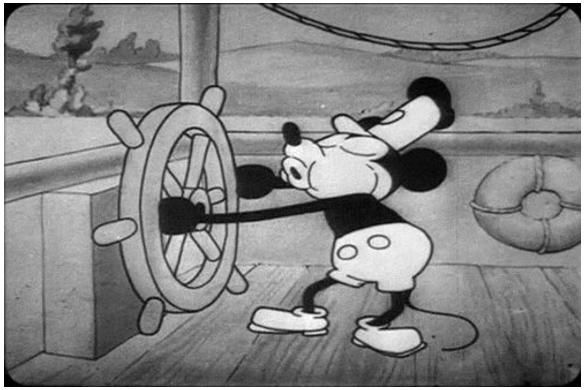
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



18 November 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

Lecture 17:
Unit 12: Torsion of shafts:
circular cross-section: *power transmission*

18 November 2025

General information

Instructor: Cosme Furlong
HL-152
(508) 831-5126
Email: cfurlong @ wpi.edu
http://www.wpi.edu/~cfurlong/es2502.html

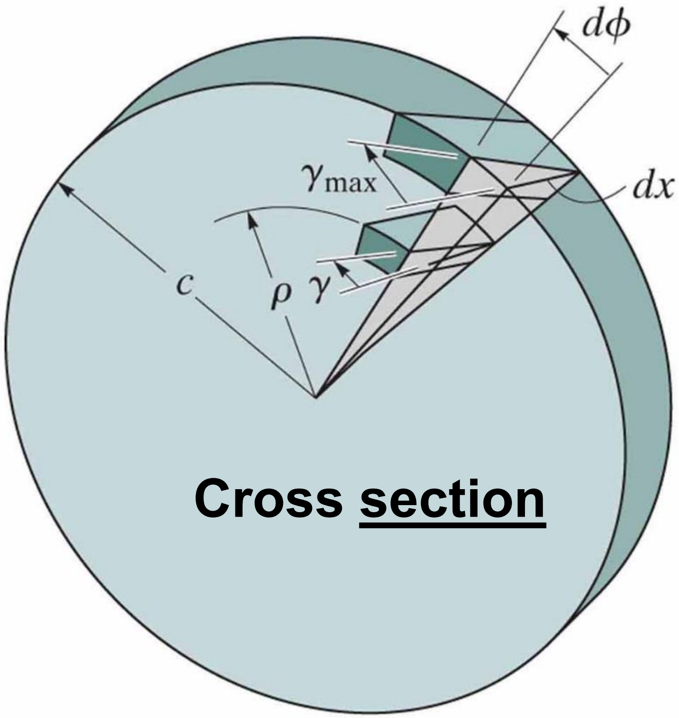
Graduate Assistants:

→ Hamed Ghavami (TA)
Email: sghavami @ wpi.edu

→ Jay Patil (GA)
Email: jpatil1 @ wpi.edu

→ Mikayla Almeida (GA)
mpalmeida @ wpi.edu

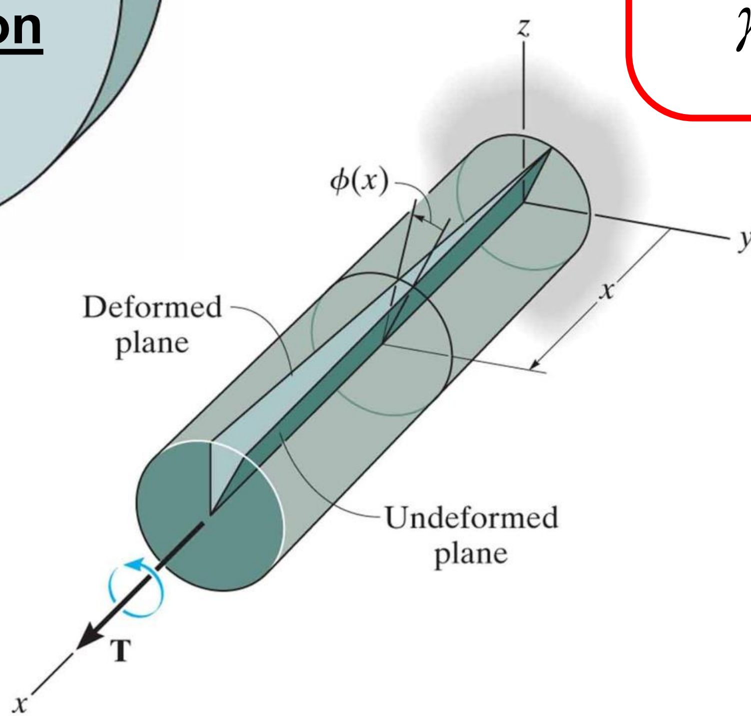
Torsion: shear strains



$$\frac{\gamma}{\rho} = \frac{\gamma_{max}}{c}$$

Shear strains vary linearly within a section:

$$\gamma = \gamma(\rho) = \rho \frac{\gamma_{max}}{c}$$



The angle of twist $\phi(x)$ increases as x increases.

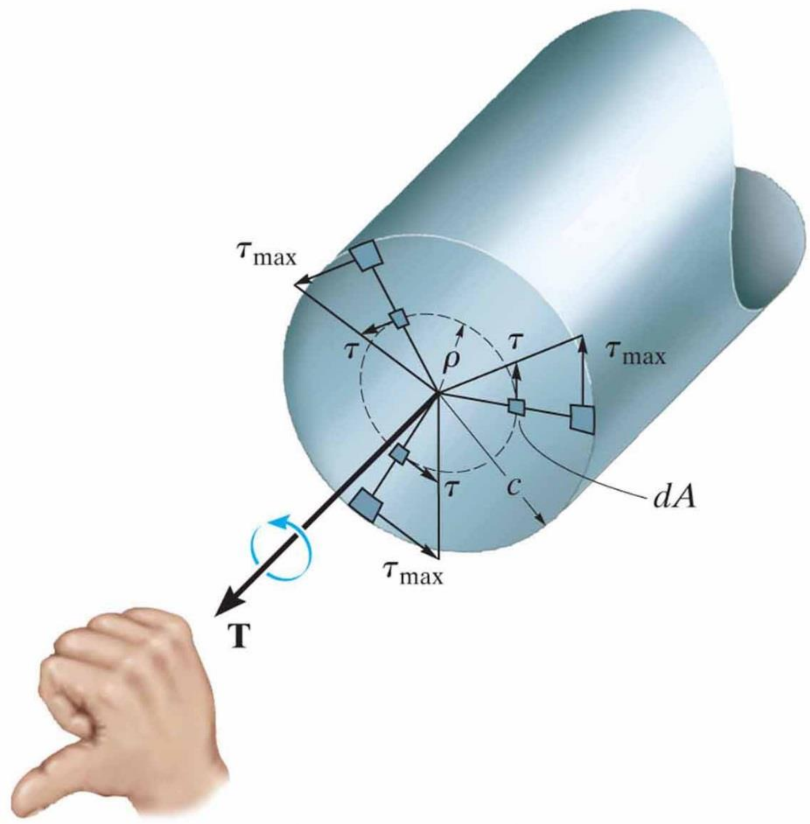
Torsion formula

According to Hook's law (linear elasticity):

$$(\tau = G \cdot \gamma)$$

Shear stresses also **vary linearly within a section:**

$$\tau = \tau(\rho) = \rho \frac{\tau_{max}}{c}$$



Differential Force:

$$dF = \tau \cdot dA$$

Differential Torque:

$$dT = \rho\,(\tau \cdot dA)$$

Torsion formula

Integrating torque:

$$T = \int_A \rho\,(\tau \cdot dA) \;=\; \int_A \rho \left(\rho \frac{\tau_{\max}}{c} \right) dA$$

$$= \frac{\tau_{\max}}{c} \int_A \rho^2 \, dA$$

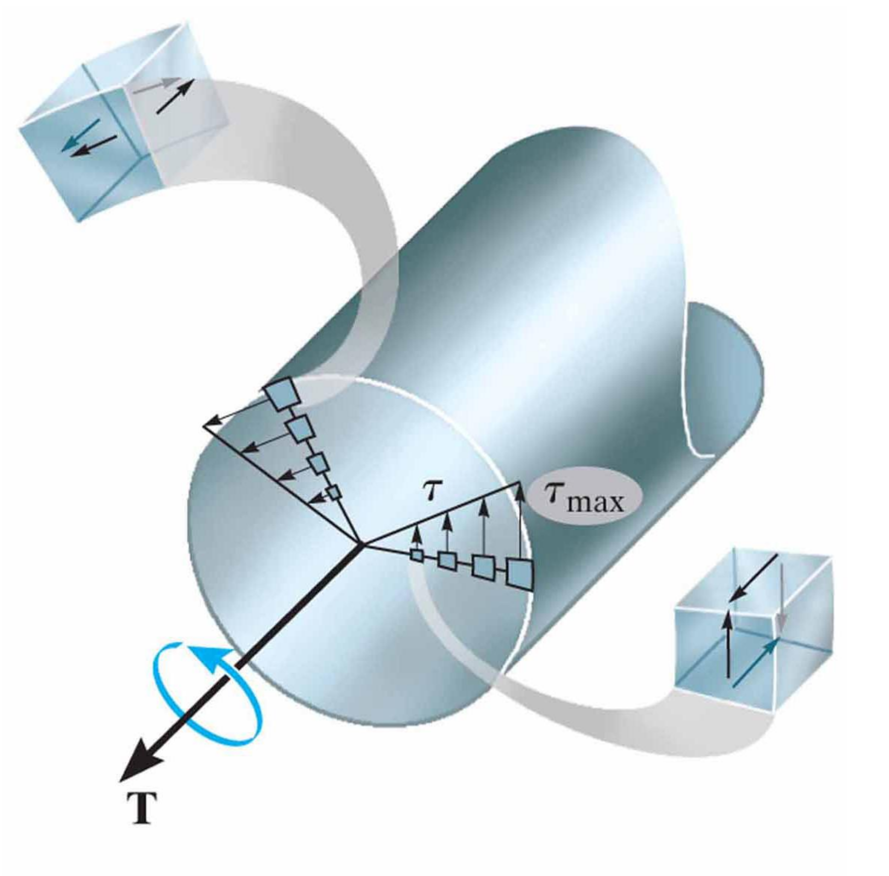
Define: $J = \int_A \rho^2 \, dA$ ← Polar area moment of inertia

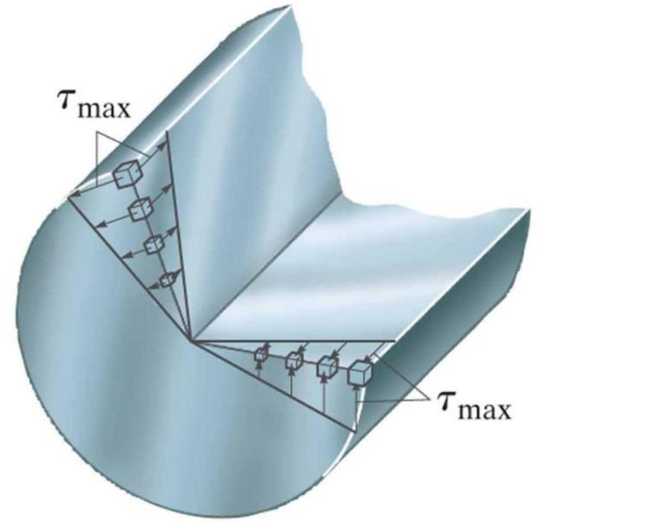
Torsion formula for stresses:
(linear elastic)

$$\tau_{\max} = \frac{T\,c}{J} \quad and \quad \tau = \tau(\rho) = \frac{T\,\rho}{J}$$

Torsion formula: solid circular bar

Linear variation of shear stress

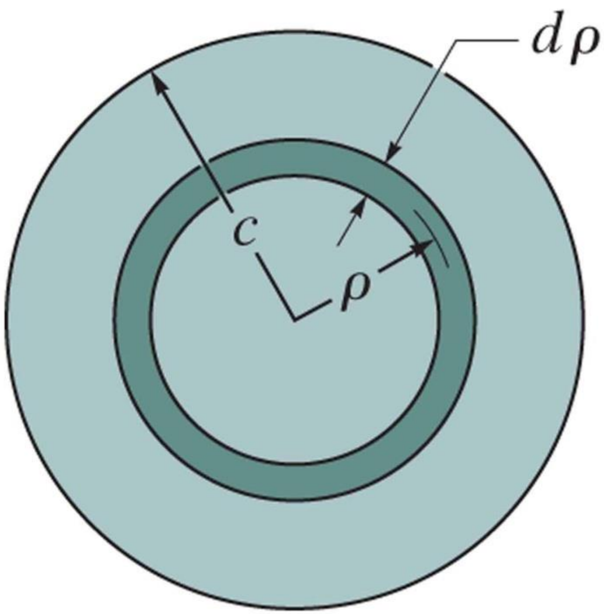




Shear stress varies linearly along each radial line of the cross section.

Torsion formula: polar area moment of inertia

Solid bar



$$J = \int_A \rho^2 \, dA$$

$$= \int_0^c \rho^2 \, (2\pi \, \rho \, d\rho)$$

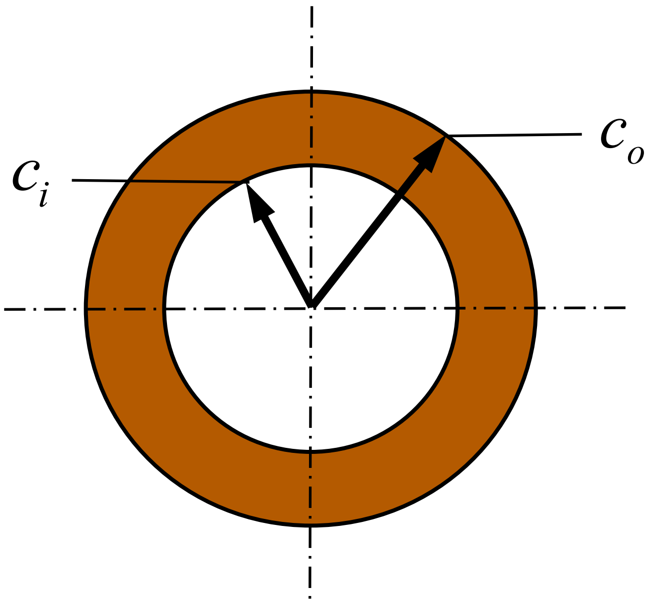
$$= 2\pi \int_0^c \rho^3 \, d\rho = \quad 2\pi \left(\frac{\rho^4}{4} \right)_0^c$$

Solid, circular, section:

$$J = \frac{\pi}{2} c^4$$

Torsion formula: polar area moment of inertia

Tubular bar



$$J = \int_A \rho^2 \, dA$$

$$= \int_{c_i}^{c_o} \rho^2 \, (2\pi \, \rho \, d\rho)$$

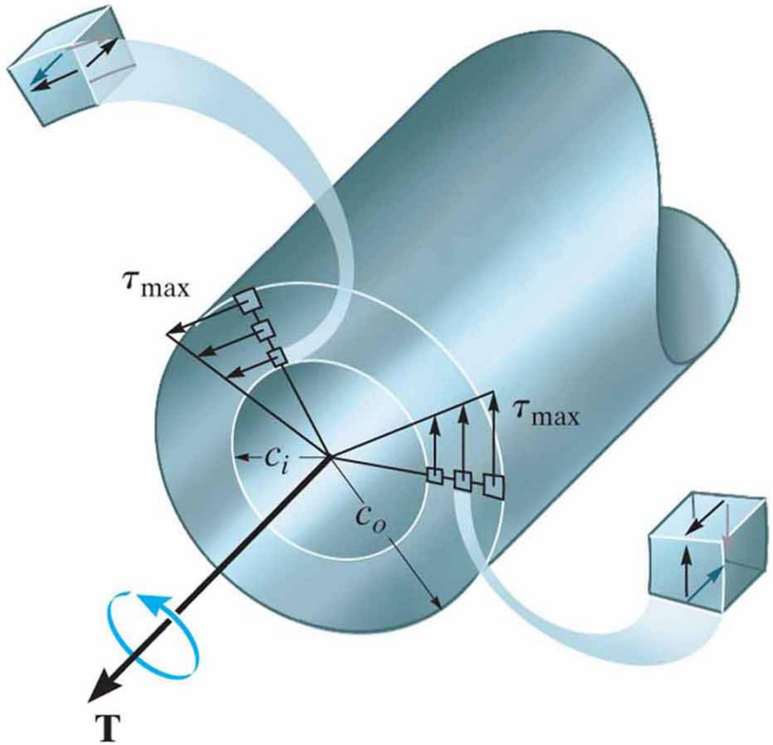
$$= 2\pi \int_{c_i}^{c_o} \rho^3 \, d\rho = \quad 2\pi \left(\frac{\rho^4}{4} \right)_{c_i}^{c_o}$$

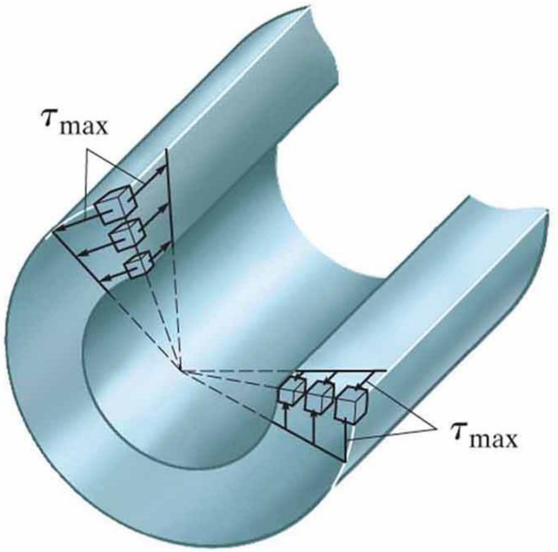
Tubular section:

$$J = \frac{\pi}{2}(c_o{}^4 - c_i{}^4)$$

Torsion formula: tubular bar

Linear variation of shear stress



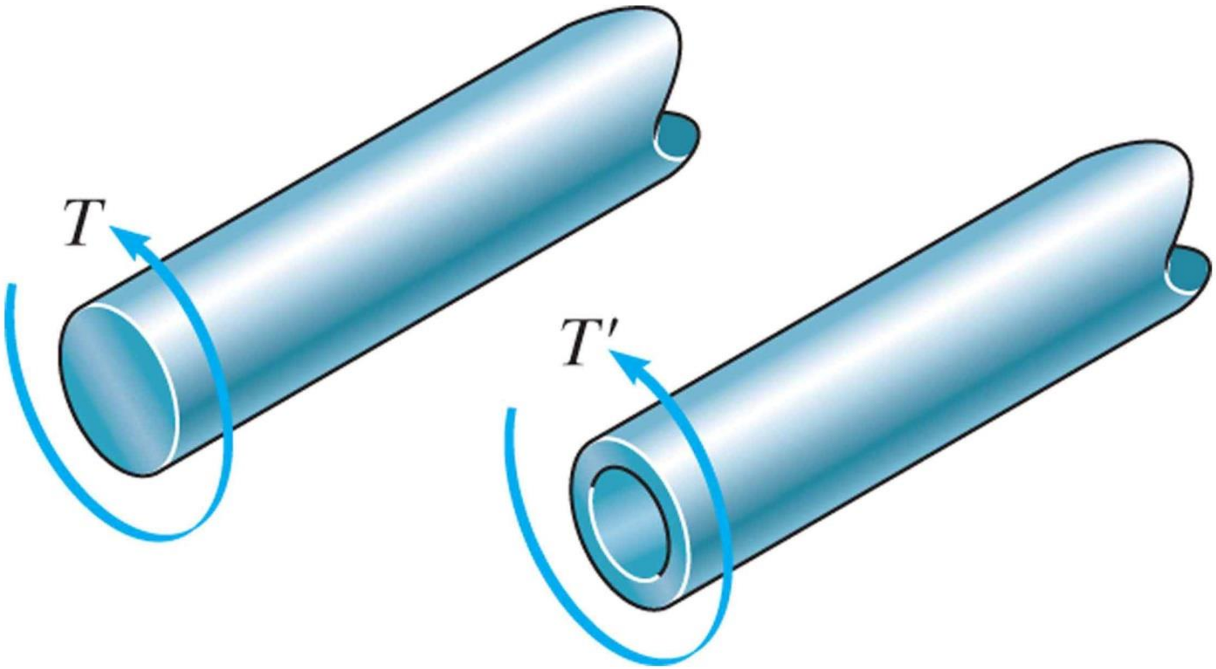


Shear stress varies linearly along each radial line of the cross section.

Torsion: example A

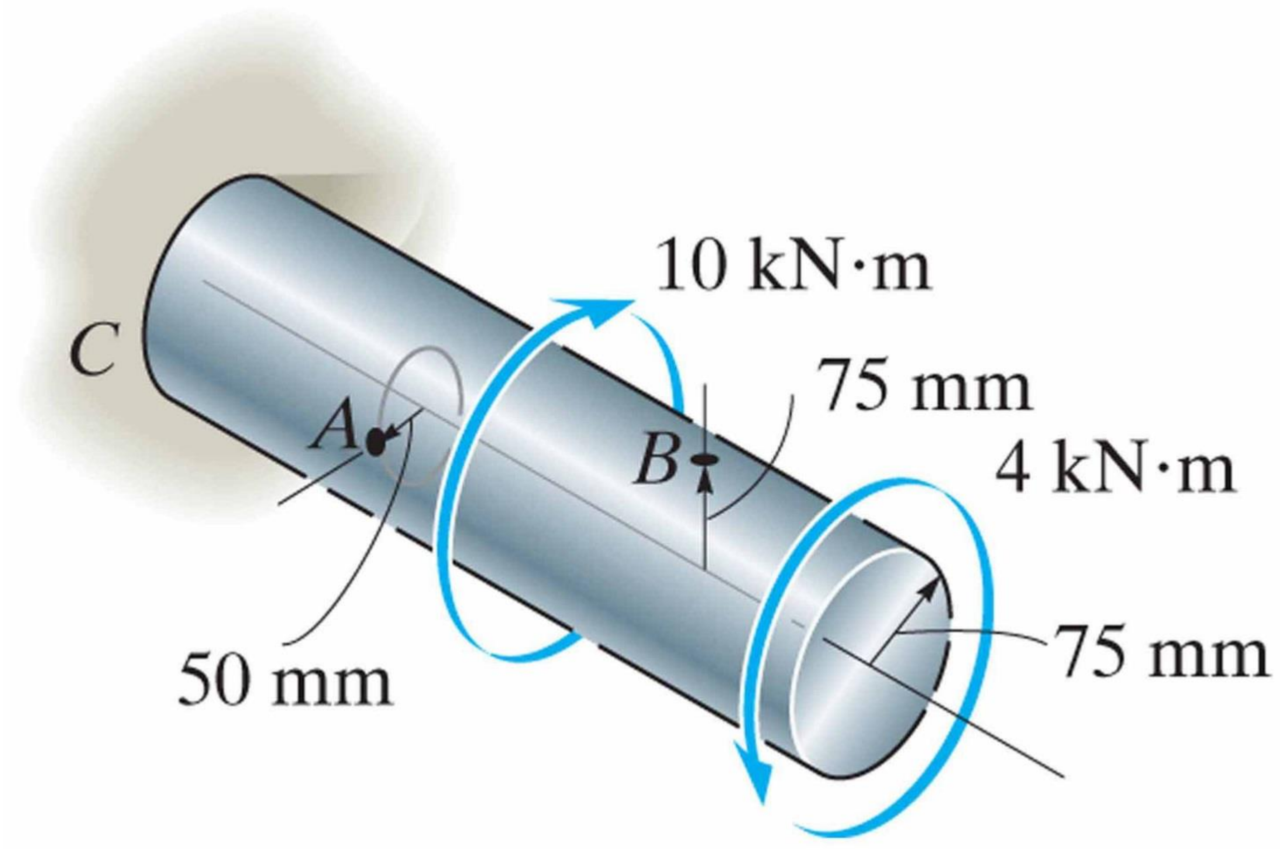
A shaft is made of a steel alloy having an allowable shear stress of $\tau_{\text{allow}} = 12\ \text{ksi}$. If the diameter of the shaft is $1.5\ \text{in.}$, determine the maximum torque $\boldsymbol{T}$ that can be transmitted.

What would be the maximum torque $\boldsymbol{T'}$ if a 1-in. diameter hole is bored through the shaft? Sketch the shear-stress distribution *along a radial line* in each case.



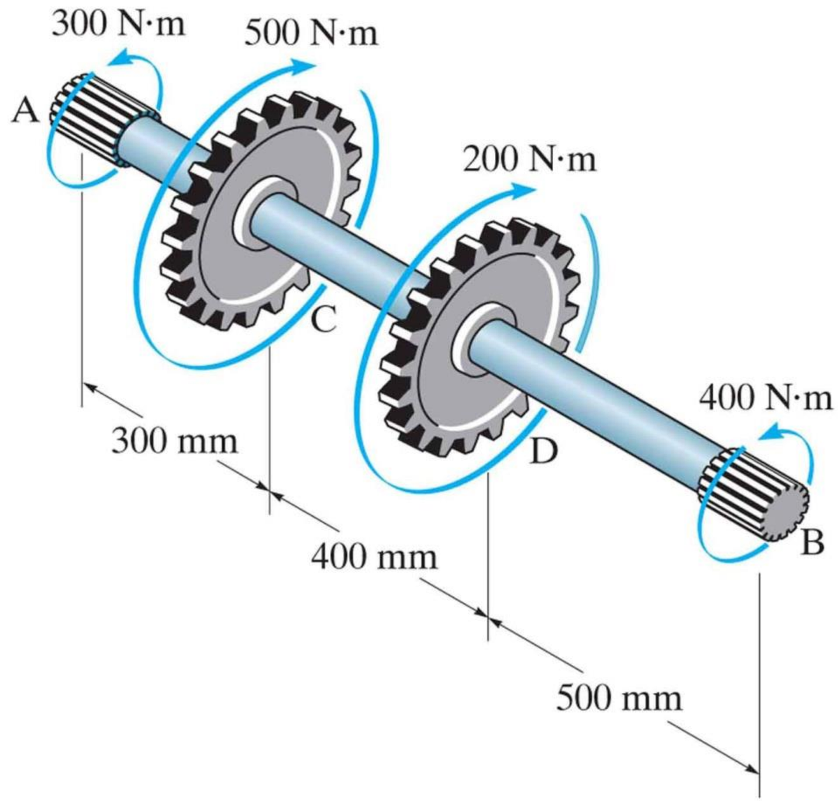
Torsion: example B

The solid shaft is fixed to the support at *C* and subjected to the torsional loadings shown. Determine the shear stress at points *A* and *B* and sketch the shear stress on volume (stress) elements located at these points.



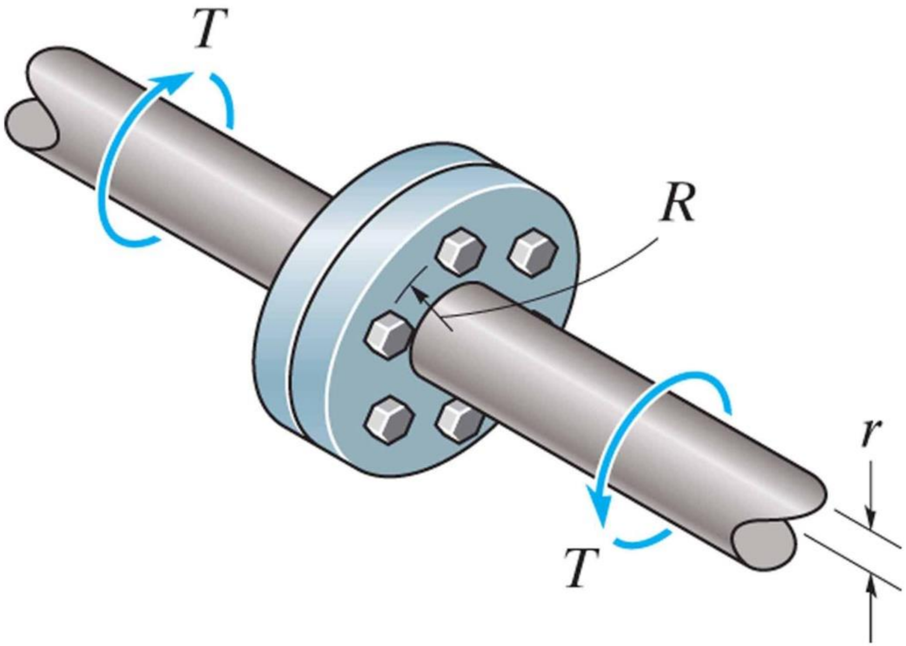
Torsion: example C

The solid 30 mm diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft.

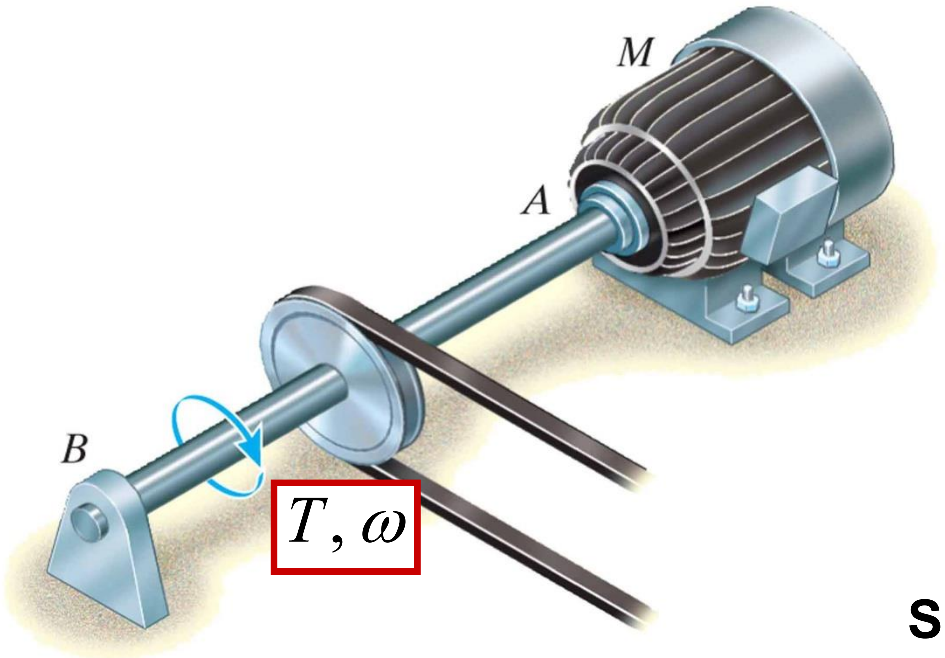


Torsion: example D

The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is *uniform*, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shear stress in the bolts. Each bolt has a diameter d.



Power transmission



$$P = T\,\omega$$

with:

$$\omega = 2\pi \cdot f$$

$$\omega\left[\frac{rad}{sec}\right]$$

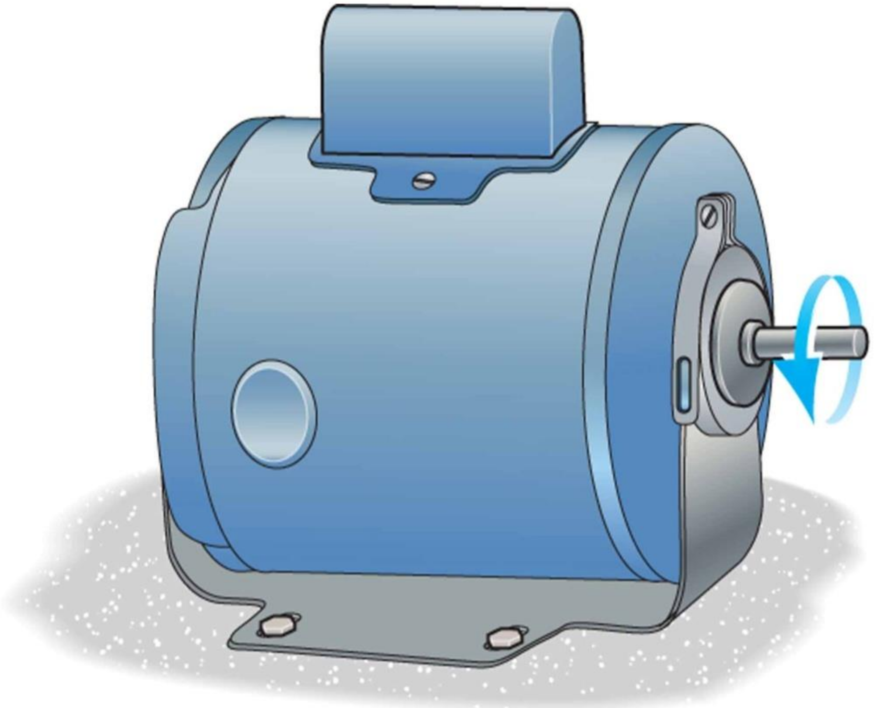
$$f\,[Hz]$$

SI: $1W = 1N \cdot \frac{m}{s}$

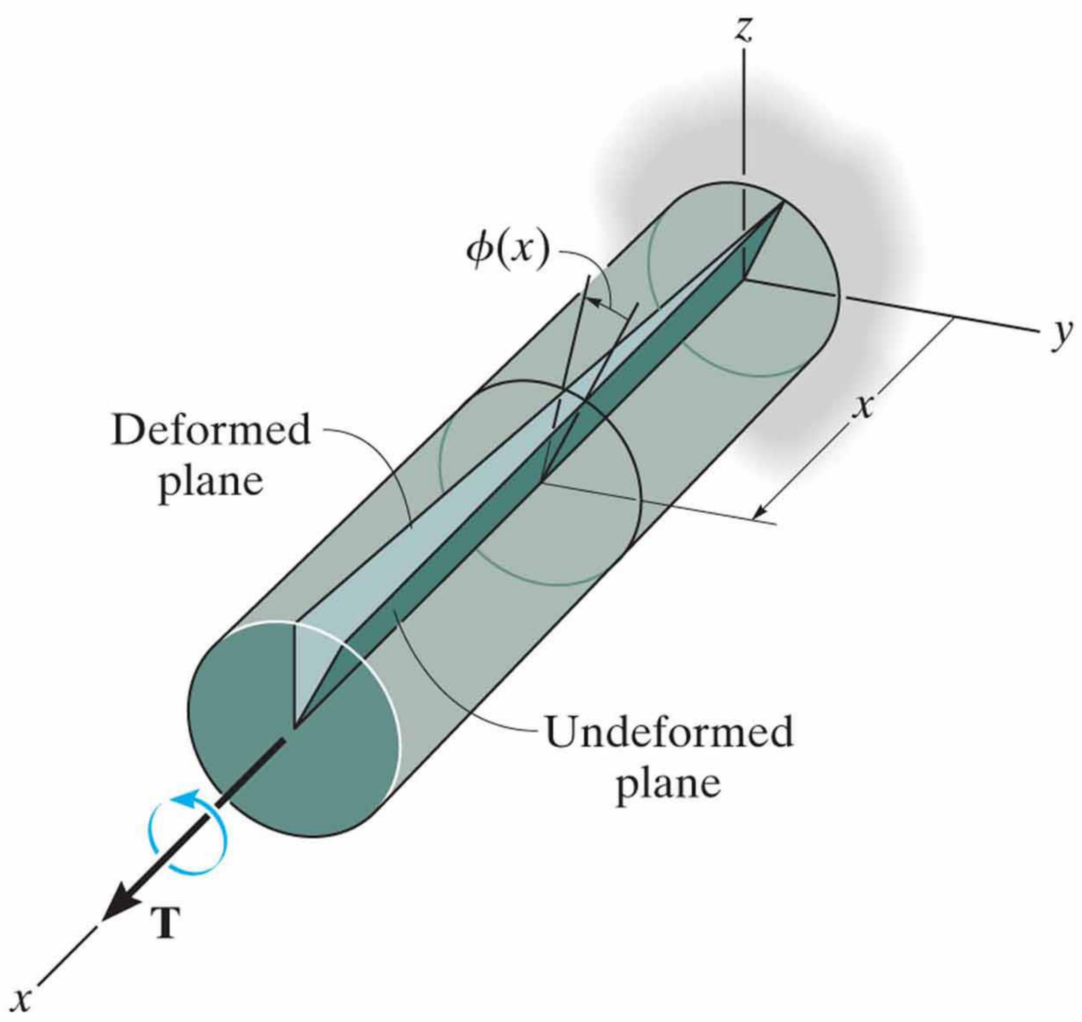
FPS: $1hp = 550\ ft \cdot \frac{lb}{s}$

Torsion: example E

The $25\ \mathrm{mm}$ diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{allow} = 75\ \mathrm{MPa}$. If the motor is operating at its maximum power of $5\ \mathrm{kW}$, determine the minimum allowable rotation of the shaft.

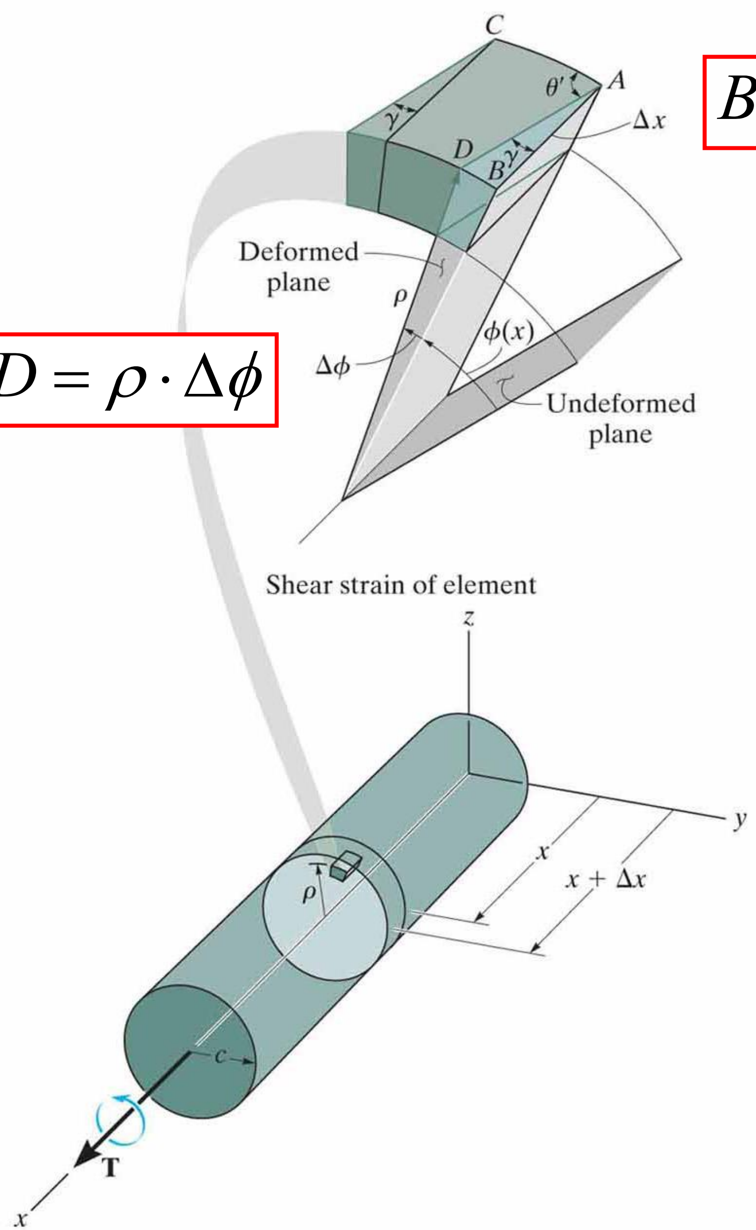


Torsion: angle of twist



The angle of twist $\phi(x)$ increases as x increases.

Torsion: angle of twist ϕ



$$BD = \gamma \cdot \Delta x$$

$$BD = \rho \cdot \Delta\phi$$

Shear strain: $\gamma = \rho \dfrac{d\phi}{dx}$

Therefore: $$d\phi = \frac{\gamma}{\rho} dx$$

Torsion: angle of twist ϕ

From before: $$d\phi = \frac{\gamma}{\rho} dx$$

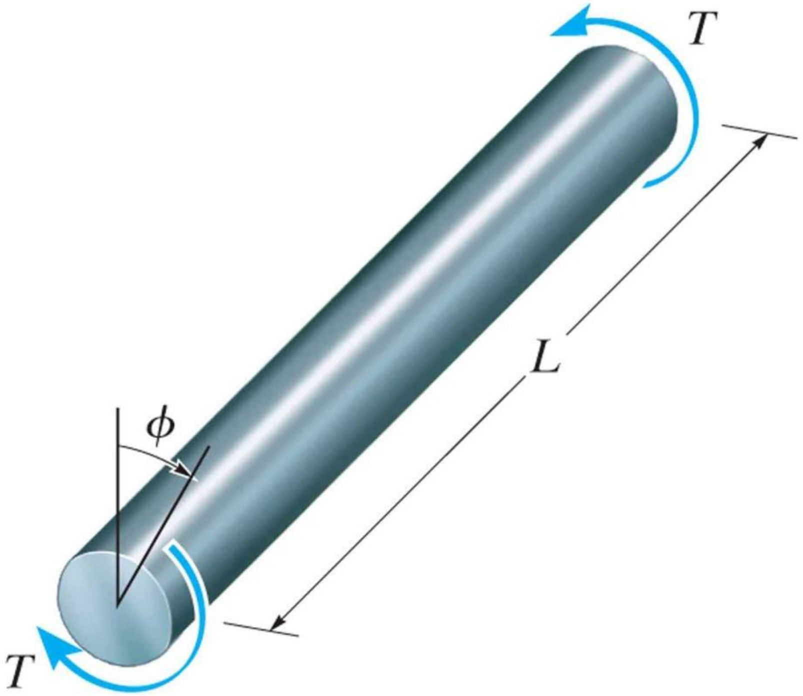
By Hook's law: $$\gamma = \frac{\tau}{G} = \frac{1}{G}\frac{T\rho}{J}$$

$$\gamma(x,\rho) = \frac{1}{G}\frac{T(x)\rho}{J(x)}$$

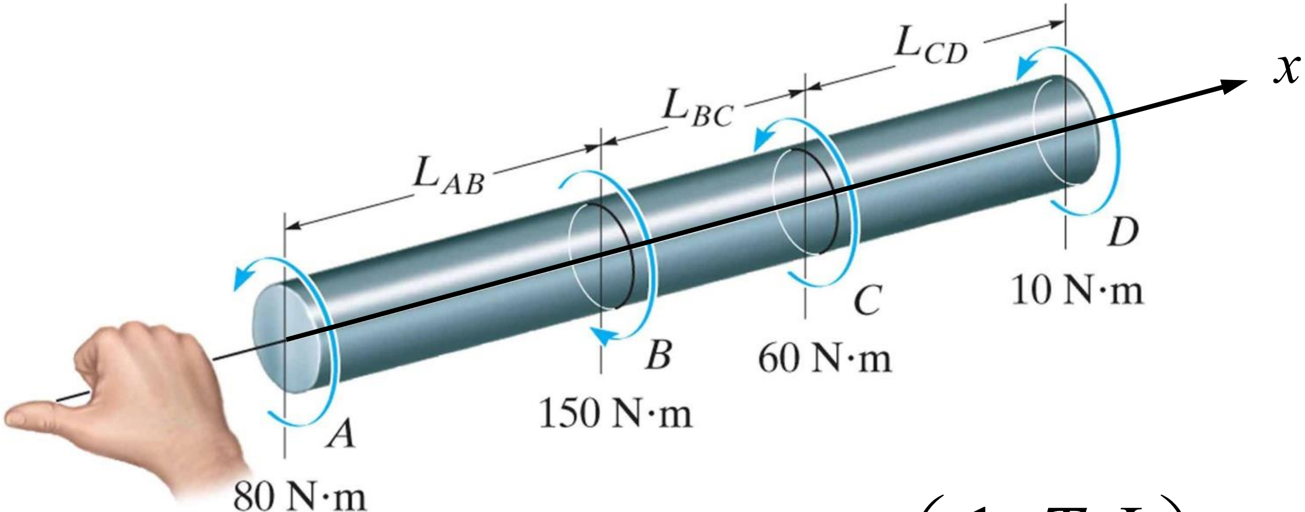
Angle of twist: $$\phi(x) = \int_0^L \frac{1}{G}\frac{T(x)}{J(x)} dx$$

Torsion: angle of twist ϕ
Constant torque and cross sectional area

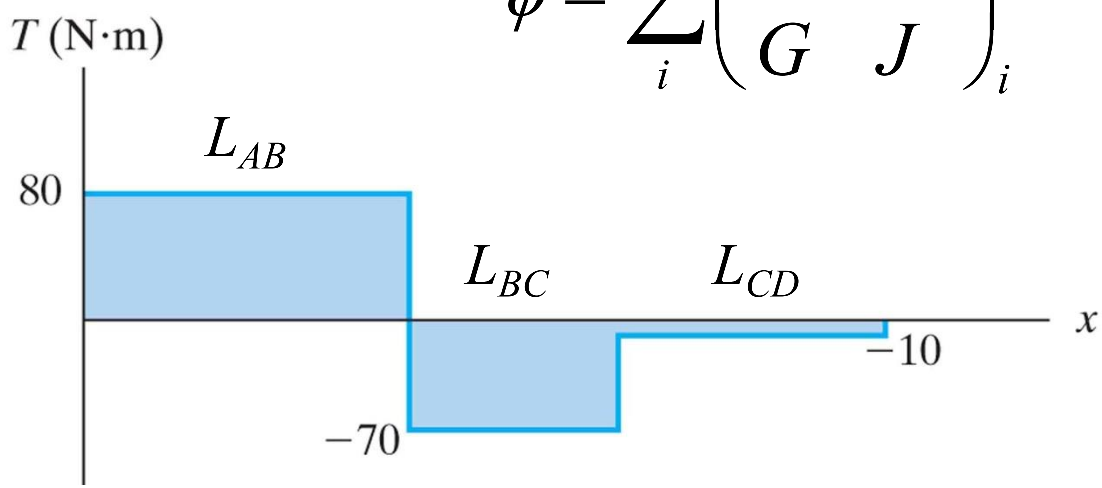
Angle of twist: $$\phi(x = L) = \frac{1}{G}\frac{T\,L}{J}$$



Torsion: angle of twist ϕ
Multiple torques

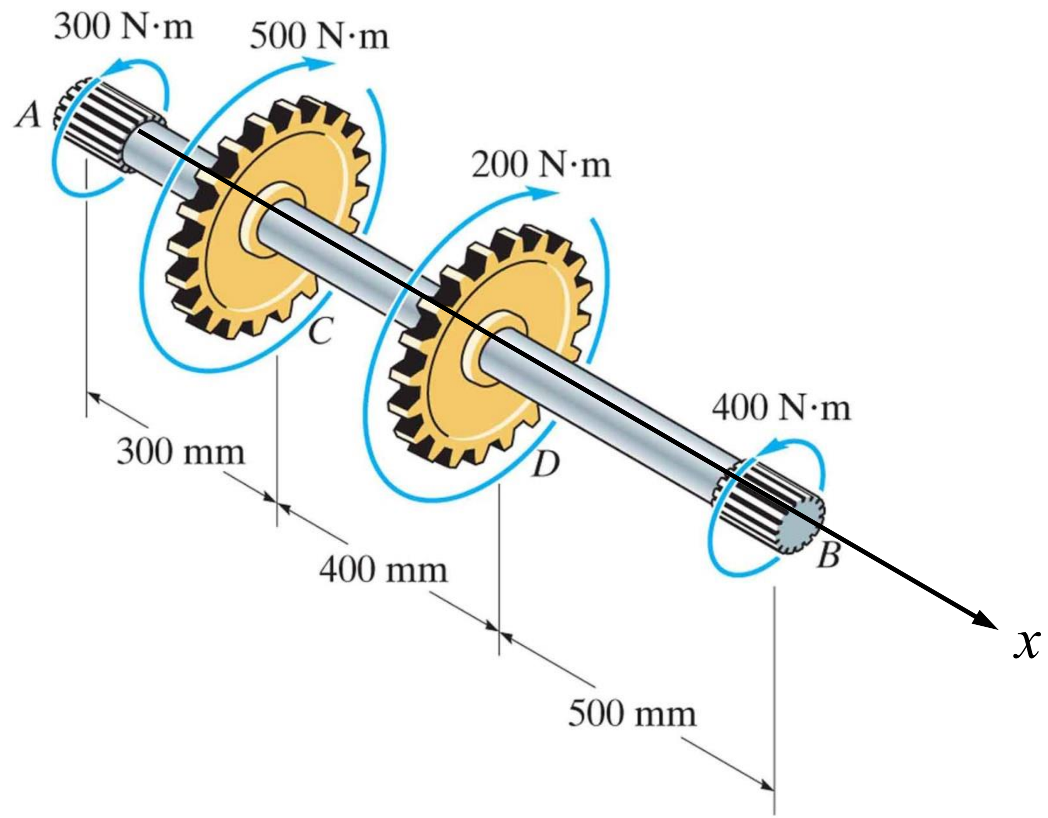


$$\phi = \sum_i \left(\frac{1}{G} \frac{T\,L}{J} \right)_i$$



Torsion: example F

The splined ends and gears attached to the A-36vsteel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of $40\ \mathrm{mm}$.



Reading assignment

- Chapter 5 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course