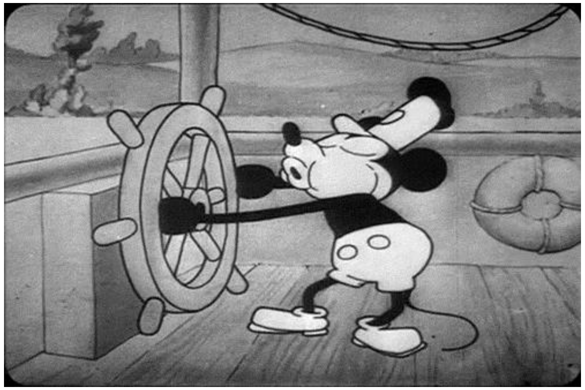
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



17 November 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

Lecture 16:
Unit 12: Torsion of shafts:
circular cross-section: *torsion formula*

17 November 2025

General information

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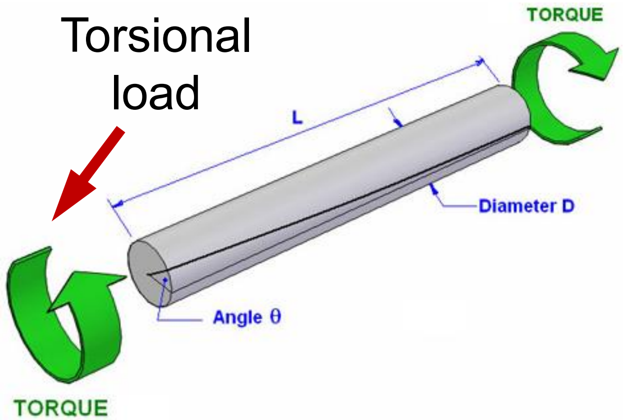
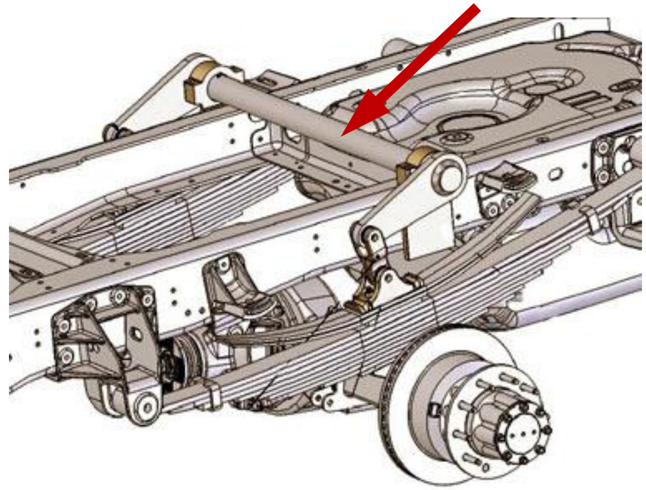
Torsion

Components subjected to torsional loads: just a few examples

Torsional load



Suspension mechanisms: *torsion bar*

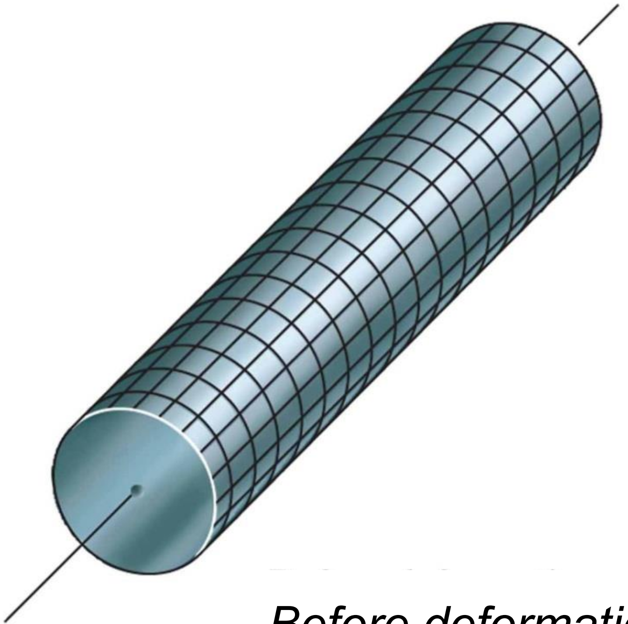


Gear trains: *shafts*

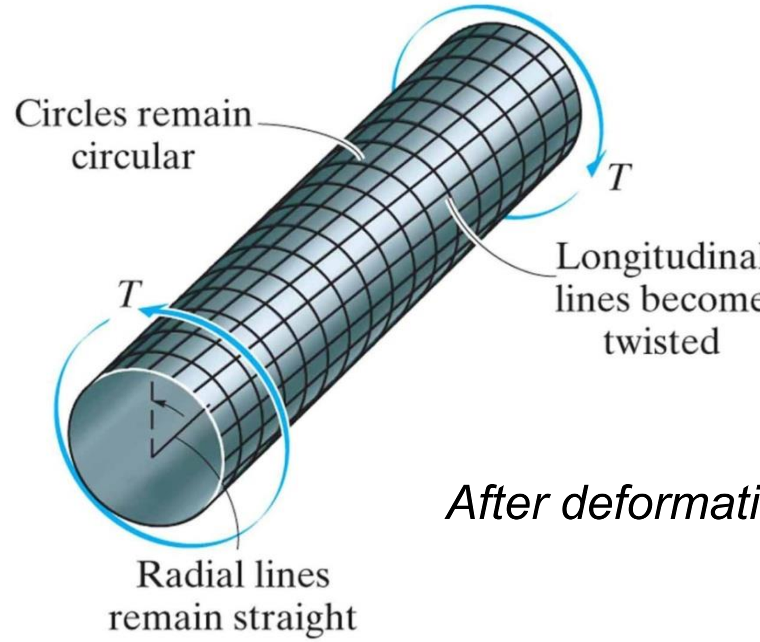
Pedals of bicycles

Torsion

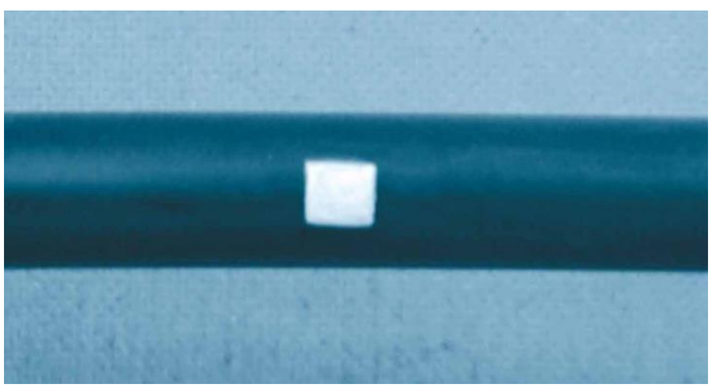
Component subjected to torsional load

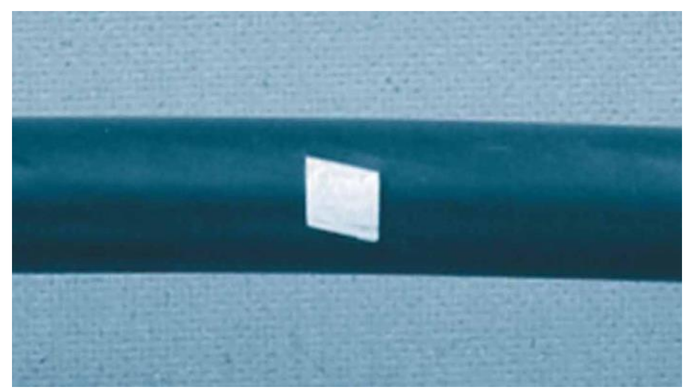


Before deformation

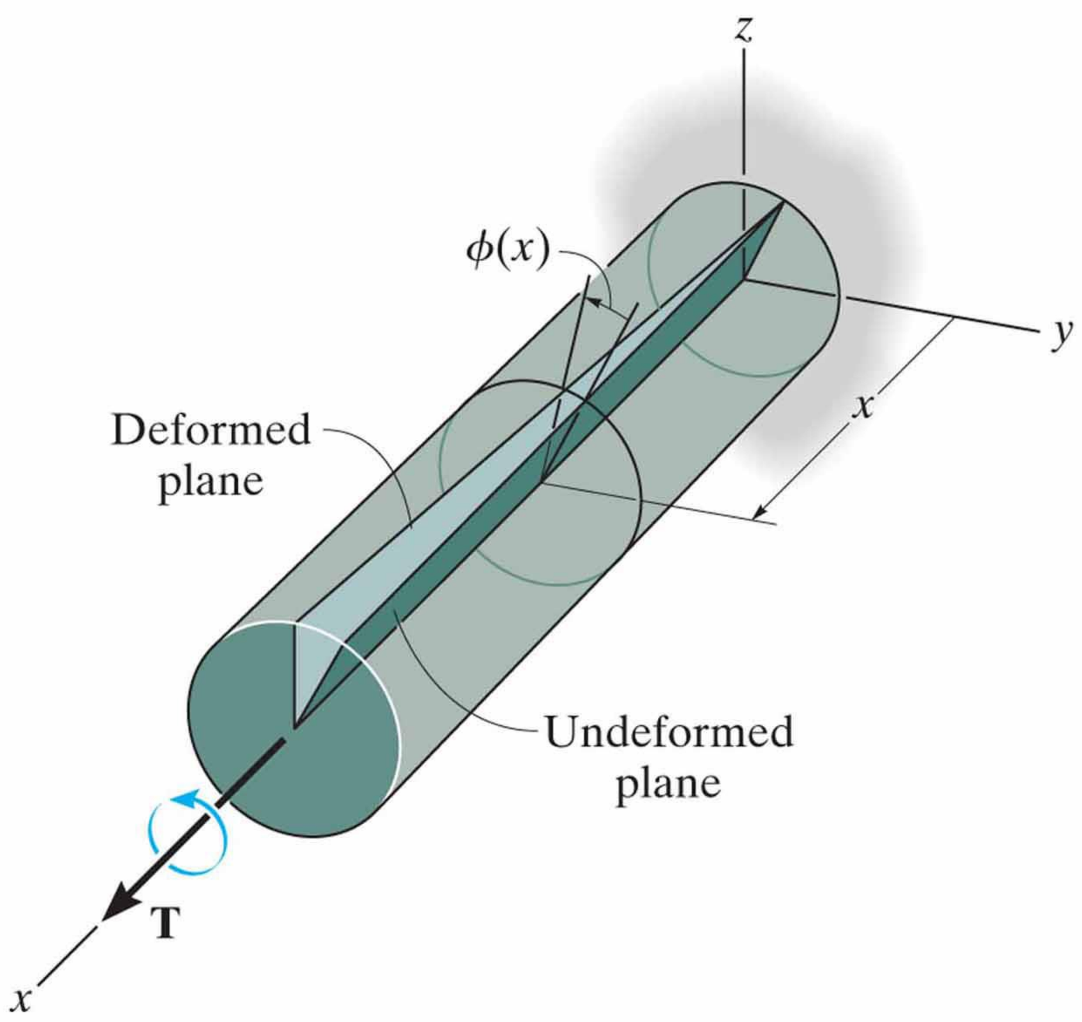


After deformation





Torsion: angle of twist

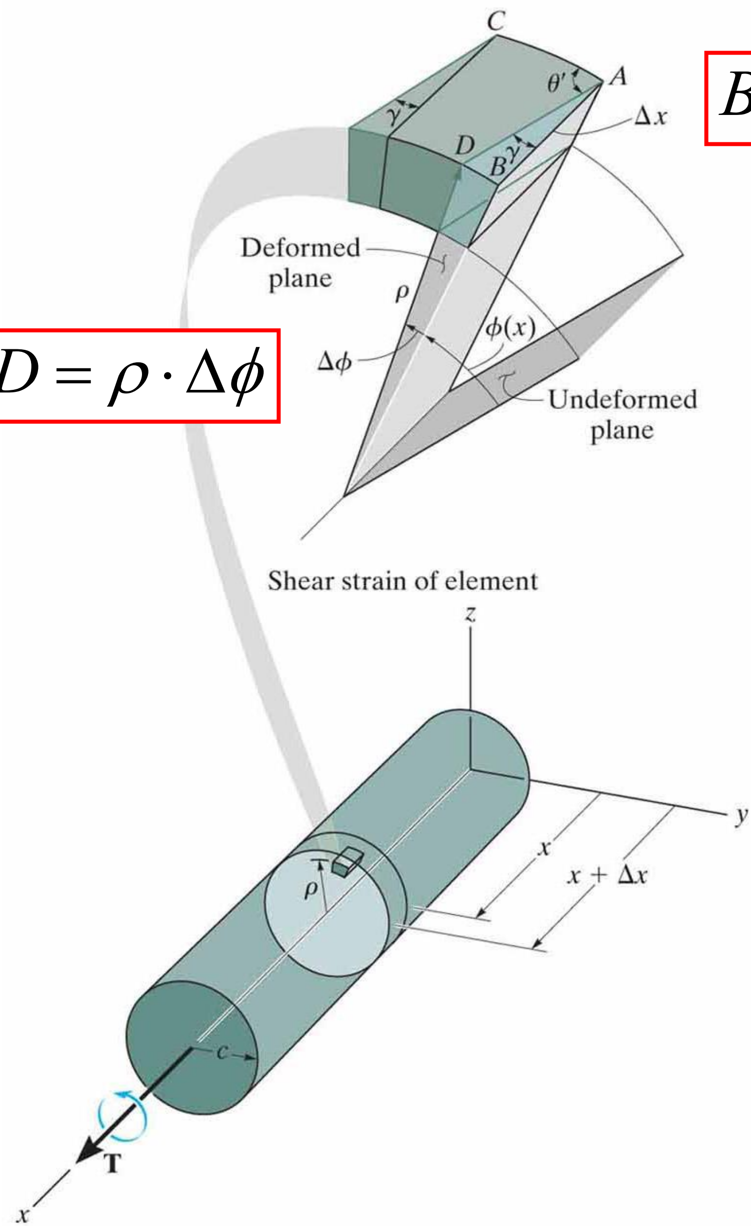


The angle of twist $\phi(x)$ increases as x increases.

Torsion: shear strains

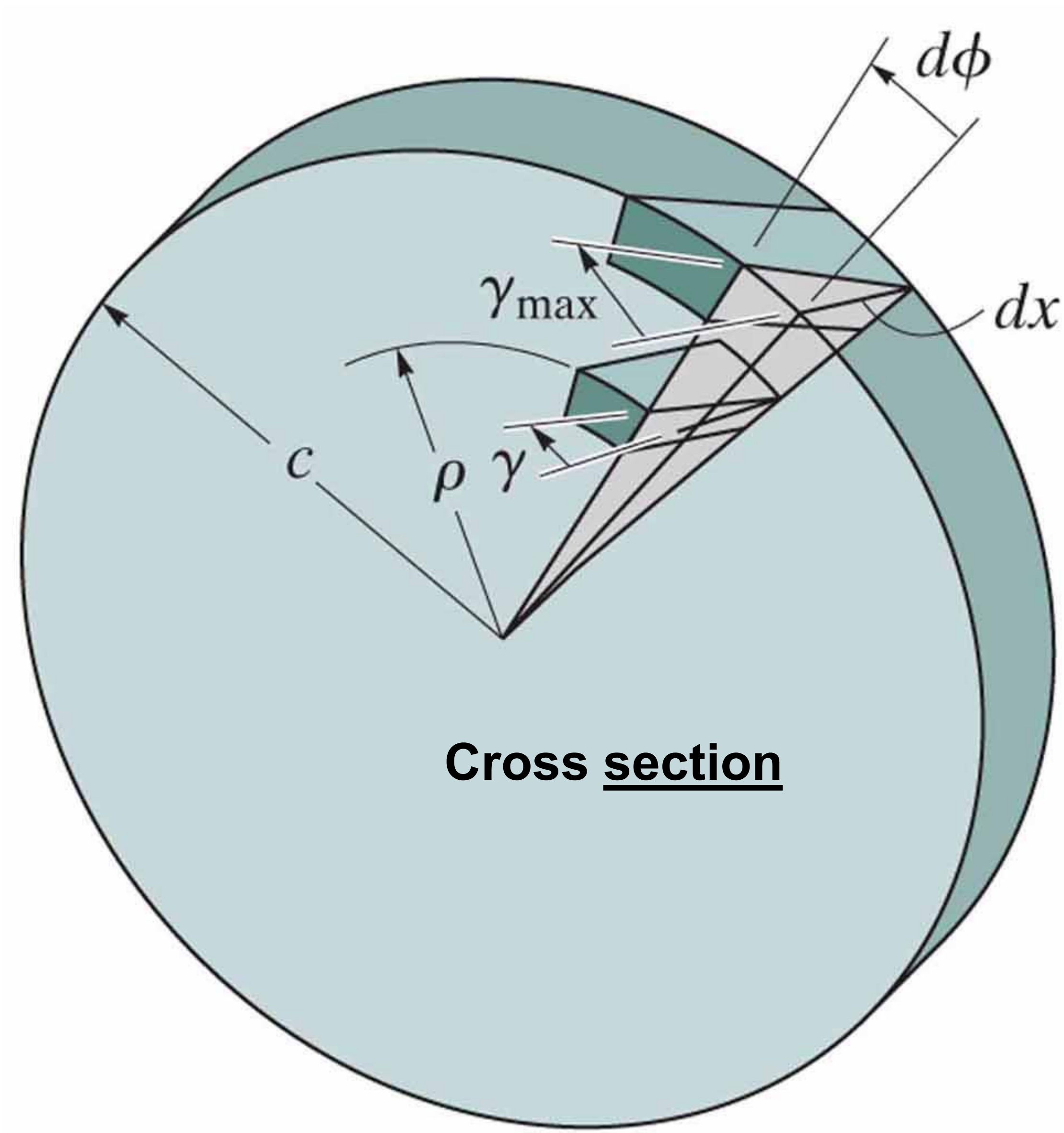
$$BD = \gamma \cdot \Delta x$$

$$BD = \rho \cdot \Delta\phi$$



Shear strain: $\gamma = \rho \dfrac{d\phi}{dx}$

Torsion: shear strains



$$\frac{\gamma}{\rho} = \frac{\gamma_{max}}{c}$$

Shear strains vary linearly within a section:

$$\gamma = \gamma(\rho) = \rho \frac{\gamma_{max}}{c}$$

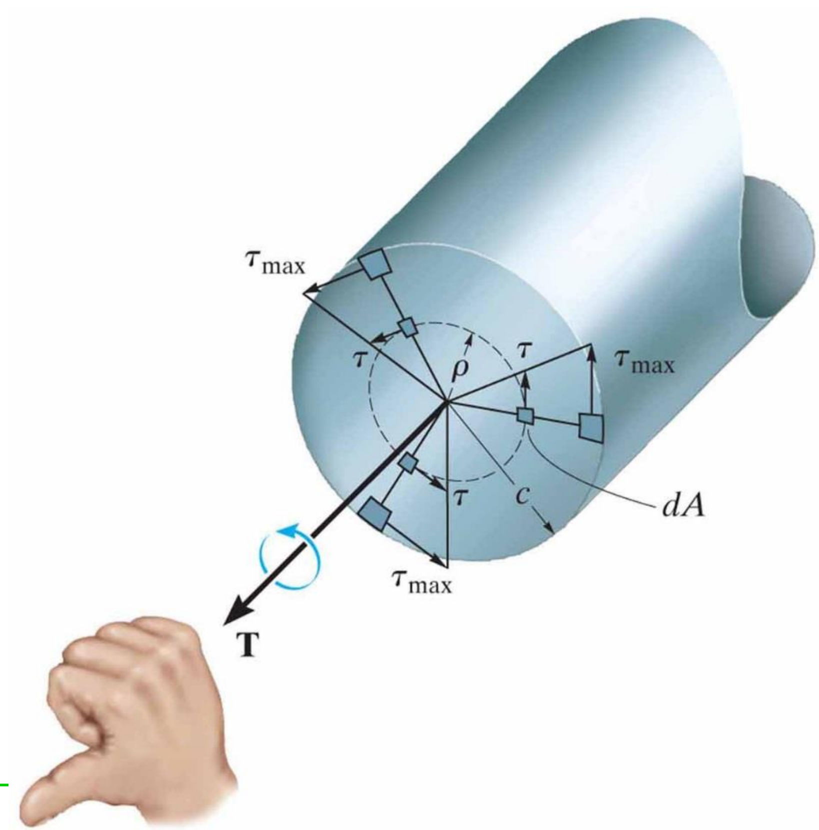
Torsion formula

According to Hook's law (linear elasticity):

$$(\tau = G \cdot \gamma)$$

Shear <u>stresses</u> also vary linearly within a section:

$$\tau = \tau(\rho) = \rho \frac{\tau_{max}}{c}$$



Differential Force:

$$dF = \tau \cdot dA$$

Differential Torque:

$$dT = \rho\,(\tau \cdot dA)$$

Torsion formula

Integrating torque: $$T = \int_A \rho\,(\tau \cdot dA) \;=\; \int_A \rho \left(\rho \frac{\tau_{\max}}{c} \right) dA$$

$$= \frac{\tau_{\max}}{c} \int_A \rho^2 \, dA$$

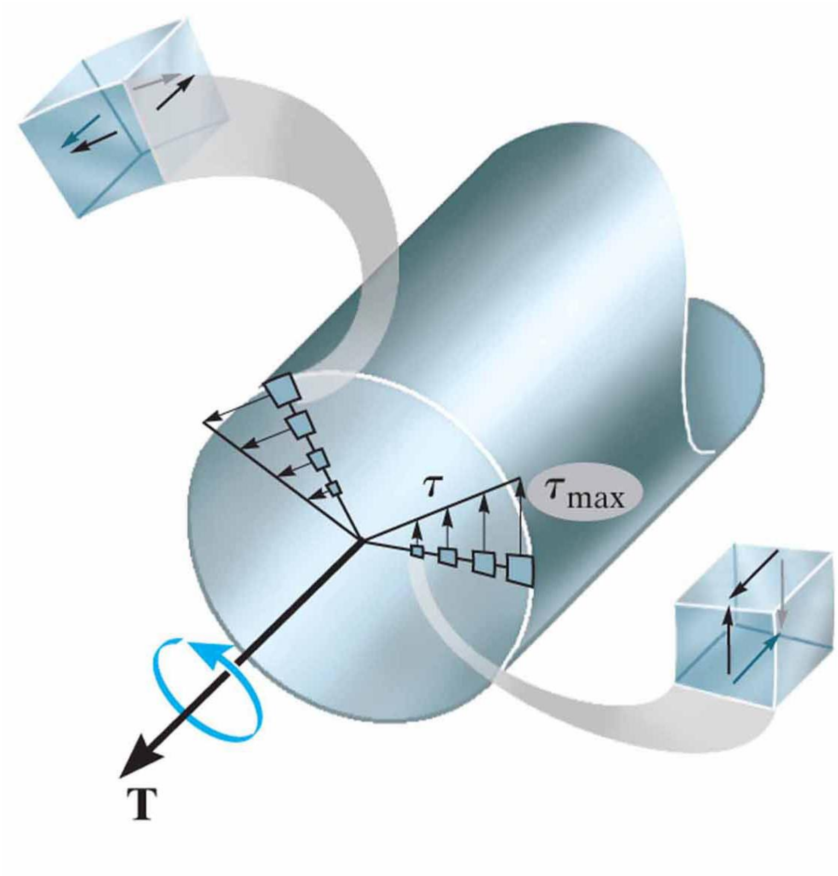
Define: $$J = \int_A \rho^2 \, dA$$ ← Polar area moment of inertia

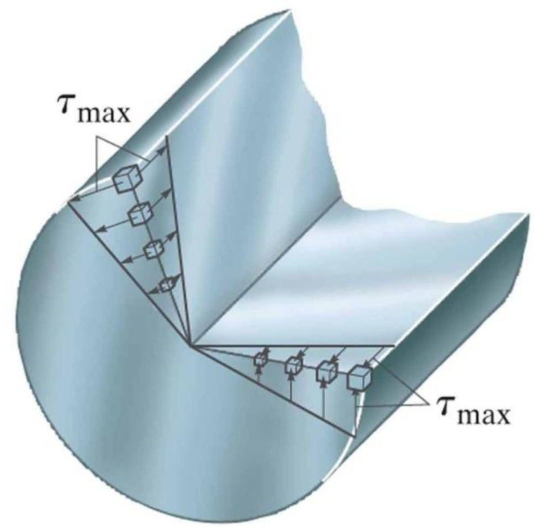
Torsion formula for stresses:
(linear elastic)

$$\tau_{\max} = \frac{T\,c}{J} \quad and \quad \tau = \tau(\rho) = \frac{T\,\rho}{J}$$

Torsion formula: solid circular bar

Linear variation of shear stress

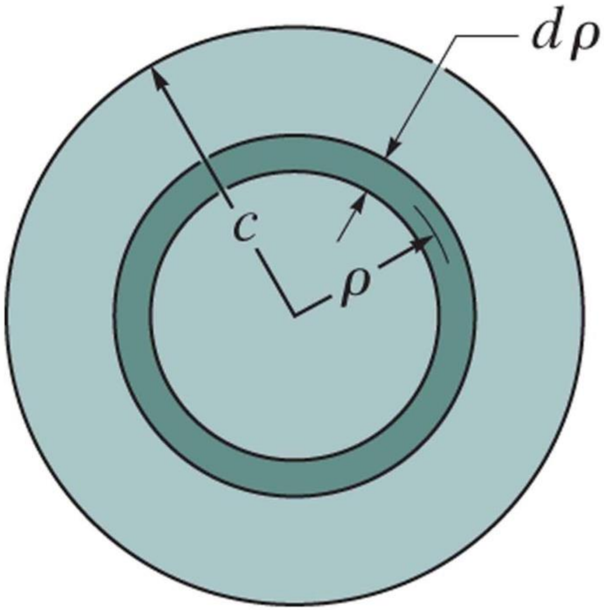




Shear stress varies linearly along each radial line of the cross section.

Torsion formula: polar area moment of inertia

Solid bar



$$J = \int_A \rho^2 \, dA$$

$$= \int_0^c \rho^2 \, (2\pi \, \rho \, d\rho)$$

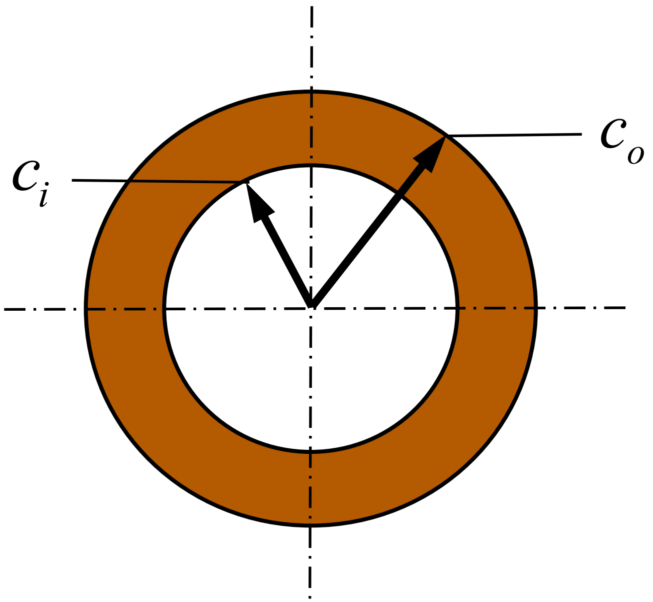
$$= 2\pi \int_0^c \rho^3 \, d\rho = \quad 2\pi \left(\frac{\rho^4}{4} \right)_0^c$$

Solid, circular, section:

$$J = \frac{\pi}{2} c^4$$

Torsion formula: polar area moment of inertia

Tubular bar



$$J = \int_A \rho^2 \, dA$$

$$= \int_{c_i}^{c_o} \rho^2 \, (2\pi \, \rho \, d\rho)$$

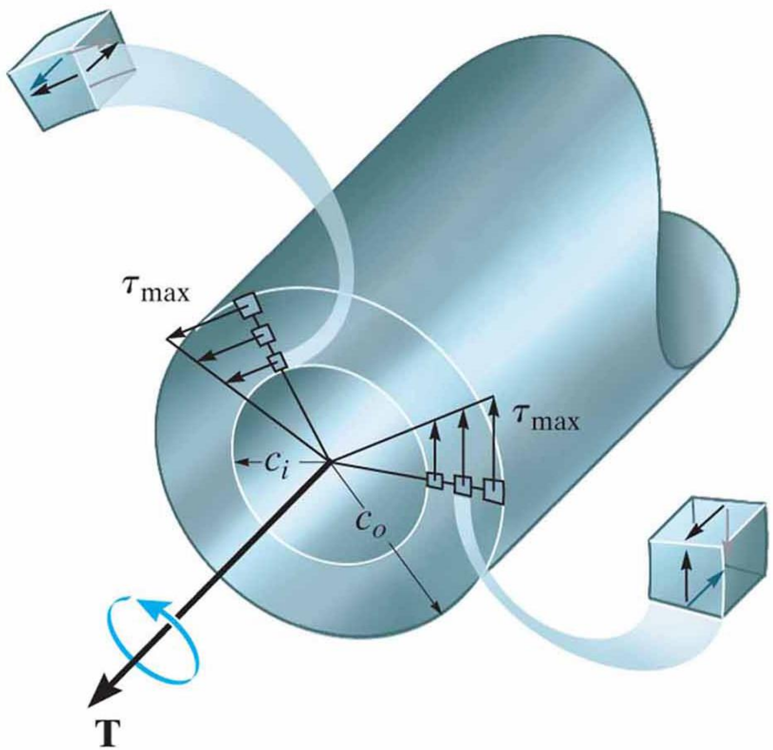
$$= 2\pi \int_{c_i}^{c_o} \rho^3 \, d\rho = \quad 2\pi \left(\frac{\rho^4}{4} \right)_{c_i}^{c_o}$$

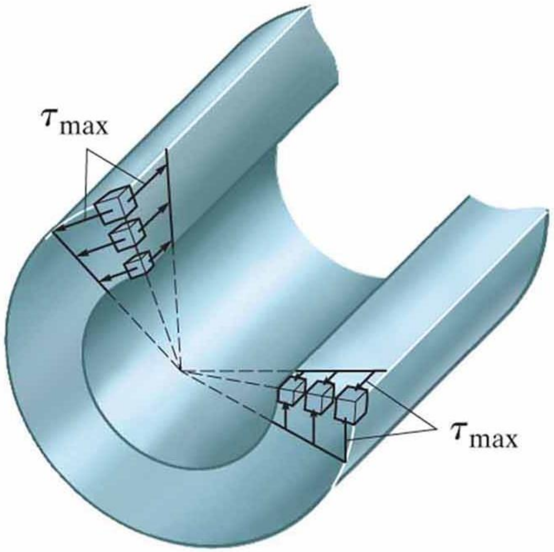
Tubular section:

$$J = \frac{\pi}{2} (c_o{}^4 - c_i{}^4)$$

Torsion formula: tubular bar

Linear variation of shear stress



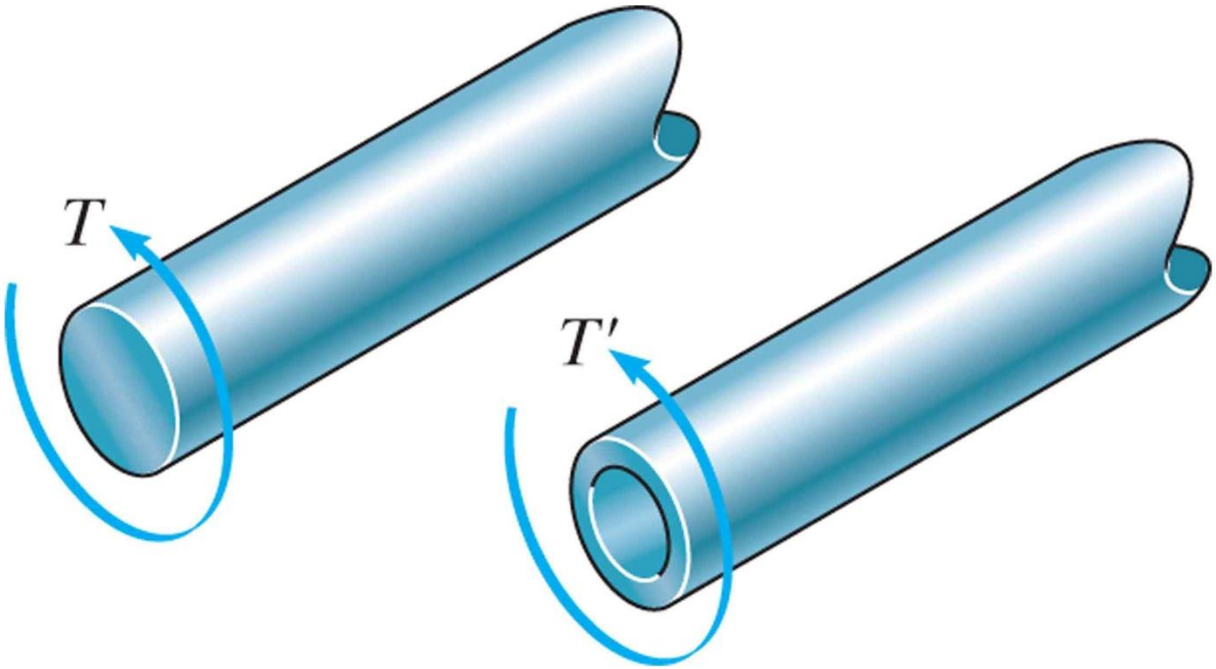


Shear stress varies linearly along each radial line of the cross section.

Torsion: example A

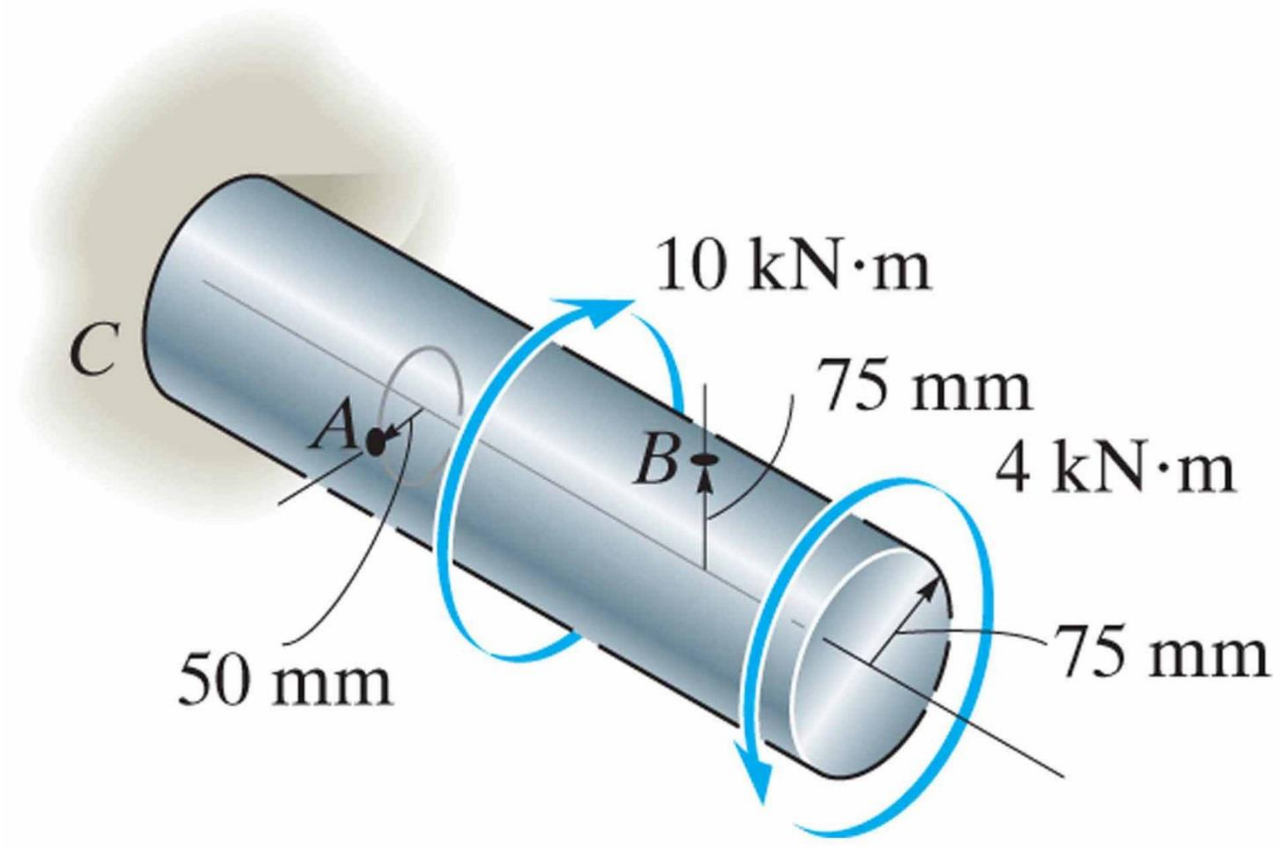
A shaft is made of a steel alloy having an allowable shear stress of $\tau_{allow} = 12\ \text{ksi}$. If the diameter of the shaft is $1.5\ \text{in.}$, determine the maximum torque T that can be transmitted.

What would be the maximum torque T' if a 1-in. diameter hole is bored through the shaft? Sketch the shear-stress distribution *along a radial line* in each case.



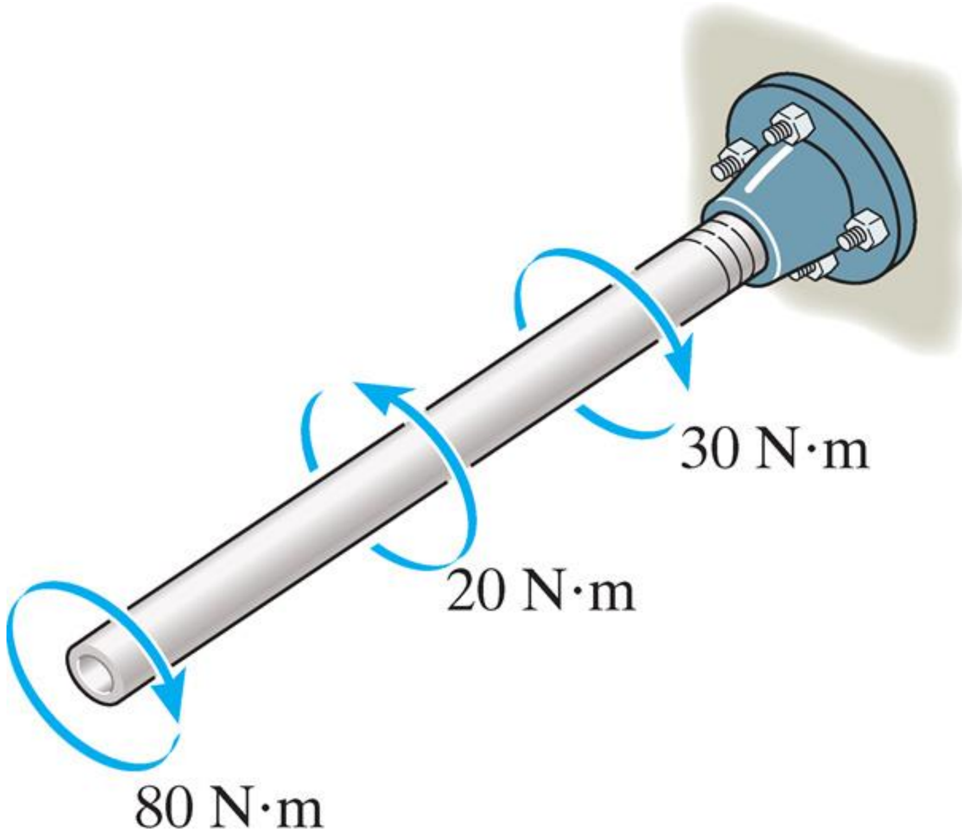
Torsion: example B

The solid shaft is fixed to the support at *C* and subjected to the torsional loadings shown. Determine the shear stress at points *A* and *B* and sketch the shear stress on volume (stress) elements located at these points.

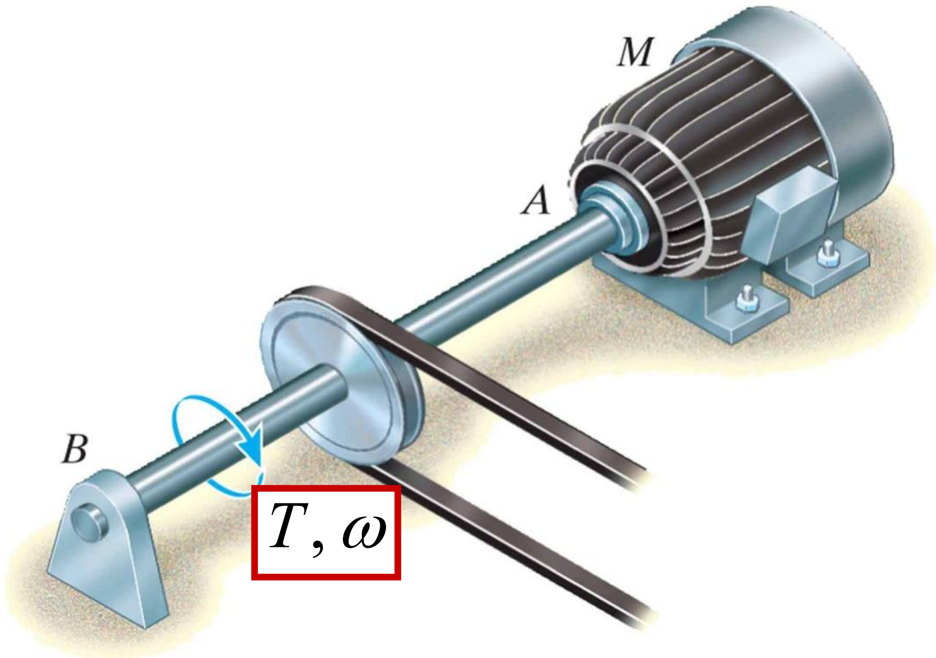


Torsion: example C

The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall and three torques are applied to it, determine the absolute maximum shear stress developed in the pipe.



Power transmission



$$P = T\,\omega$$

with:

$$\omega = 2\pi \cdot f$$

$\omega\left[\dfrac{rad}{sec}\right]$

$f\,[Hz]$

Reading assignment

- Chapter 5 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course