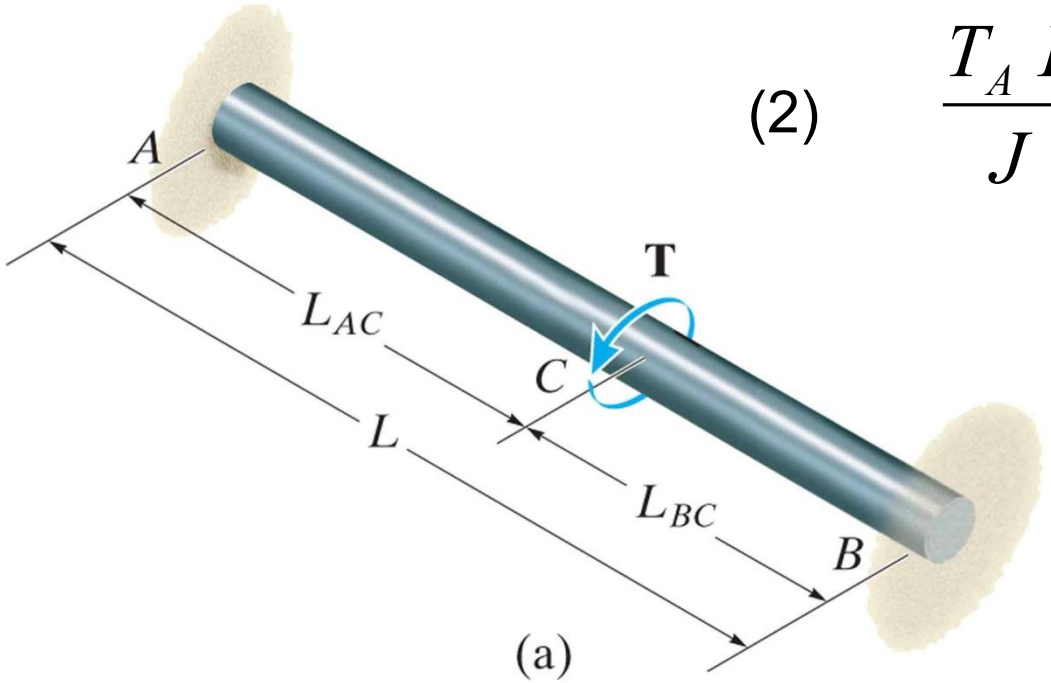


Statically indeterminate torque-loaded members

Solution with: <u>two equations for two unknowns</u> (T_A and T_B):

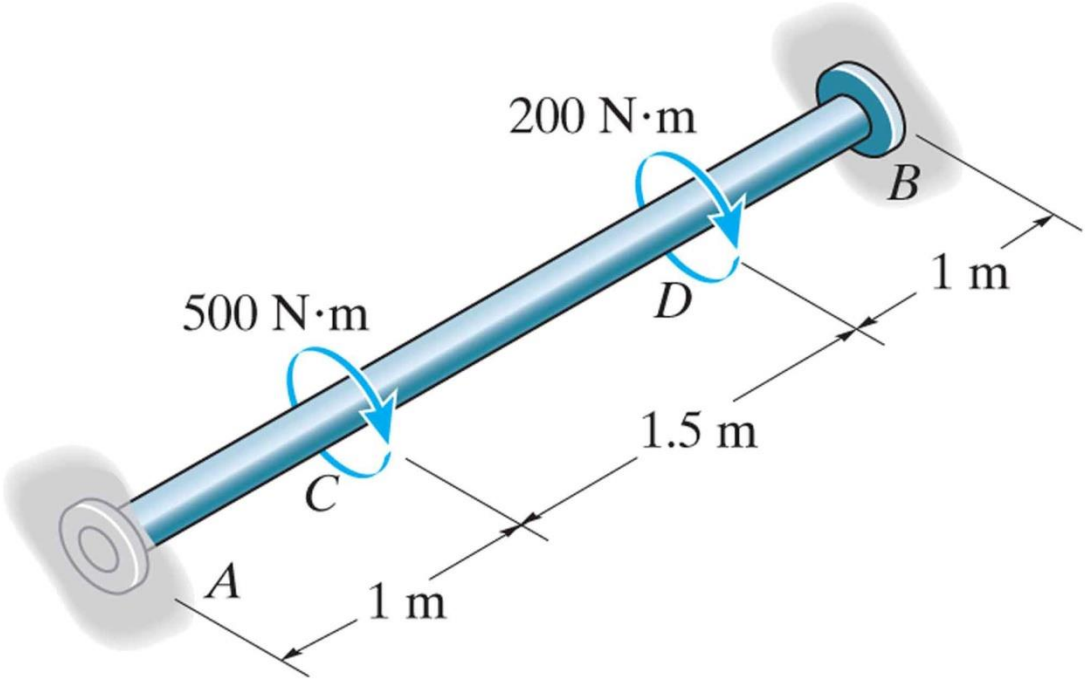
(1) $$T - T_A - T_B = 0$$

(2) $$\frac{T_A\, L_{AC}}{J\, G} - \frac{T_B\, L_{BC}}{J\, G} = 0$$



Statically indeterminate torque-loaded members: example A

The A-36 steel shaft has a diameter of $60\ \text{mm}$ and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft



Approach:

1) Apply equilibrium equations
2) Apply compatibility equations (*two possible methods*)
3) Solve for stresses

Reading assignment

- Chapter 5 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course