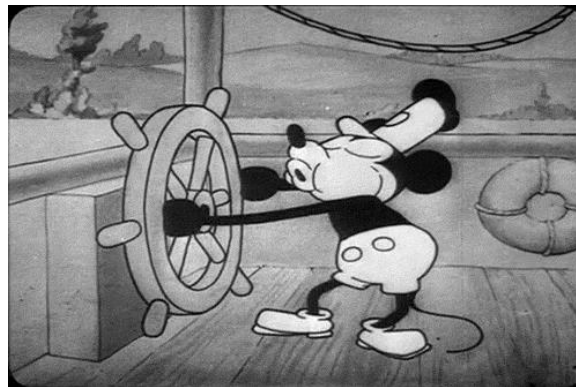


WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS ES-2502, B'2025

We will get started soon...



05 November 2025



WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS ES-2502, B'2025

Lecture 10:
Unit 6: tension/compression of slender
longitudinal bars: general case

05 November 2025



General information

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Strain: example A

A concrete cylinder having a diameter of 6.0 in and a gauge length of 12 in is tested in compression. The results of the test are reported in the table as load versus contraction. Draw stress-strain diagram and estimate modulus of elasticity.

Displacement, in	Load, kip
0.0000	0.0
0.0006	5.0
0.0012	9.5
0.0020	16.5
0.0026	20.5
0.0036	25.5
0.0040	30.0
0.0045	34.5
0.0050	38.5
0.0062	46.5
0.0070	50.0
0.0075	53.0



Strain: example A

Compute stress and strain table:

Displacement, in	Load, kip
0.0000	0.0
0.0006	5.0
0.0012	9.5
0.0020	16.5
0.0026	20.5
0.0036	25.5
0.0040	30.0
0.0045	34.5
0.0050	38.5
0.0062	46.5
0.0070	50.0
0.0075	53.0

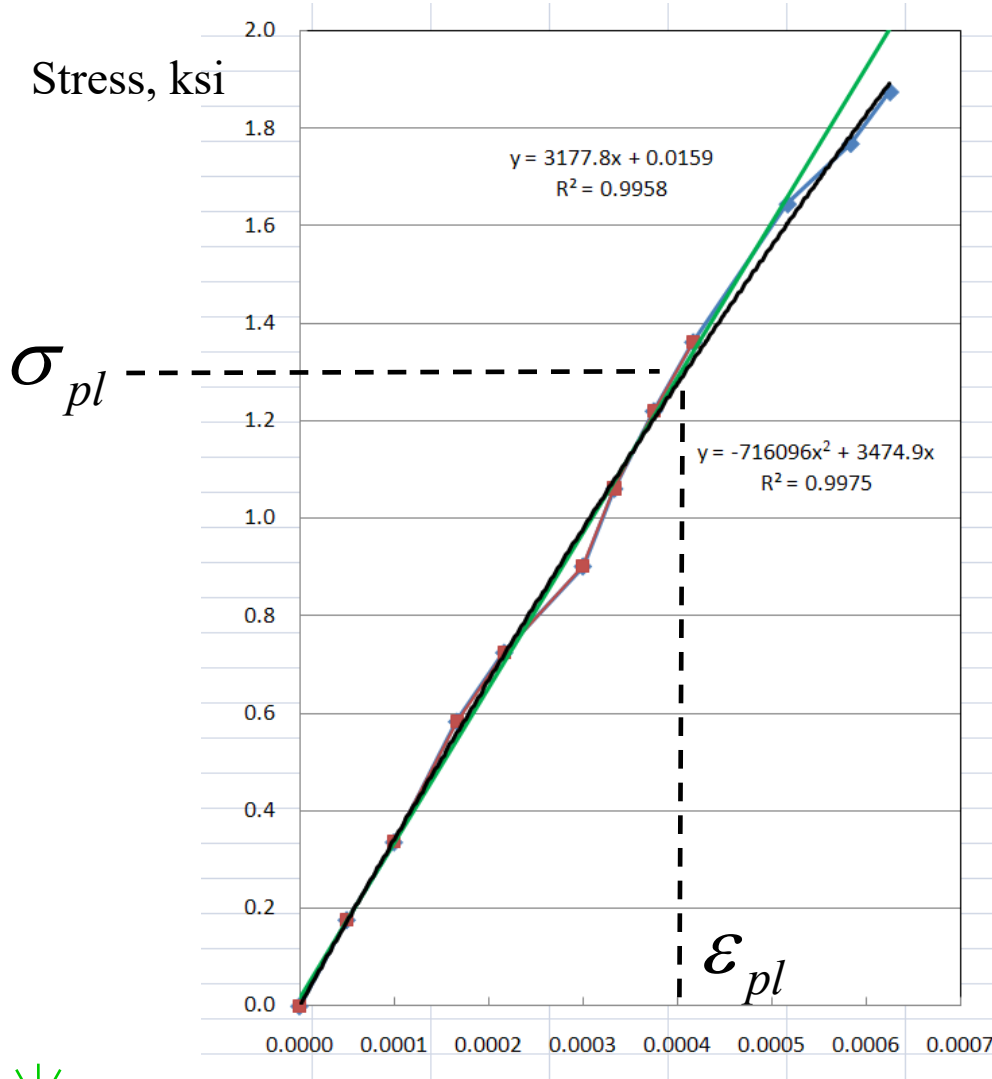


Strain, in/in	Stress, kpsi
0.000000	0.000
0.000050	0.177
0.000100	0.336
0.000167	0.584
0.000217	0.725
0.000300	0.902
0.000333	1.061
0.000375	1.220
0.000417	1.362
0.000517	1.645
0.000583	1.768
0.000625	1.874



Strain: example A

Plot data and estimate yield point:



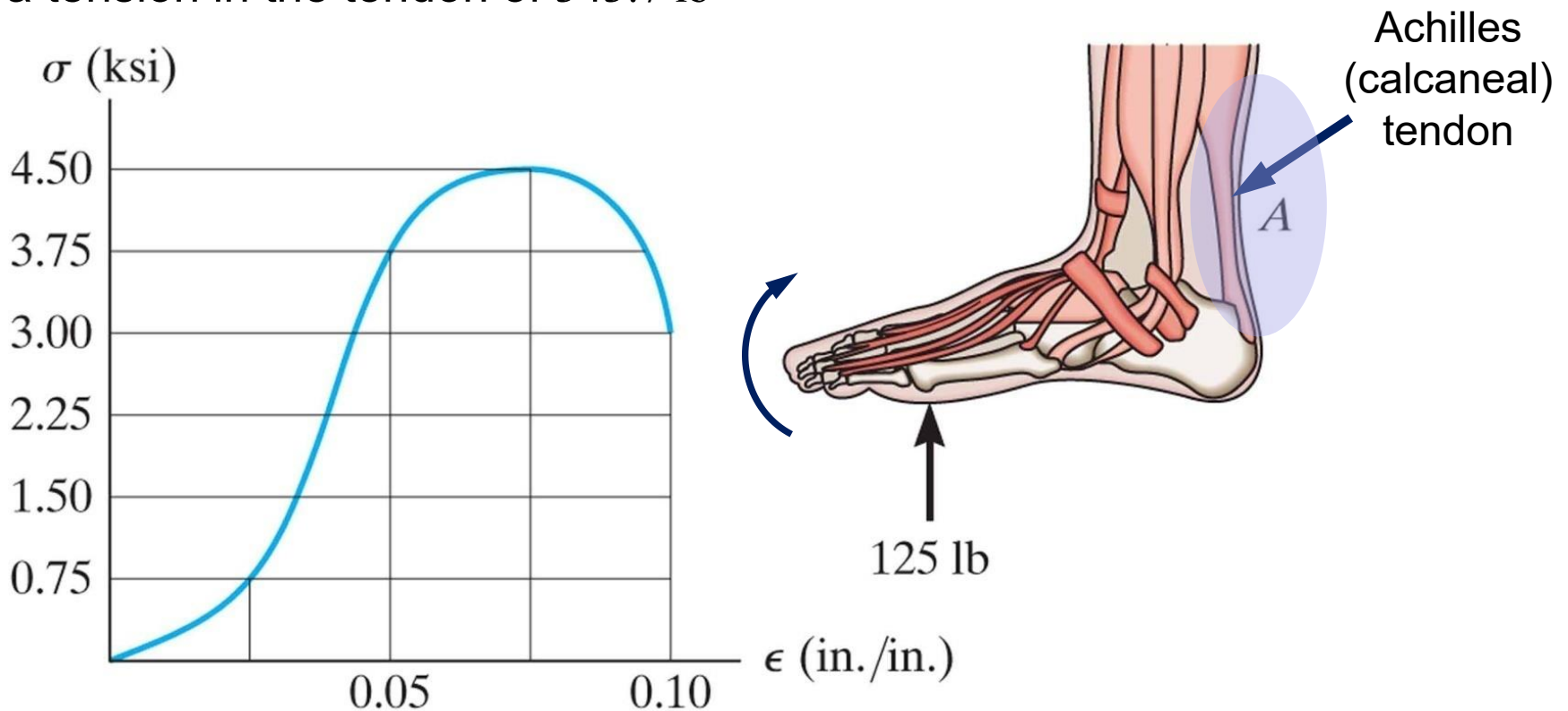
$$E = \frac{\sigma_{pl} - 0}{\epsilon_{pl} - 0} \approx 3.27 \times 10^3 \text{ ksi}$$

Strain, in/in



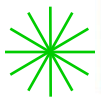
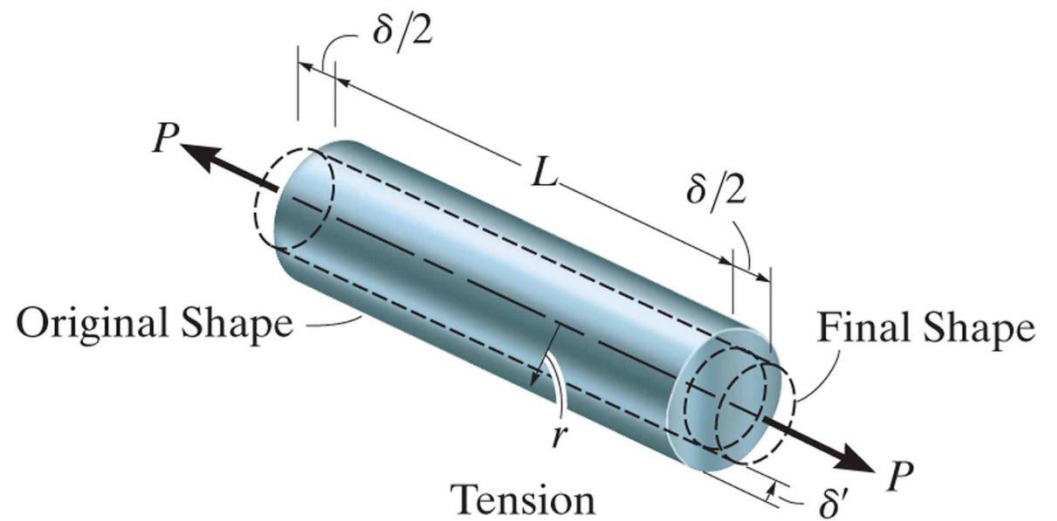
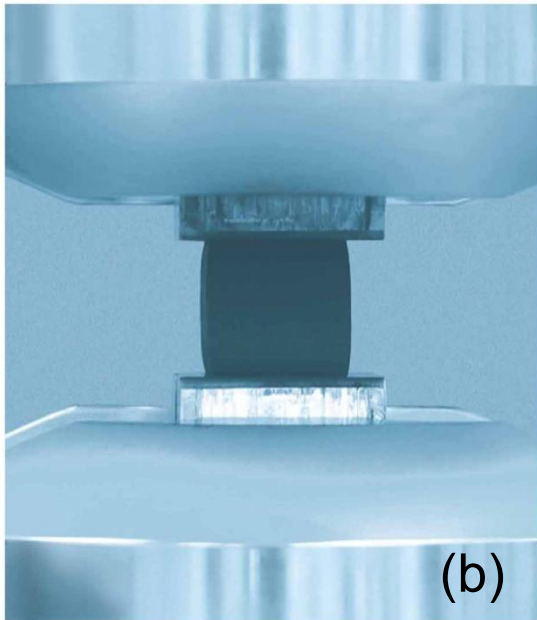
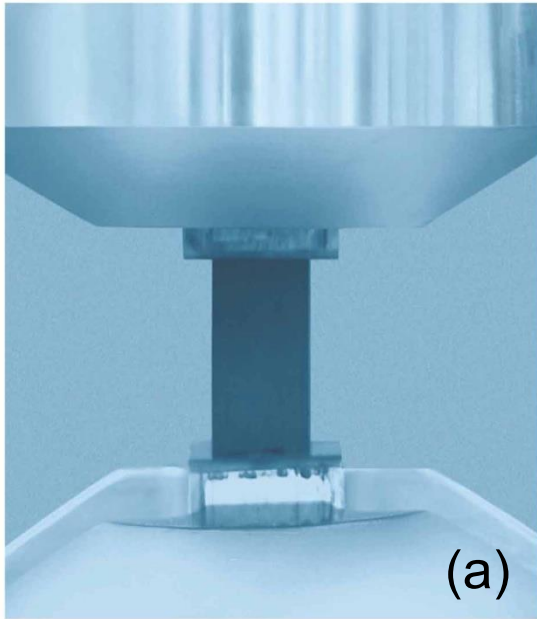
Strain: example B

The σ – ϵ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 6.5 in and an approximate cross-sectional area of 0.23 in² determine its elongation if the foot supports a load of 125 lb, which causes a tension in the tendon of 343.7 lb



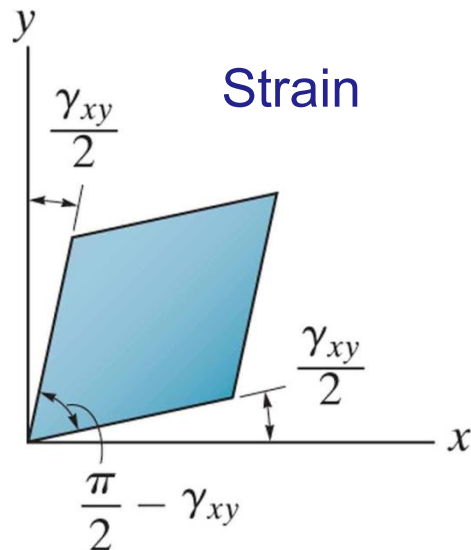
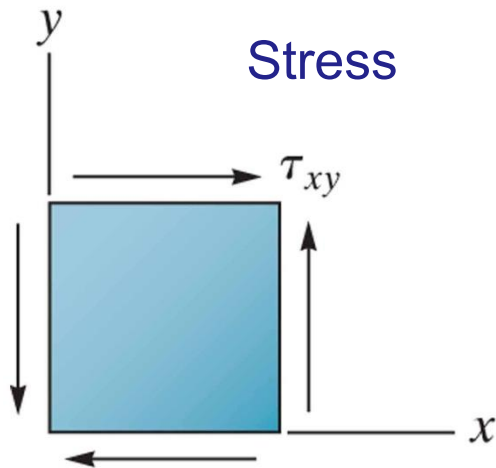
Poisson's ratio:

$$\text{Poisson's ratio: } \nu = - \frac{\epsilon_{lateral}}{\epsilon_{longitudinal}}$$



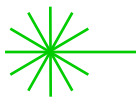
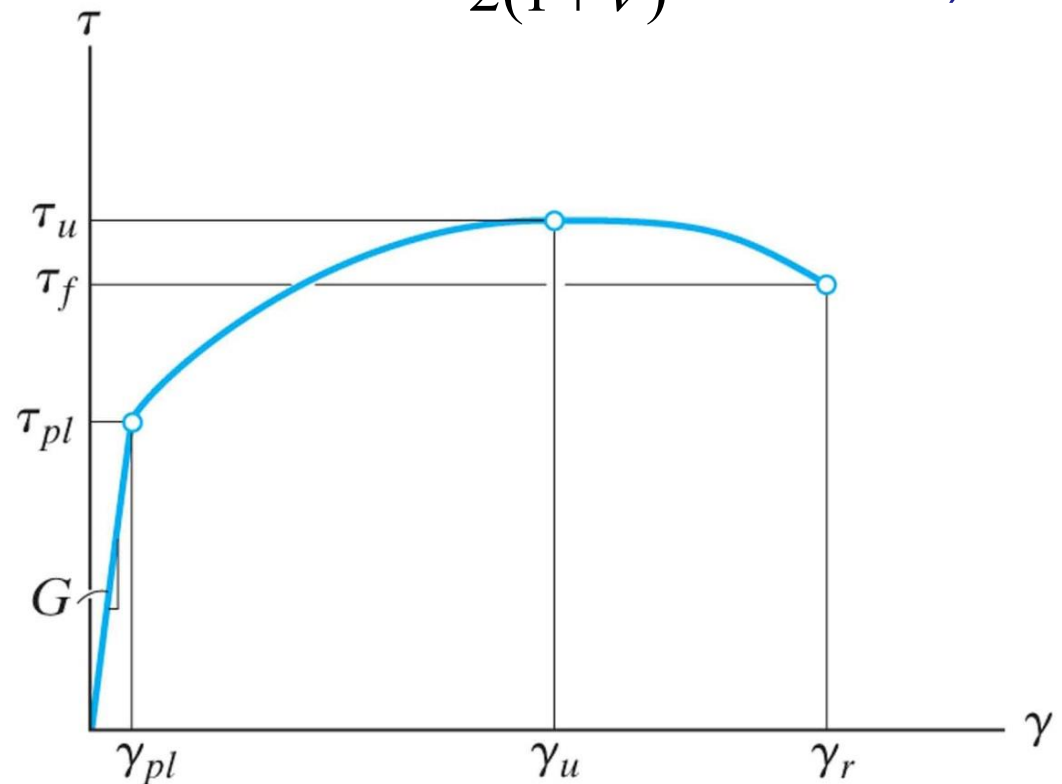
Shear stress ↔ strain

Pure shear



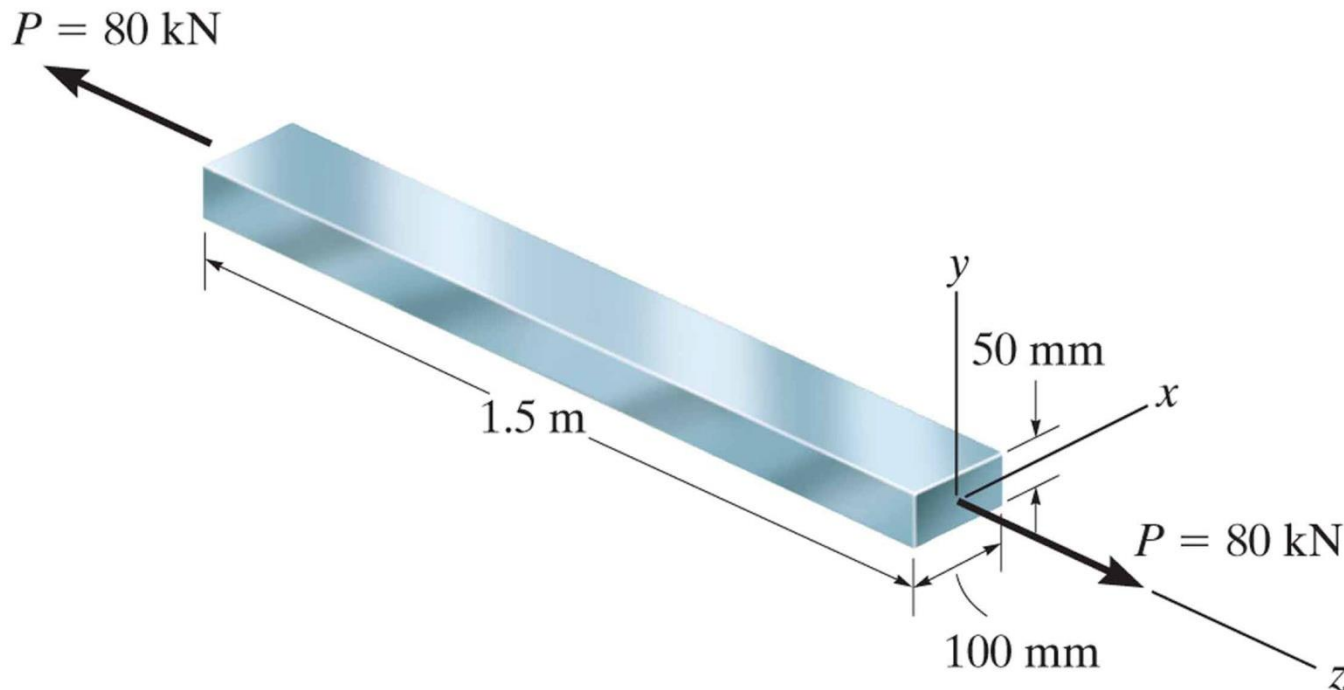
Hook's law for shear: $\tau = G \gamma$

with $G = \frac{E}{2(1+\nu)}$ (shear modulus)



Strain: example C

A bar made of ASTM A-36 steel has the dimensions shown. If the axial force of $P = 80 \text{ kN}$ is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.



Axial load

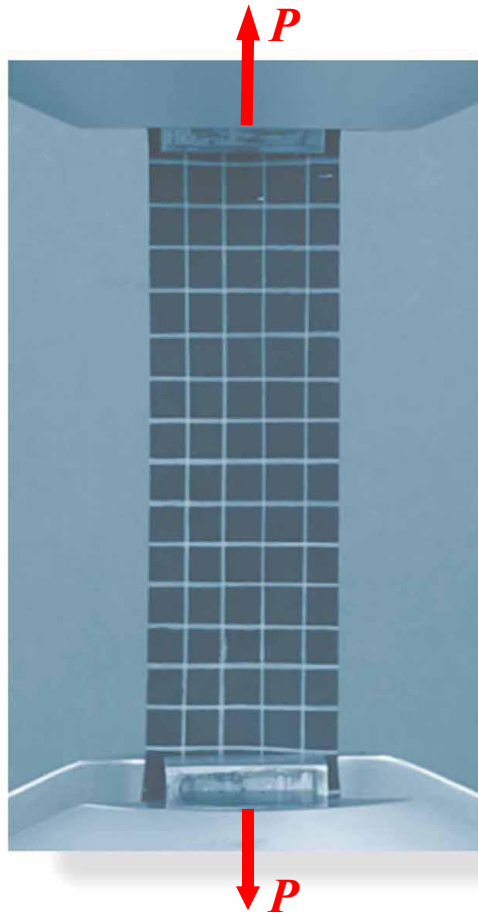


Figure: 04-01-UN-A

Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant's principle.

Note distortion lines: follow Saint-Venant's principle

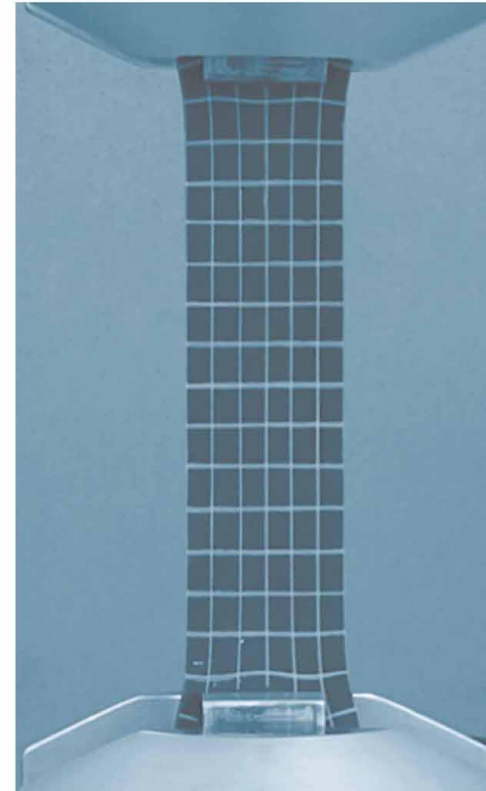


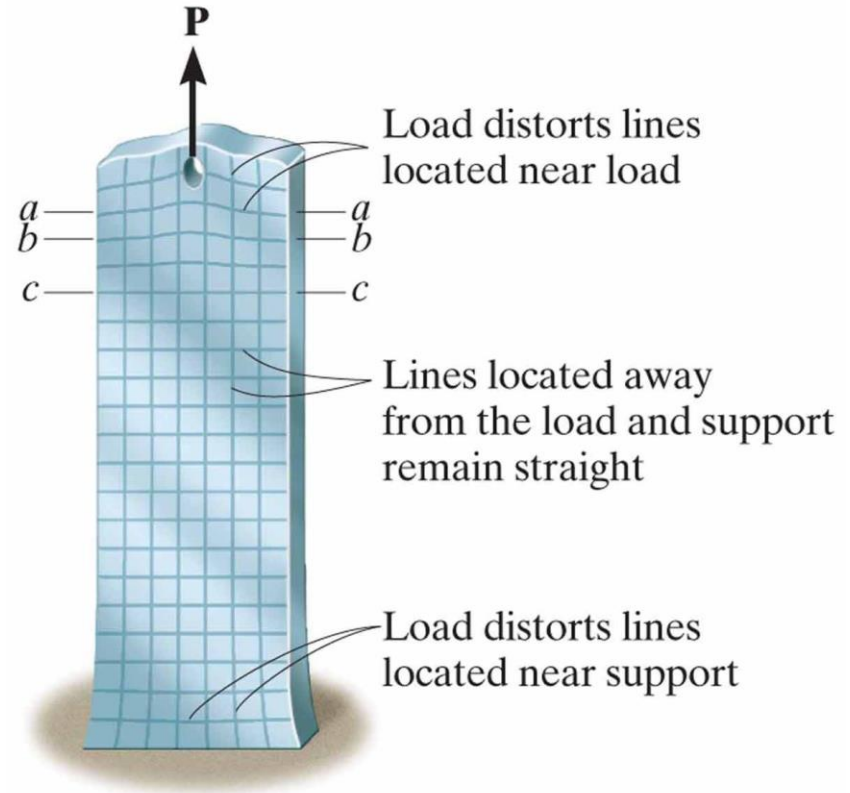
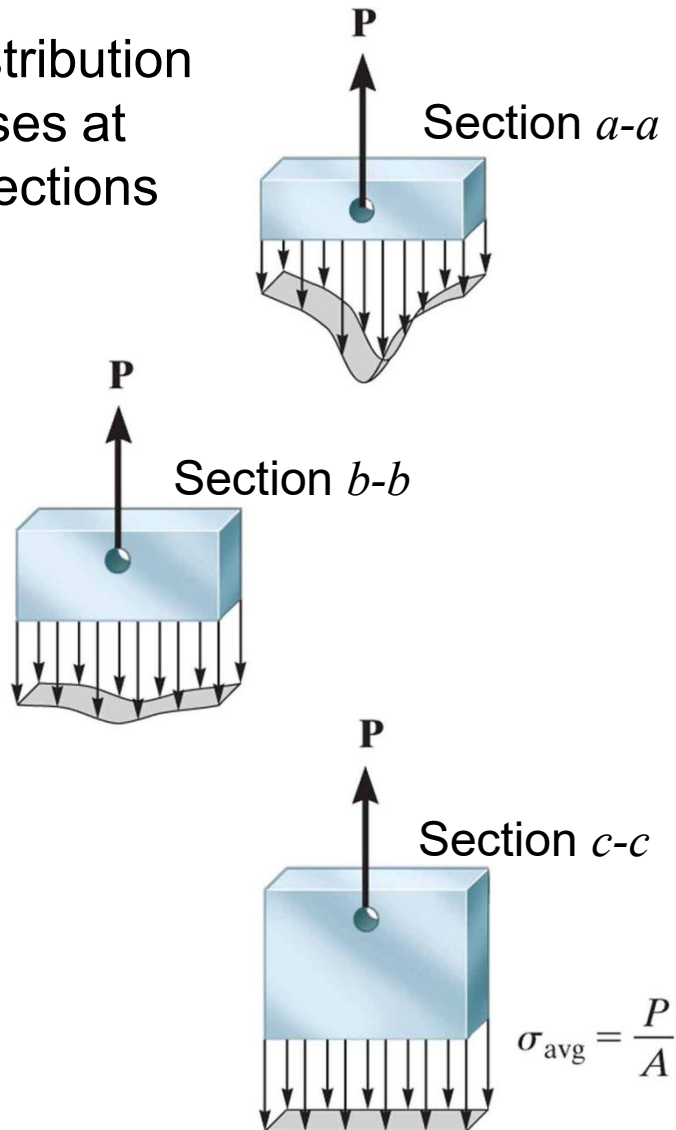
Figure: 04-01-UN-B

Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant's principle.

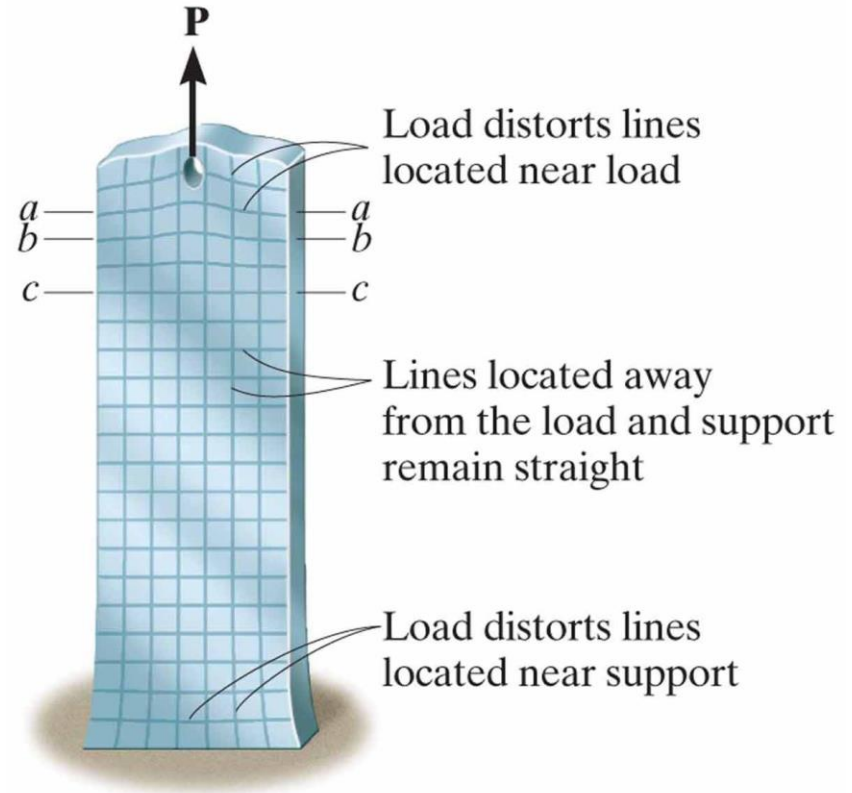
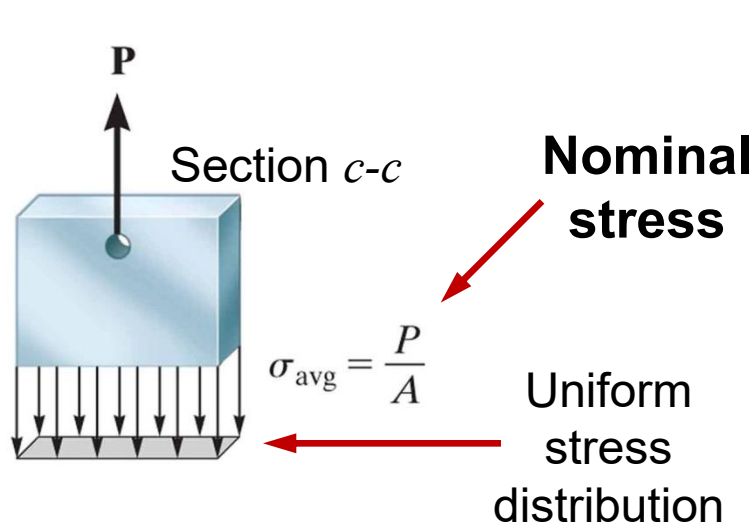


Axial load: Saint-Venant's principle

Internal distribution
of stresses at
various sections



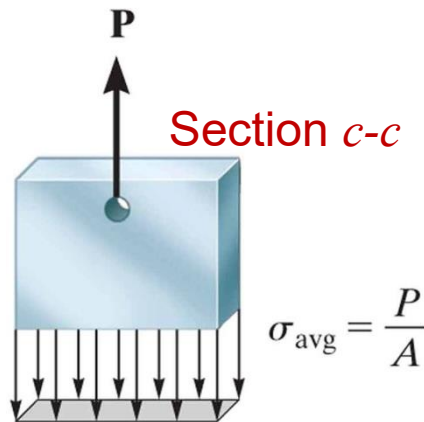
Axial load: Saint-Venant's principle



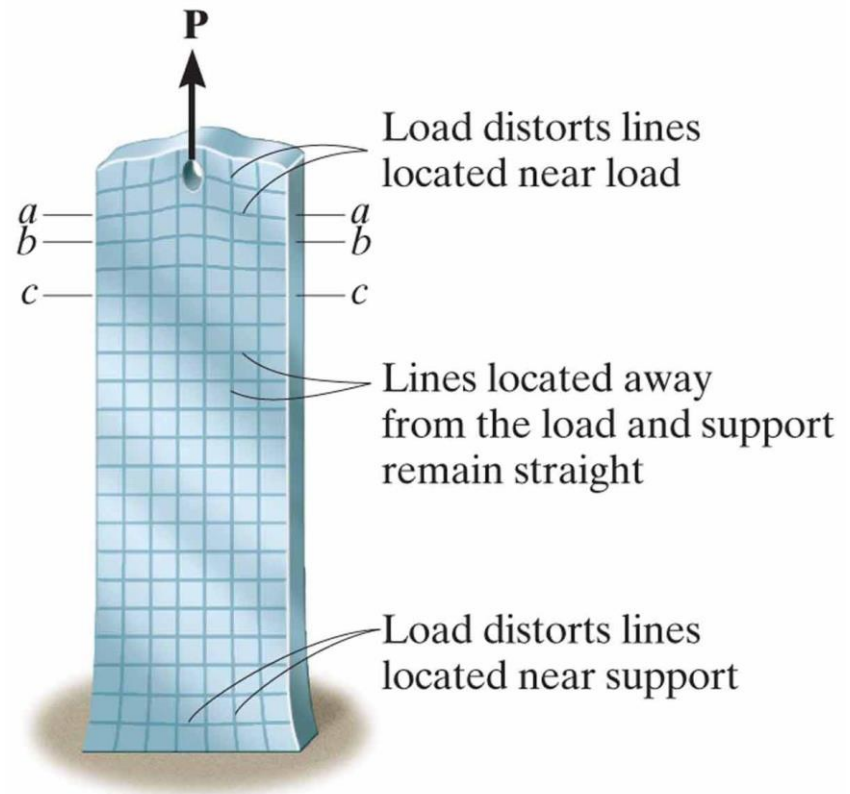
Axial load: Saint-Venant's principle

In your analyses, select locations (sections / points) located away from regions that are subjected to load application (to eliminate “end” effects)

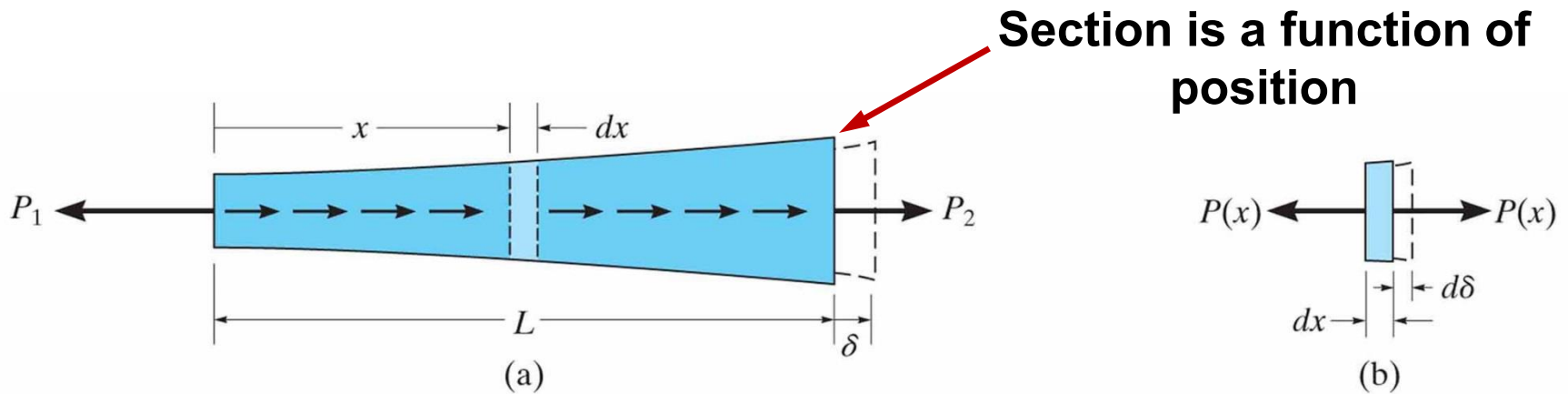
Saint-Venant's principle: stresses and strains within a section will approach their nominal values as the section locates away from regions of load application



Nominal stress



Elastic deformation of an axially loaded member



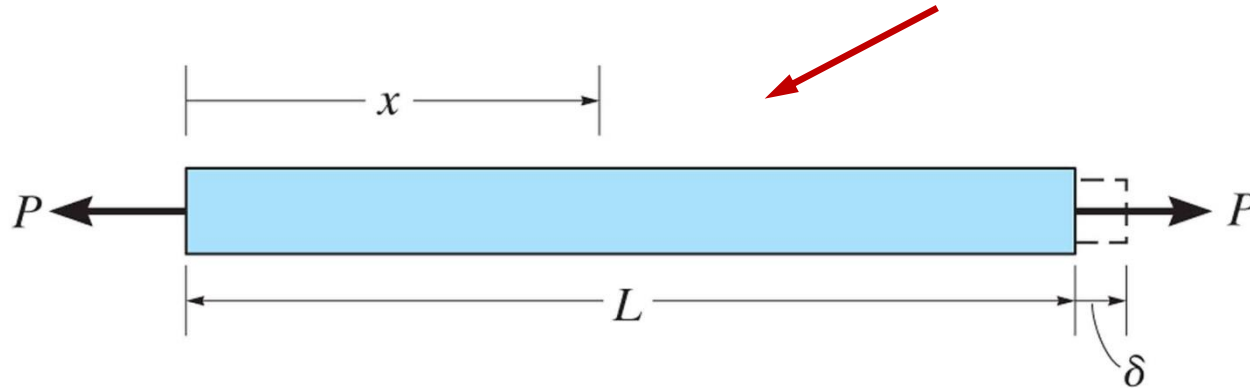
$$\sigma = \frac{P(x)}{A(x)} \quad \text{and} \quad \varepsilon = \frac{d\delta}{dx}$$

$$\text{Therefore, } d\delta = \frac{P(x) dx}{A(x) E} \longrightarrow \delta = \int_0^L \frac{P(x)}{A(x) E} dx$$



Elastic deformation of an axially loaded member

Constant load and cross-sectional area



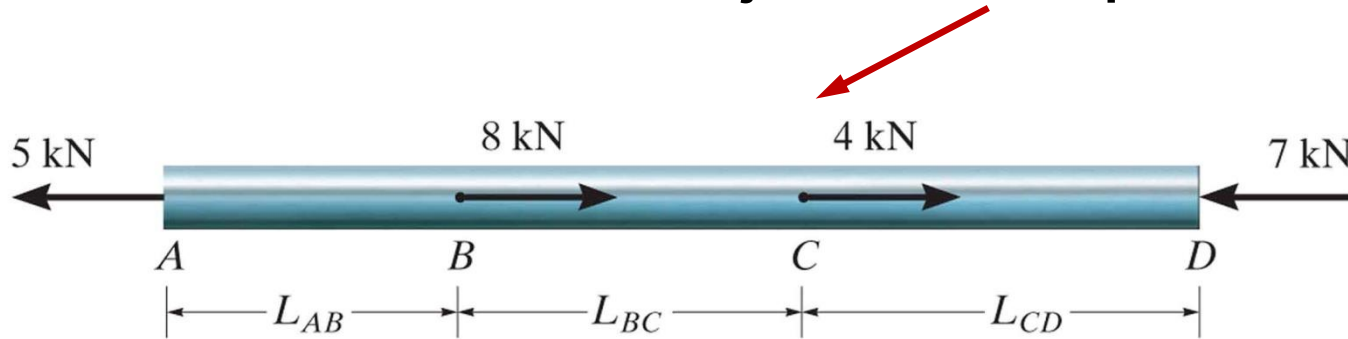
Elastic deformation:

$$\delta = \int_0^L \frac{P(x)}{A(x) E} dx = \frac{P}{A E} \int_0^L dx = \frac{P L}{A E}$$



Elastic deformation of an axially loaded member

Bar subjected to multiple axial loads



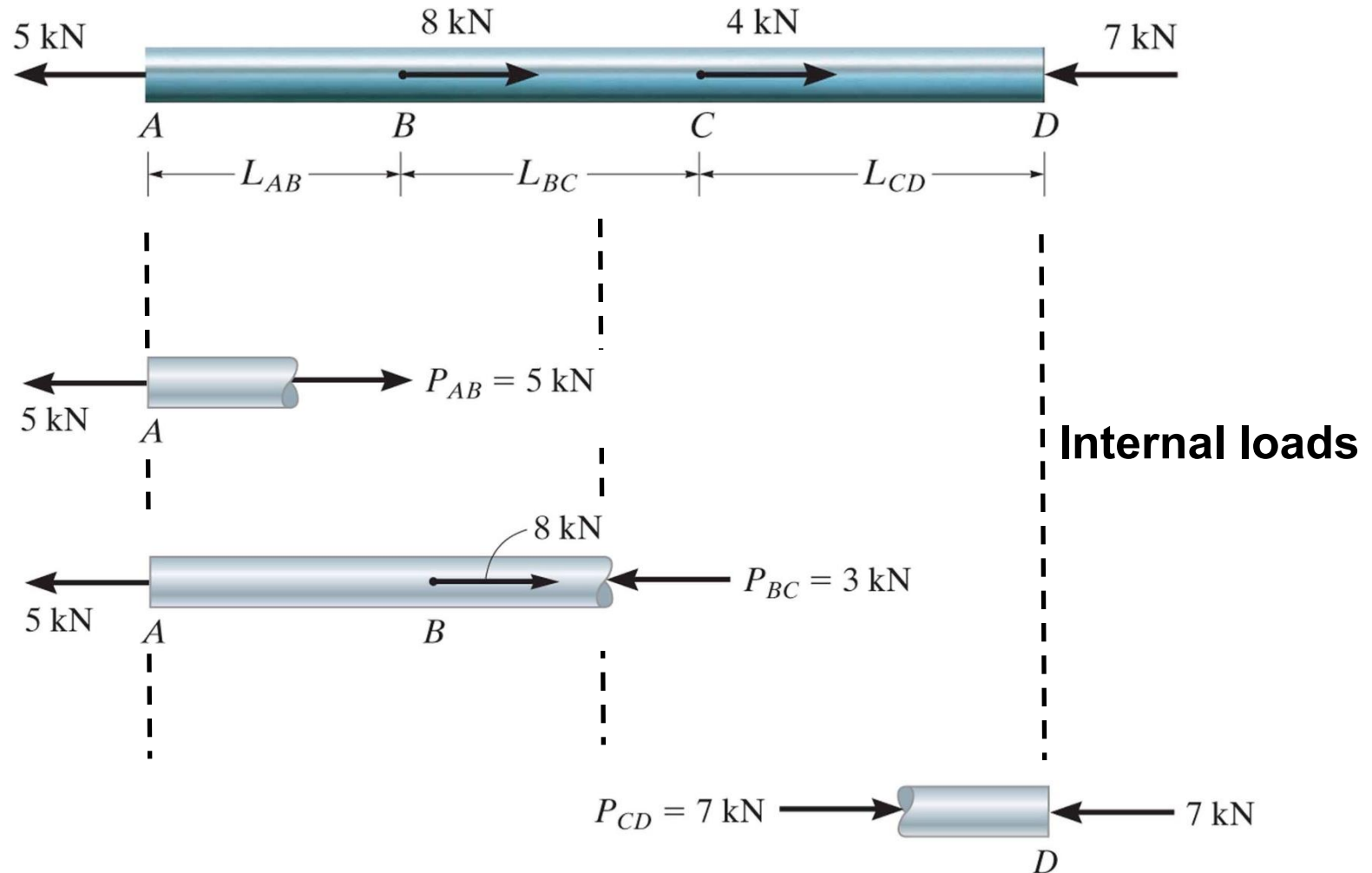
Elastic deformation:

$$\delta = \sum_i \left(\frac{P L}{A E} \right)_i$$



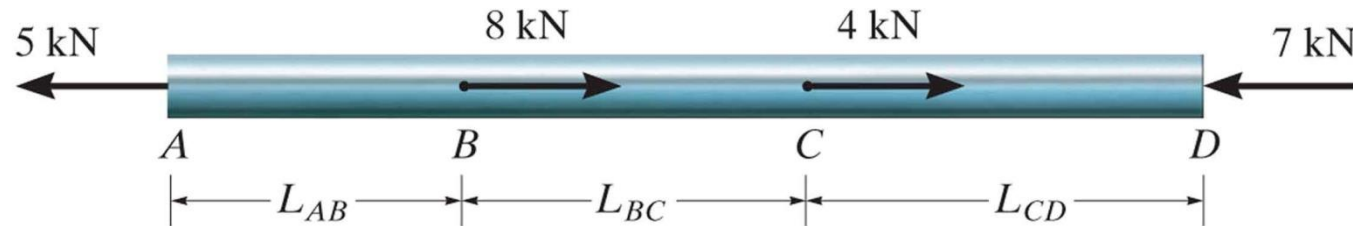
Elastic deformation of an axially loaded member

Procedure for analysis



Elastic deformation of an axially loaded member

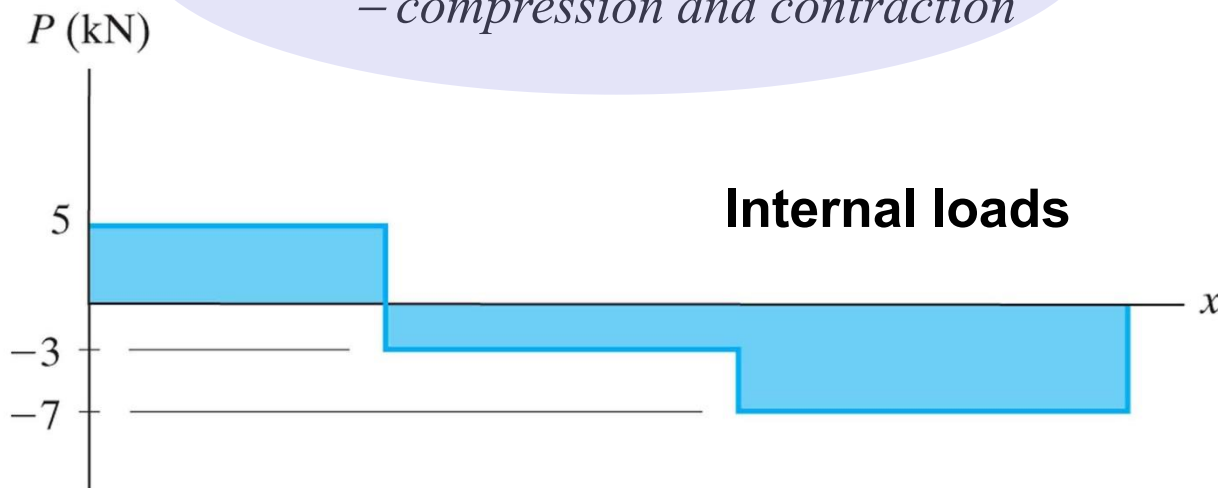
Procedure for analysis



Sign convention:

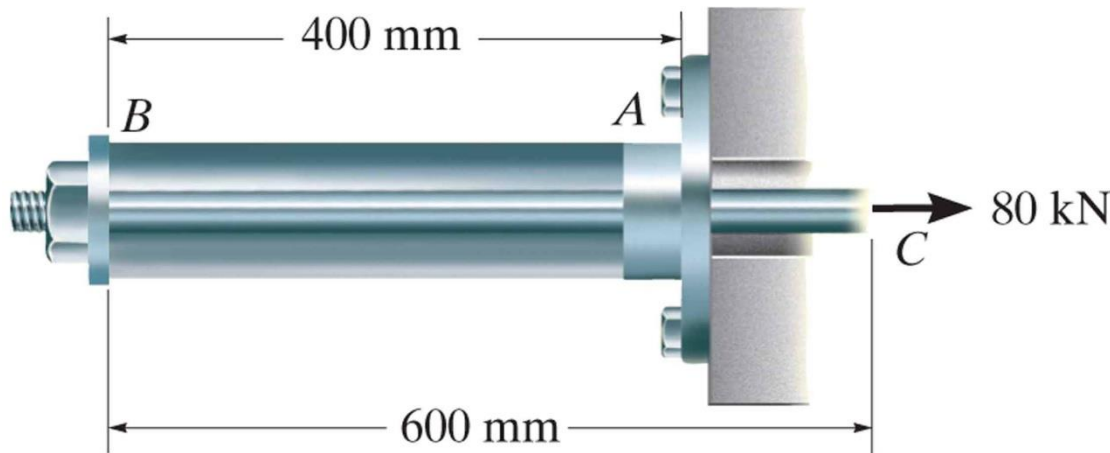
+ *tension and elongation*

– *compression and contraction*



Axial load: example D

The assembly shown consists of an aluminum tube AB having a cross sectional area of 400 mm^2 . A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end C of the rod. Elastic modules: $E_{\text{steel}} = 200 \text{ GPa}$ and $E_{\text{alum}} = 70 \text{ GPa}$



Approach:

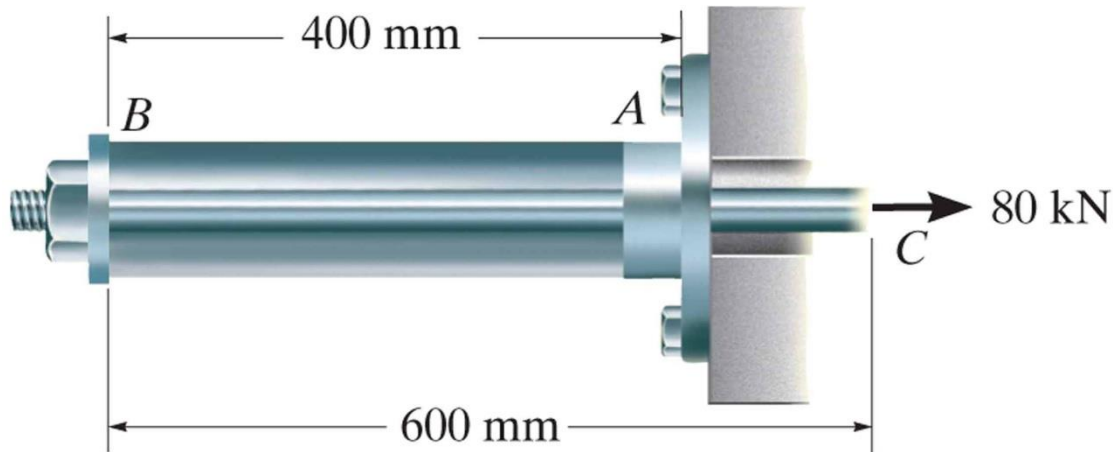
- 1) Determine internal loading
- 2) Compute displacement



Axial load: example D

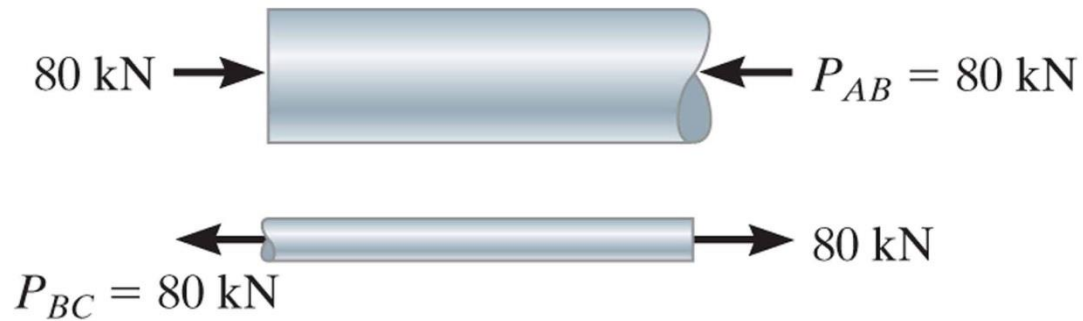
Displacement of C :

$$\delta_C = \delta_B + \delta_{C/B}$$

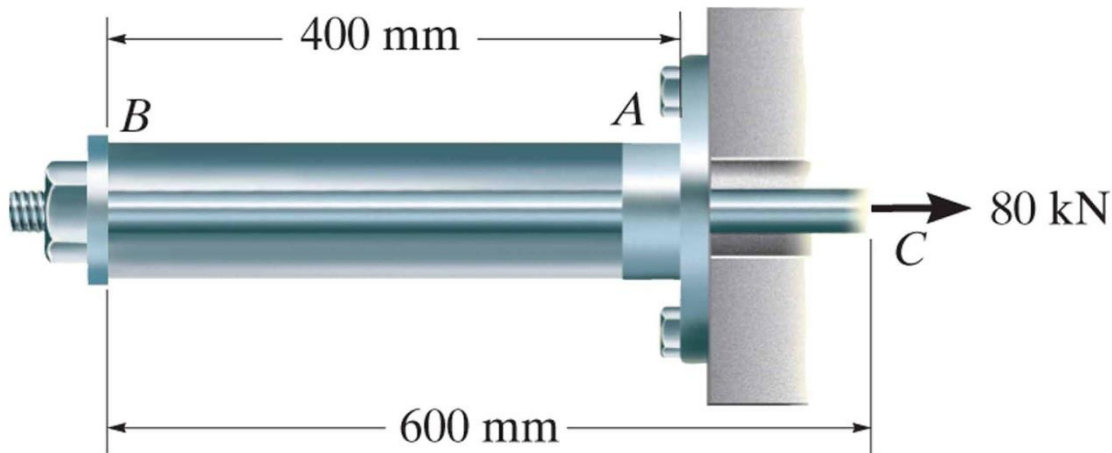


Axial load: example D

(1) Internal loading



(2) → find displacement at C



Reading assignment

- Chapters 3 and 4 of textbook
- Review notes and text: ES2001, ES2501



Homework assignment

- As indicated on webpage of our course

