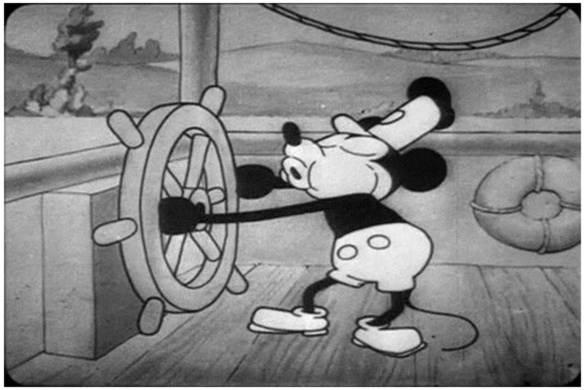
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



06 November 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

Lecture 11:
Unit 6: tension/compression of slender longitudinal bars: *statically indeterminate*

06 November 2025

General information

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Axial load

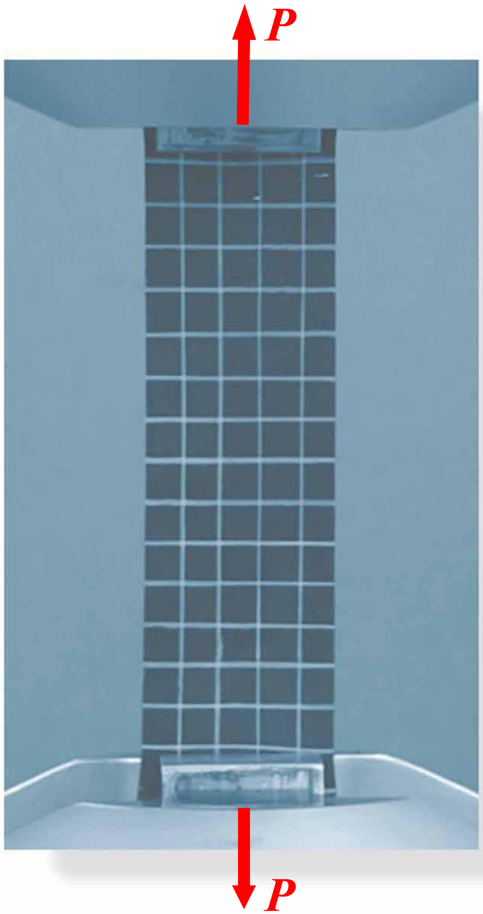


Figure: 04-01-UN-A
Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant's principle.

Note distortion lines: follow Saint-Venant's principle

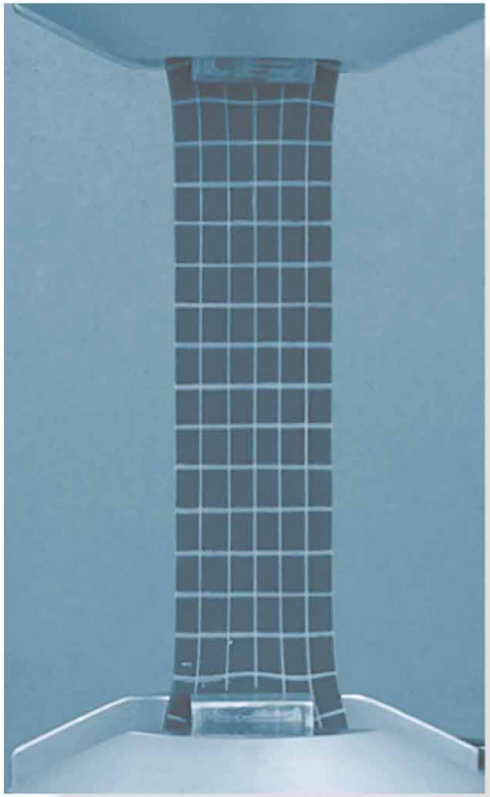
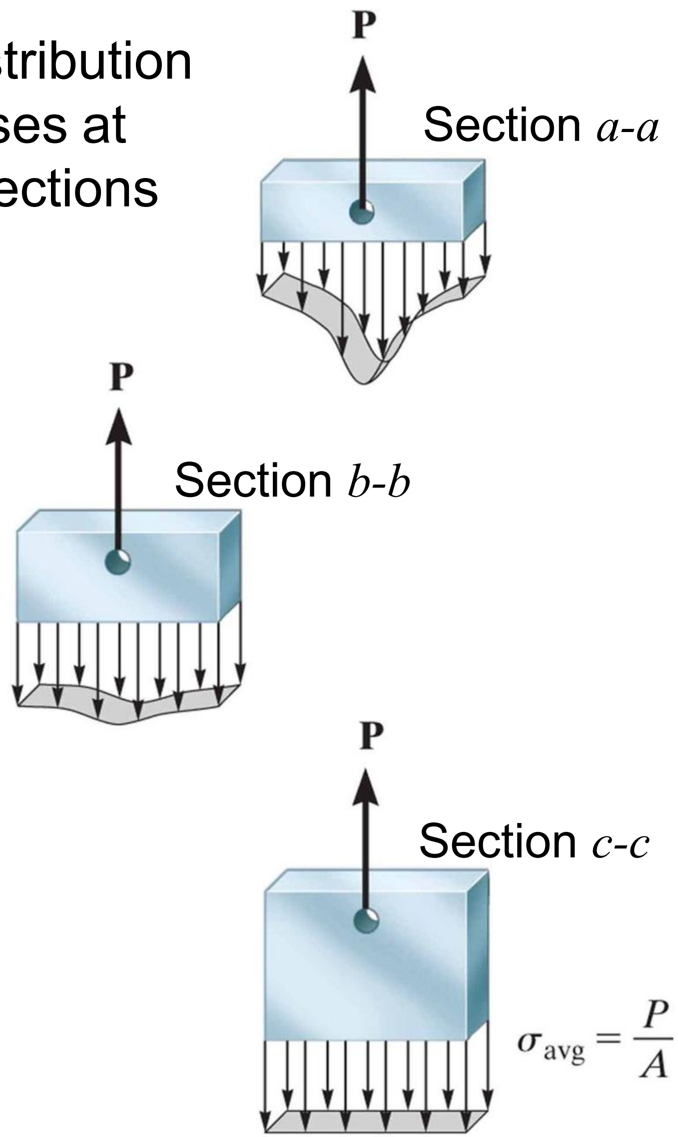
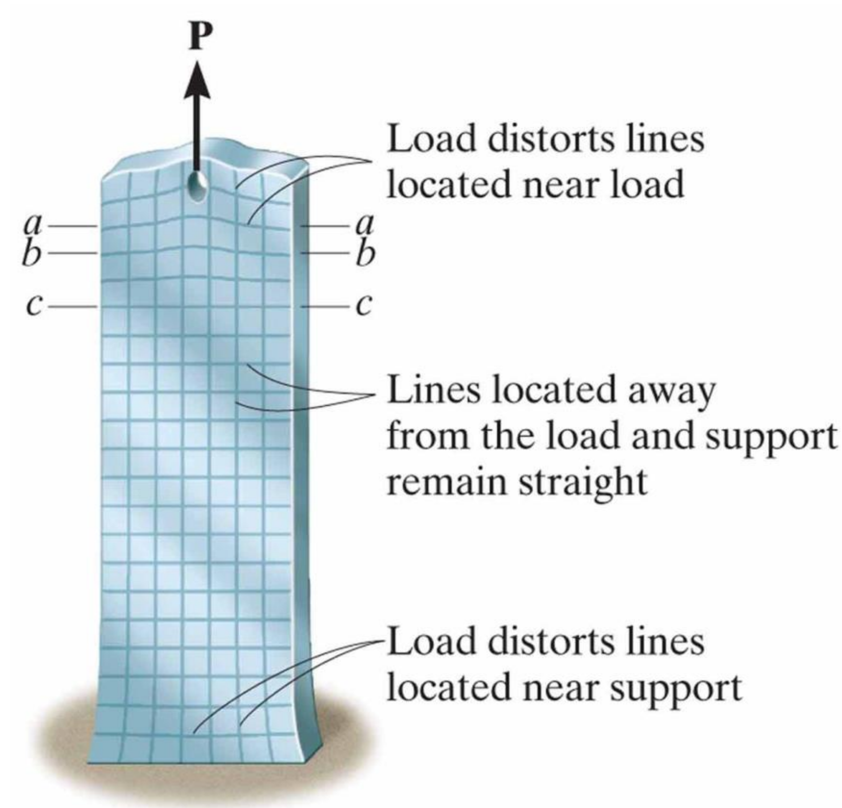


Figure: 04-01-UN-B
Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant's principle.

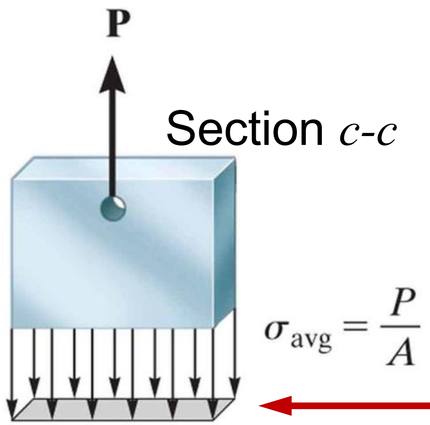
Axial load: Saint-Venant's principle

Internal distribution of stresses at various sections



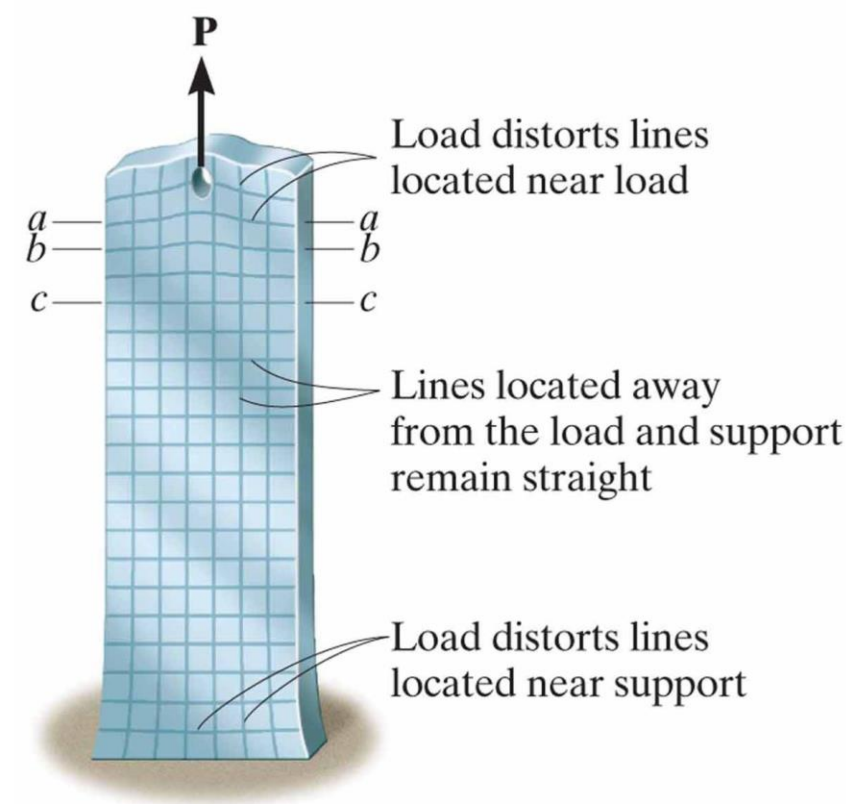


Axial load: Saint-Venant's principle



Nominal stress

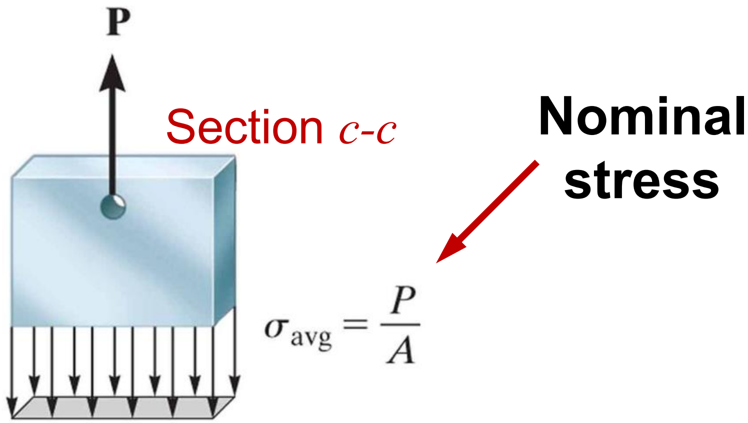
Uniform stress distribution

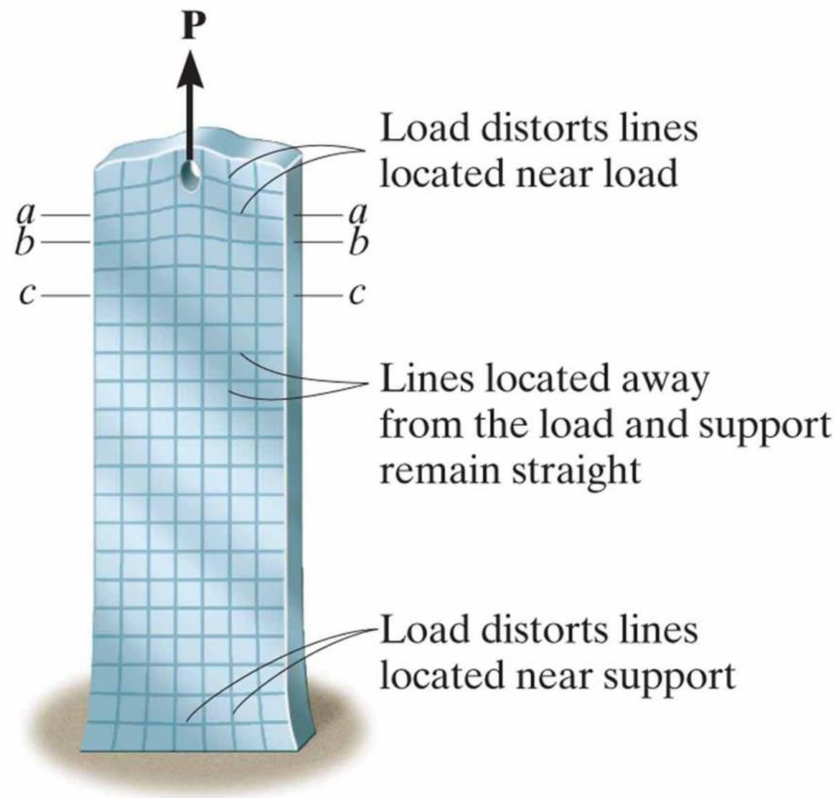


Axial load: Saint-Venant's principle

In your analyses, select locations (sections / points) located away from regions that are subjected to load application (to eliminate "end" effects)

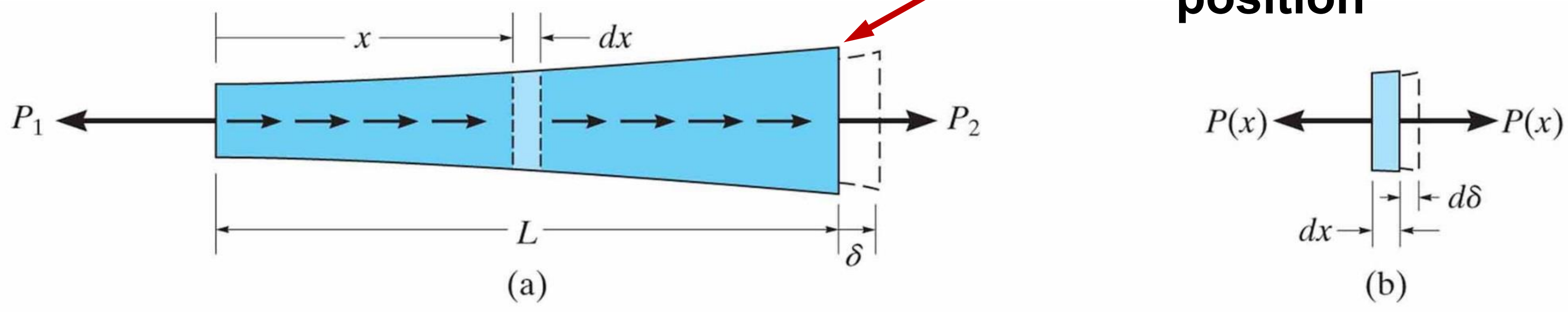
Saint-Venant's principle: stresses and strains within a section will approach their nominal values as the section locates away from regions of load application





Elastic deformation of an axially loaded member

Section is a function of position

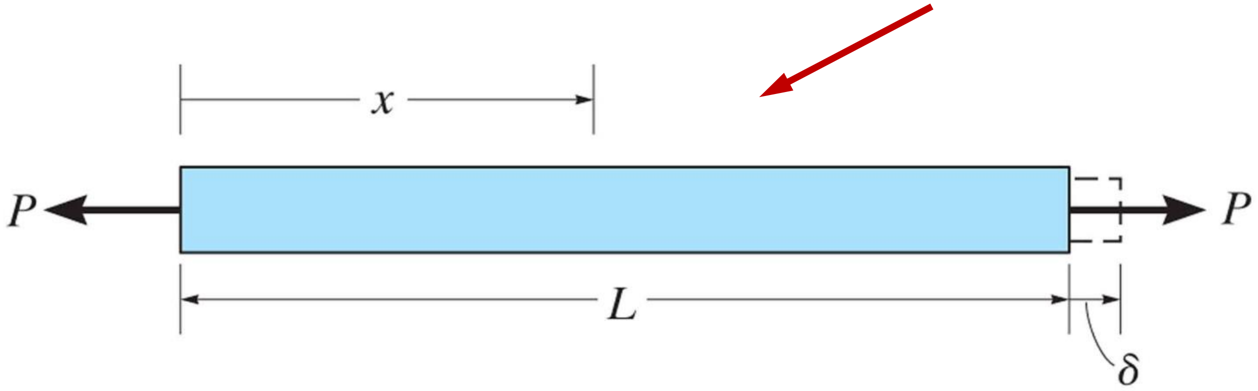


$$\sigma = \frac{P(x)}{A(x)} \qquad and \qquad \varepsilon = \frac{d\delta}{dx}$$

Therefore, $d\delta = \dfrac{P(x)\,dx}{A(x)\,E} \longrightarrow \delta = \displaystyle\int_0^L \frac{P(x)}{A(x)\,E}\,dx$

Elastic deformation of an axially loaded member

Constant load and cross-sectional area

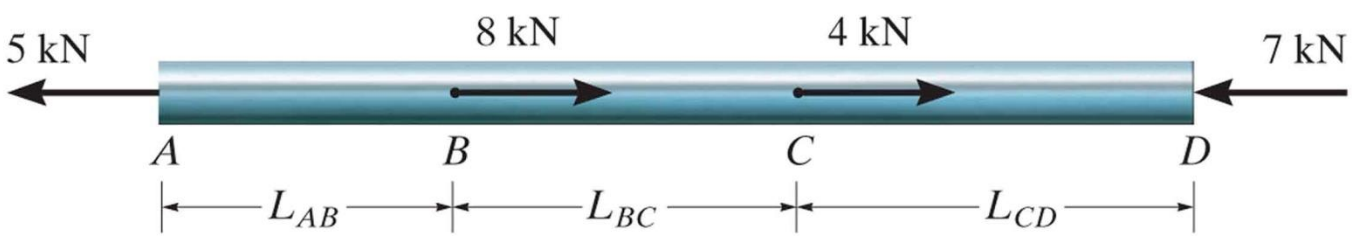


Elastic deformation:

$$\delta = \int_0^L \frac{P(x)}{A(x)\,E}\,dx = \frac{P}{A\,E}\int_0^L dx = \frac{P\,L}{A\,E}$$

Elastic deformation of an axially loaded member

Bar subjected to multiple axial loads

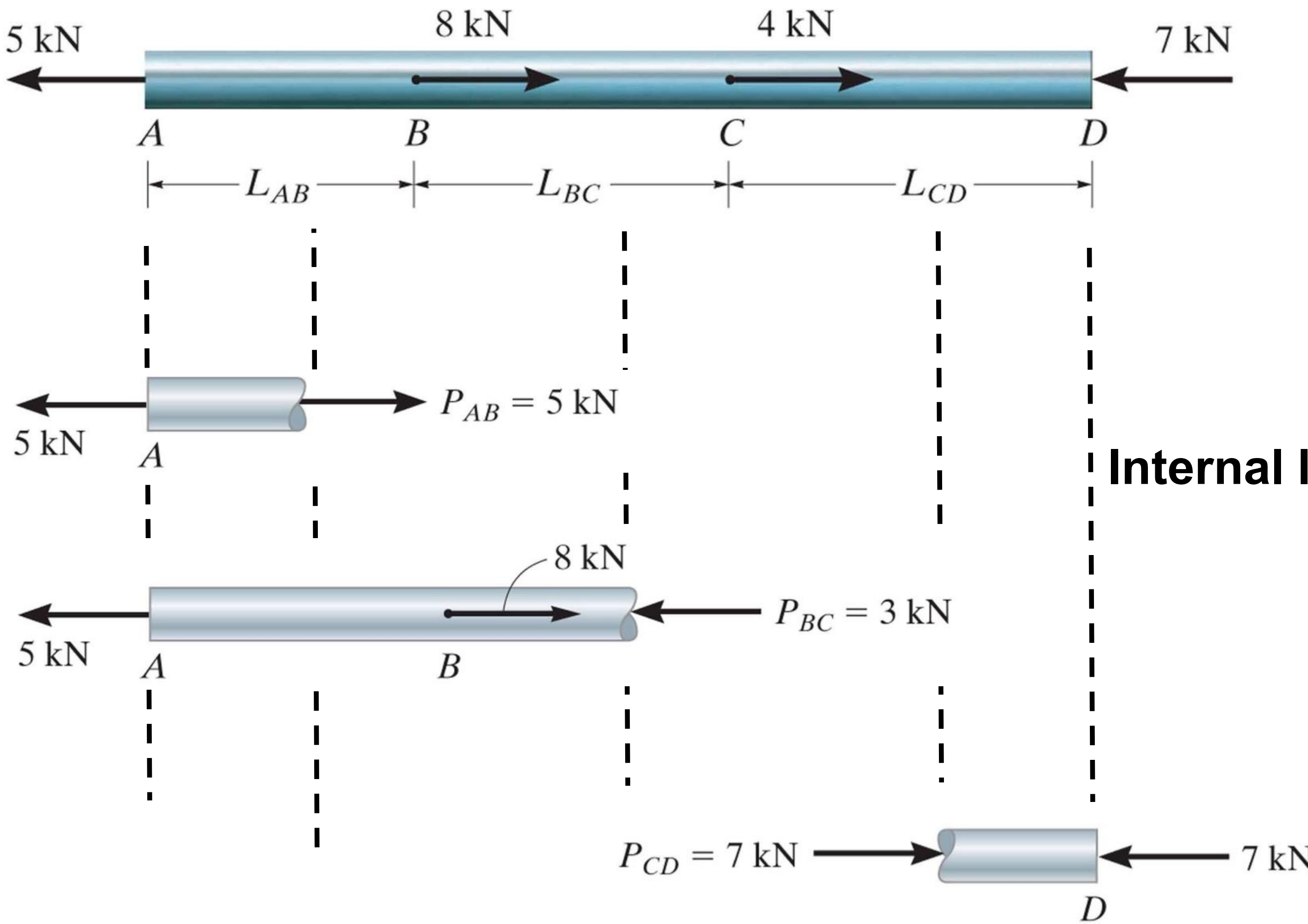


Elastic deformation:

$$\delta = \sum_i \left(\frac{P\,L}{A\,E} \right)_i$$

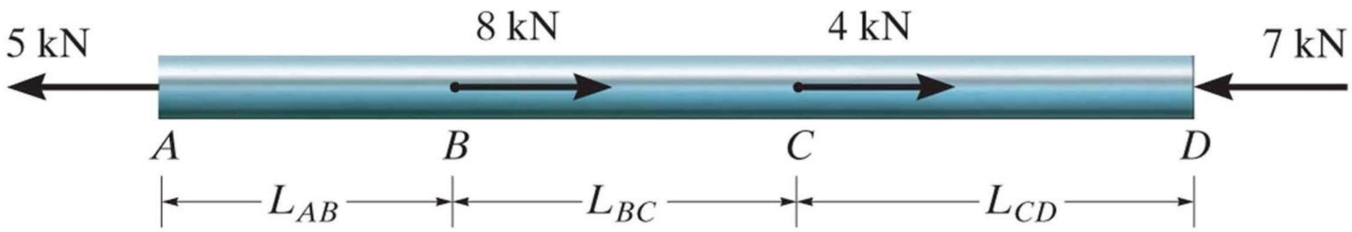
Elastic deformation of an axially loaded member

Procedure for analysis



Internal loads

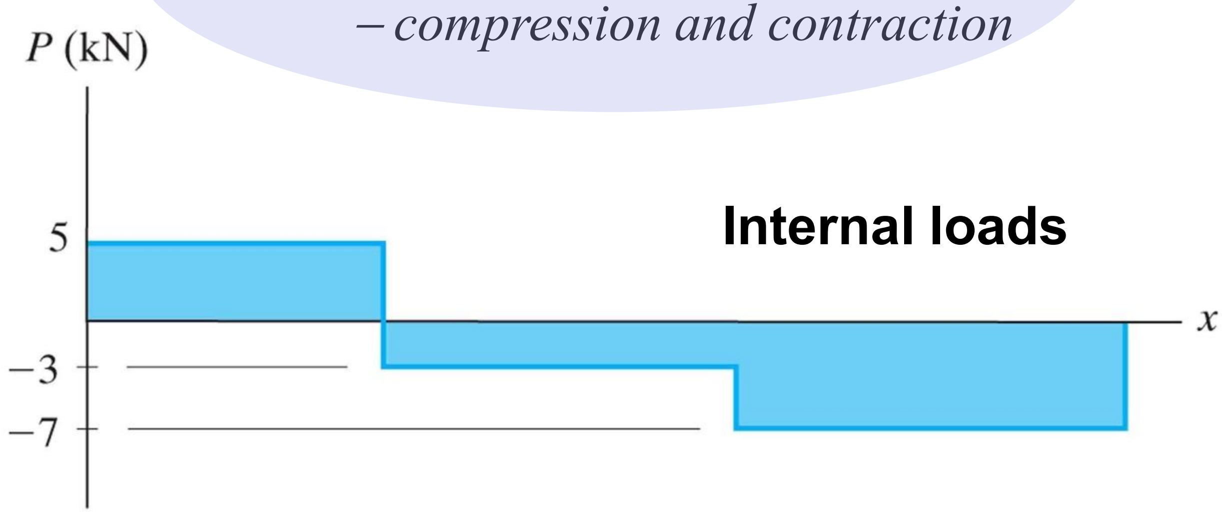
Elastic deformation of an axially loaded member
Procedure for analysis



Sign convention:

$+$ *tension and elongation*

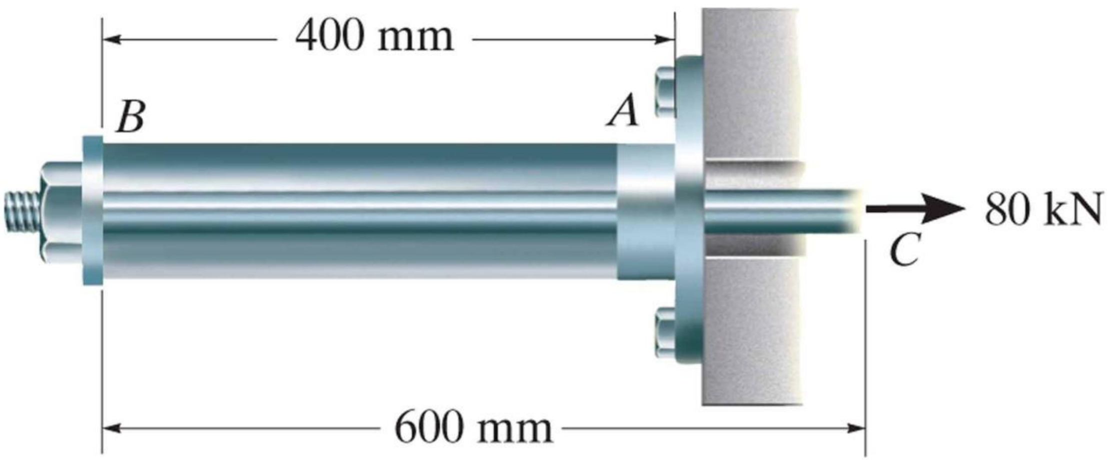
$-$ *compression and contraction*



Internal loads

Axial load: example D

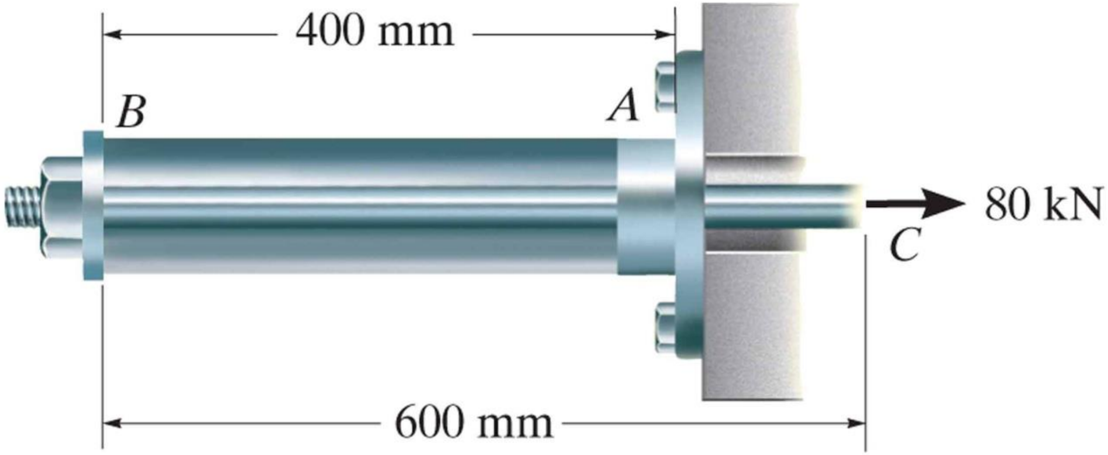
The assembly shown consists of an aluminum tube AB having a cross sectional area of $400\ \text{mm}^2$. A steel rod having a diameter of $10\ \text{mm}$ is attached to a rigid collar and passes through the tube. If a tensile load of $80\ \text{kN}$ is applied to the rod, determine the displacement of the end C of the rod. Elastic modules: $E_{\text{steel}} = 200\ \text{GPa}$ and $E_{\text{alum}} = 200\ \text{GPa}$



Approach:

1) Determine internal loading
2) Compute displacement

Axial load: example D

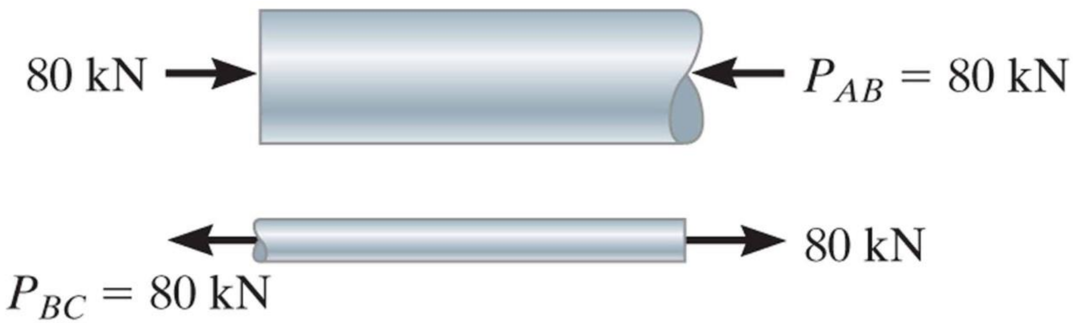


Displacement of C:

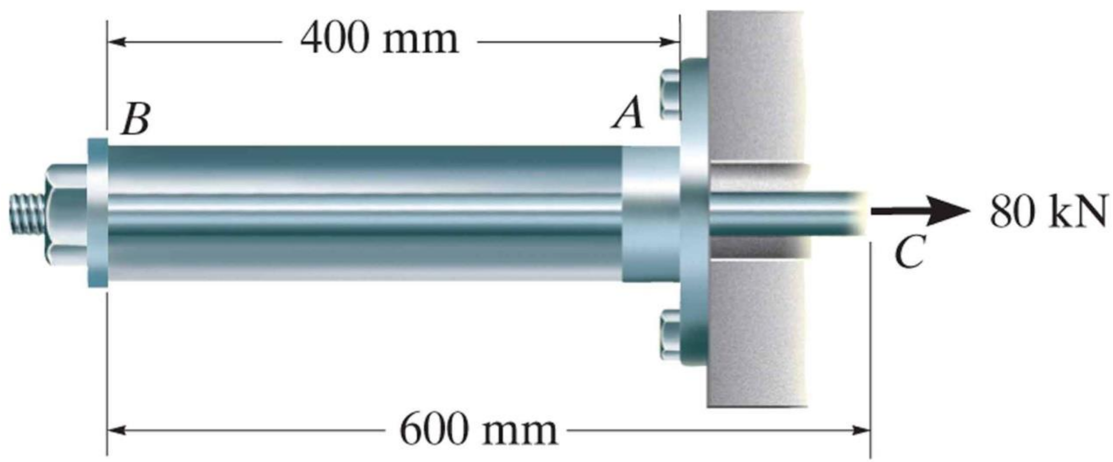
$$\delta_C = \delta_B + \delta_{C/B}$$

Axial load: example D

(1) Internal loading

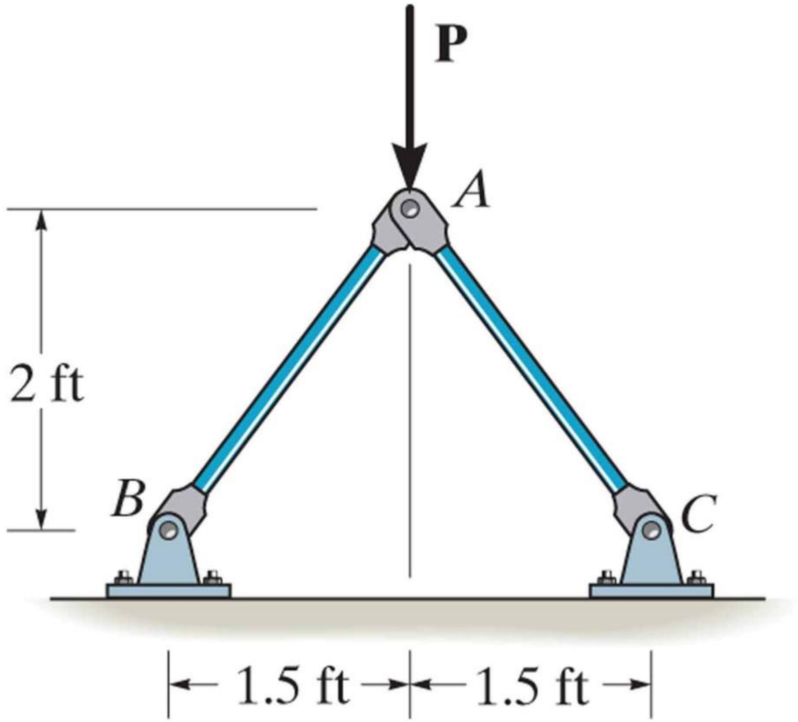


(2) → find displacement at C



Axial load: example E

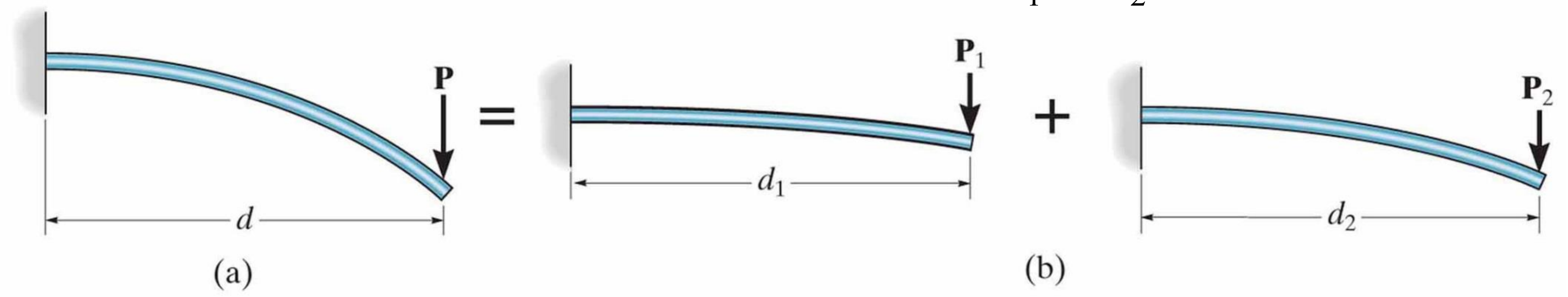
The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of $1.50\ \text{in}^2$. If a vertical force of is applied to point A, determine its vertical displacement at A.



Principle of superposition

Applied when a component is subjected to complicated loading conditions → break a complex problem into series of simple problems

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$$



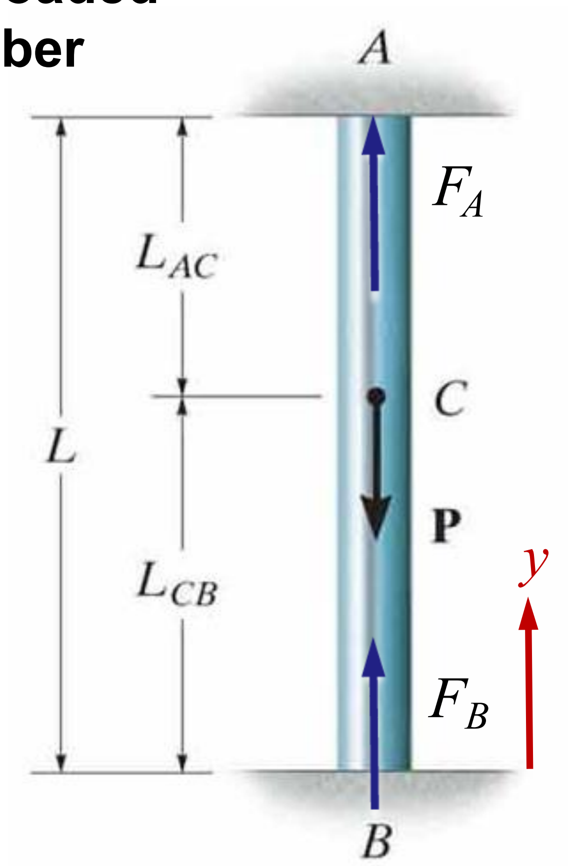
Can only be applied for:

(a) small deformations;

(b) deformations in the elastic (linear) range of the $\sigma-\varepsilon$ diagram

Statically indeterminate axially loaded member

Axially loaded member



In this case, only one equilibrium equation:

$$+\uparrow \quad \sum F_y = 0 \, ;$$

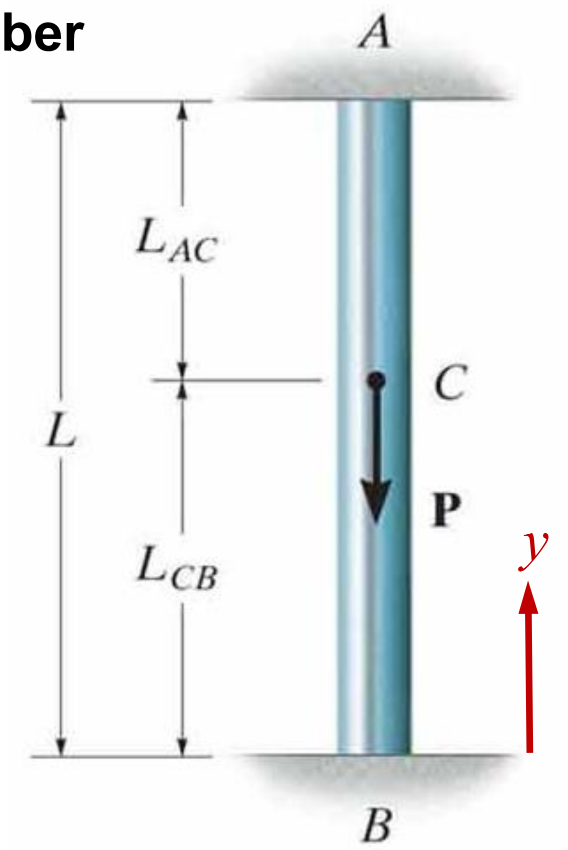
$$F_B + F_A - P = 0 \qquad (1)$$

→ Statically indeterminate problem

Need additional equations!!

Statically indeterminate axially loaded member

Axially loaded member



Additional equations are obtained by applying:

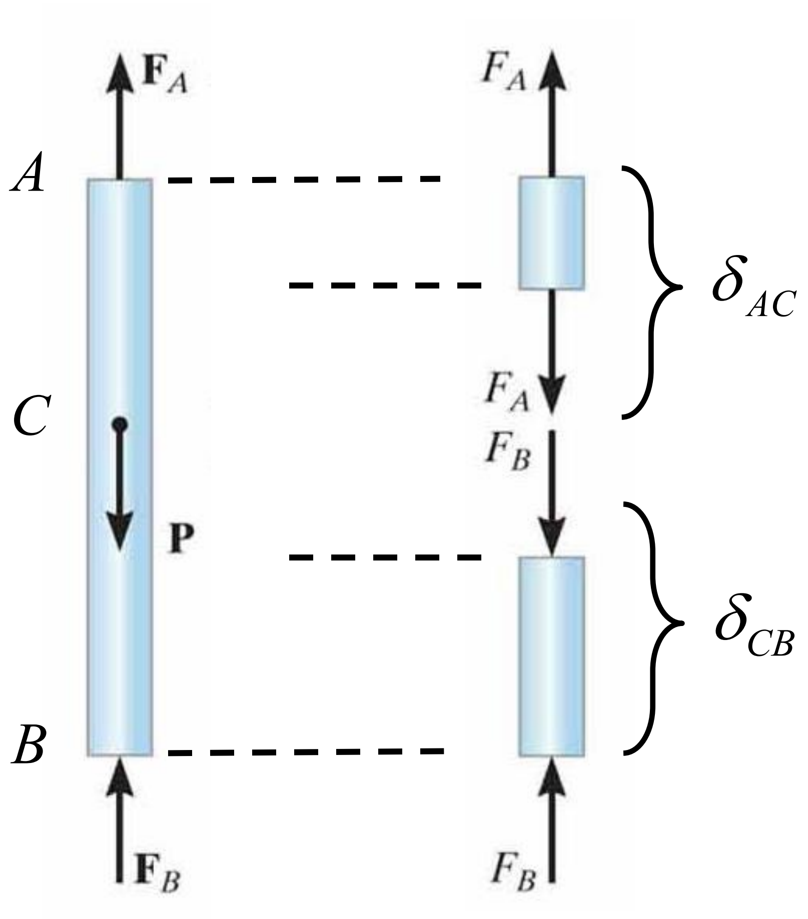
Compatibility or kinematic equations

↑

Load-displacement equations

$$\delta_{A/B} = 0$$

Statically indeterminate axially loaded member

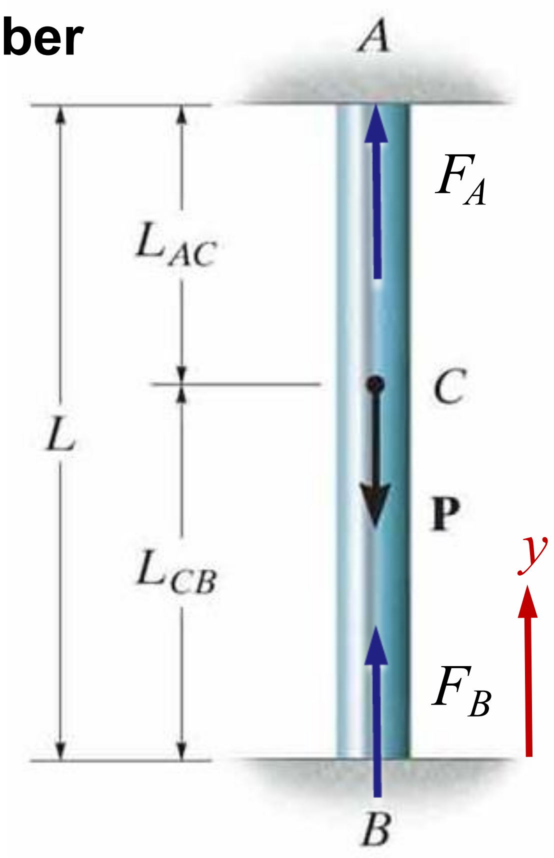


***Compatibility or kinematic equations*:**

$$\frac{F_A L_{AC}}{A E} - \frac{F_B L_{CB}}{A E} = 0 \qquad (2)$$

Statically indeterminate axially loaded member

Axially loaded member



Forces ***are obtained by solving system of equations*:**

Equilibrium

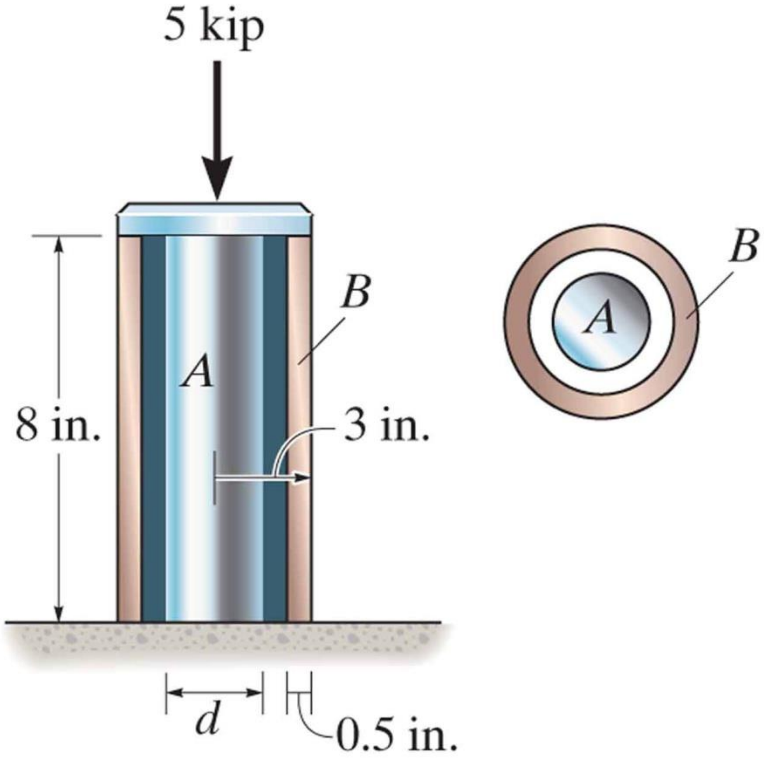
$$F_B + F_A - P = 0 \qquad (1)$$

$$\frac{F_A\, L_{AC}}{A\, E} - \frac{F_B\, L_{CB}}{A\, E} = 0 \qquad (2)$$

Compatibility

Axial load: example F

The 304 stainless steel post A has a diameter of $d = 2.0\text{ in}$ and is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the average normal stress developed in the post and the tube.



Approach:

1) Apply equilibrium equations
2) Apply compatibility equations
3) Solve for stresses

Reading assignment

- Chapters 3 and 4 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- As indicated on webpage of our course