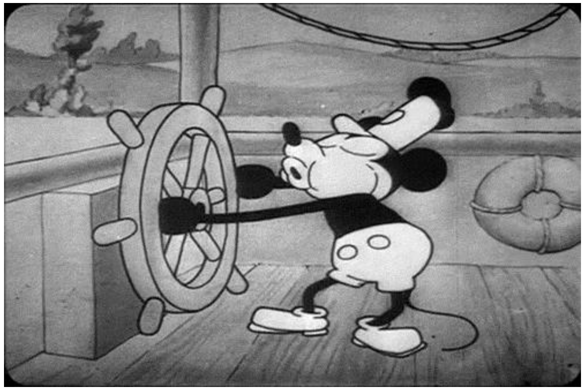
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, B'2025

We will get started soon...



22 October 2025

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

STRESS ANALYSIS
ES-2502, D'2020

Lecture 03:
Unit 3: definition of normal and shear stress

22 October 2025

General information

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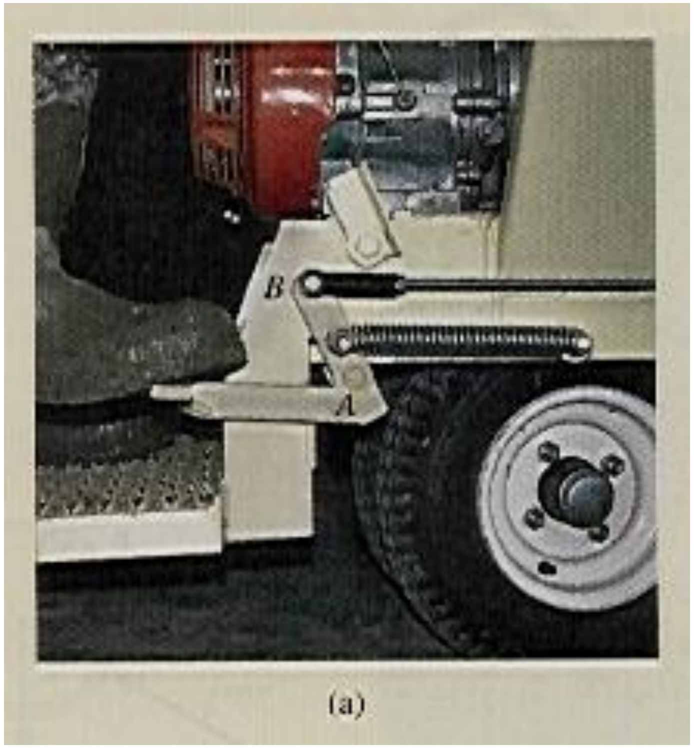
Email: jpatil1 @ wpi.edu

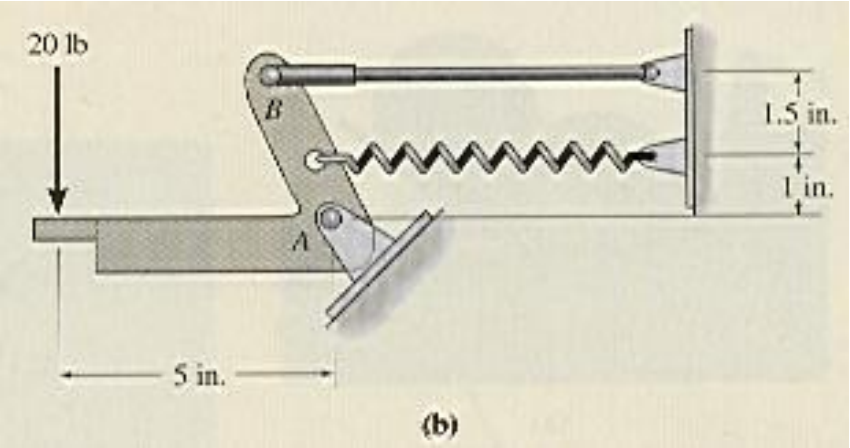
Free-body diagrams

Operator applies 20-lb to pedal stretching spring by 1.5 in.

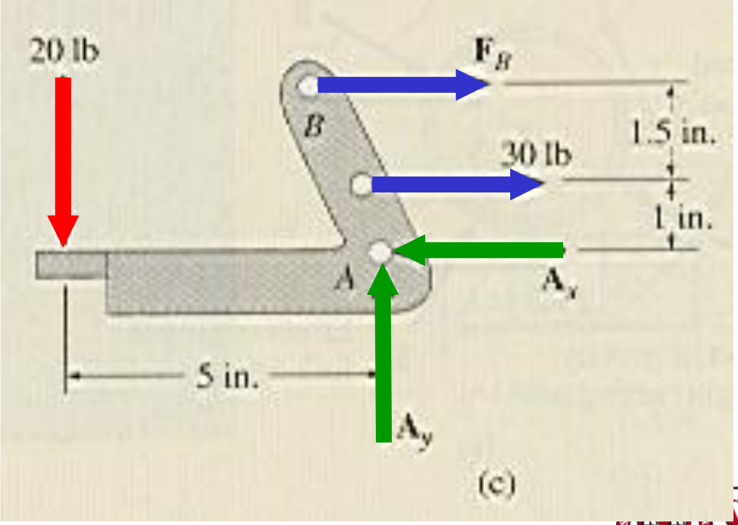


Actual mechanism

Schematic representation:



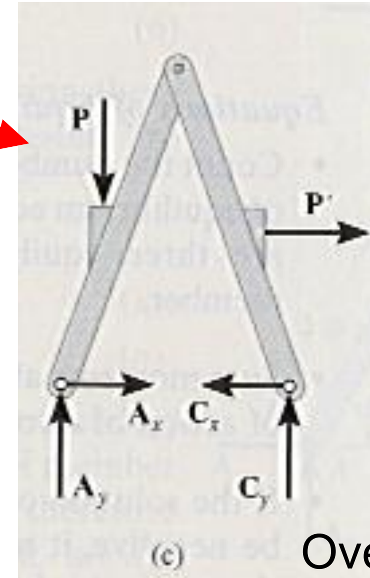
Free-body diagram:



Force analysis. Free-body diagrams

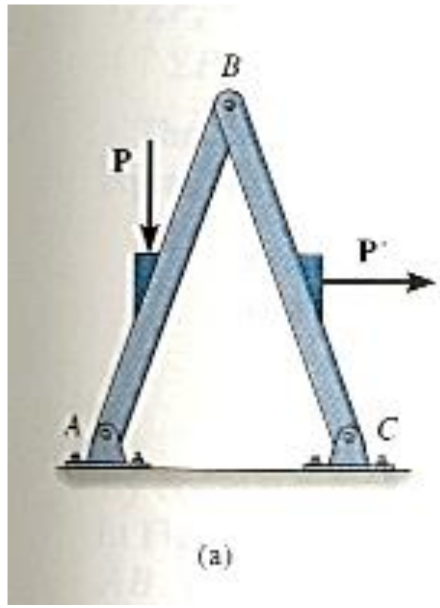
Number of unknowns ?

Equilib. Equations ?

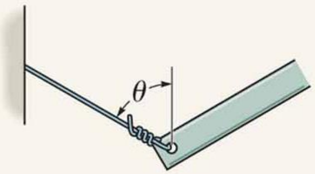
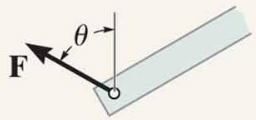
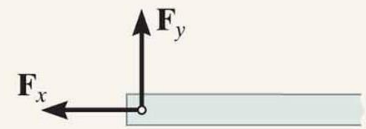
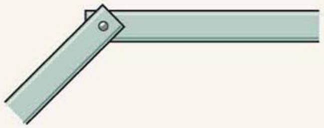
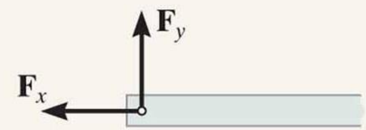
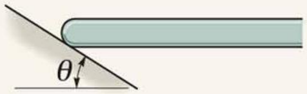
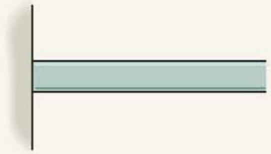
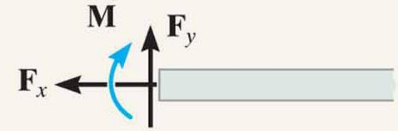


Overall FBD

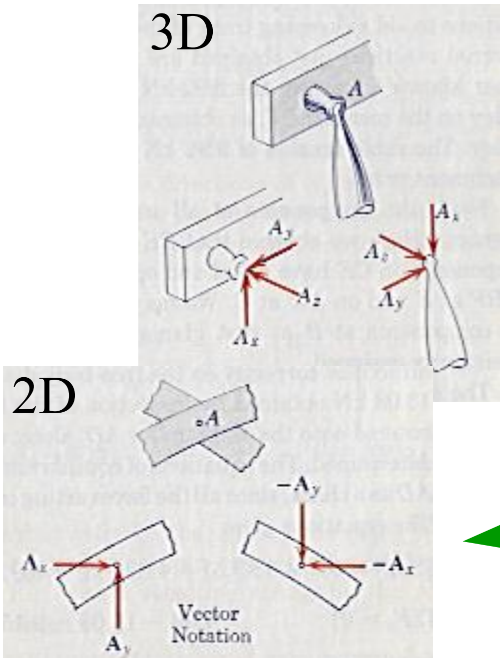
Is this a statically indetermined case ?



Force analysis. Free-body diagrams

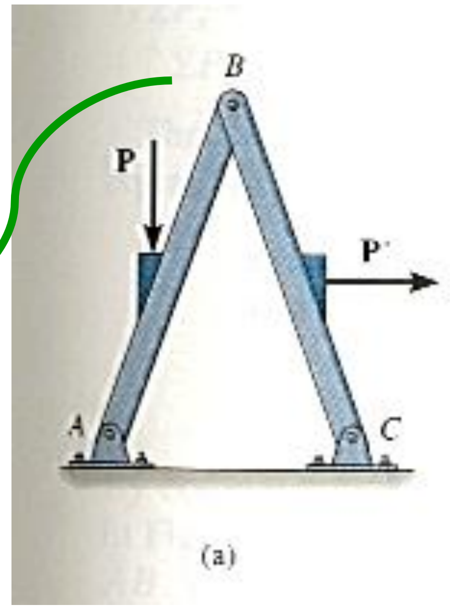
Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

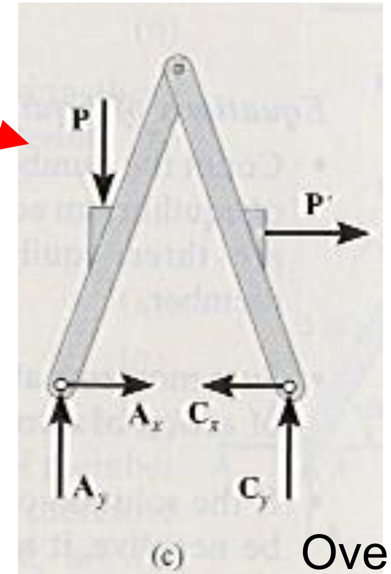
Force analysis. Free-body diagrams



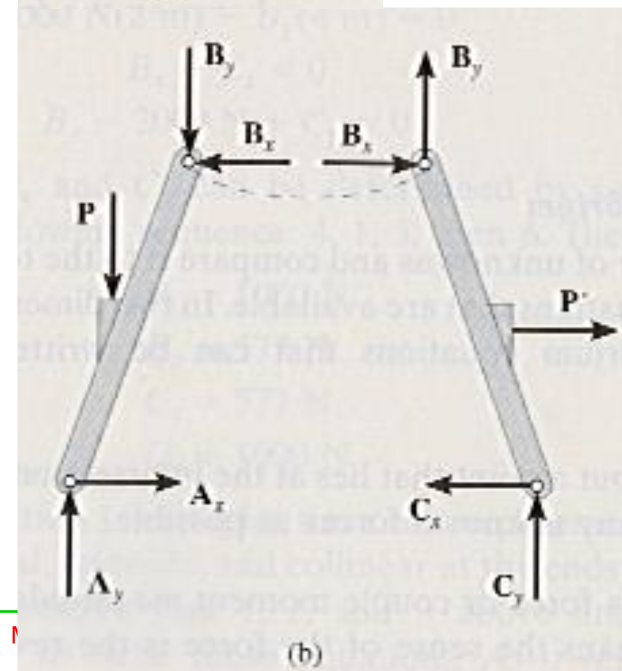
Number of unknowns ?

Equilib. Equations ?





Overall FBD

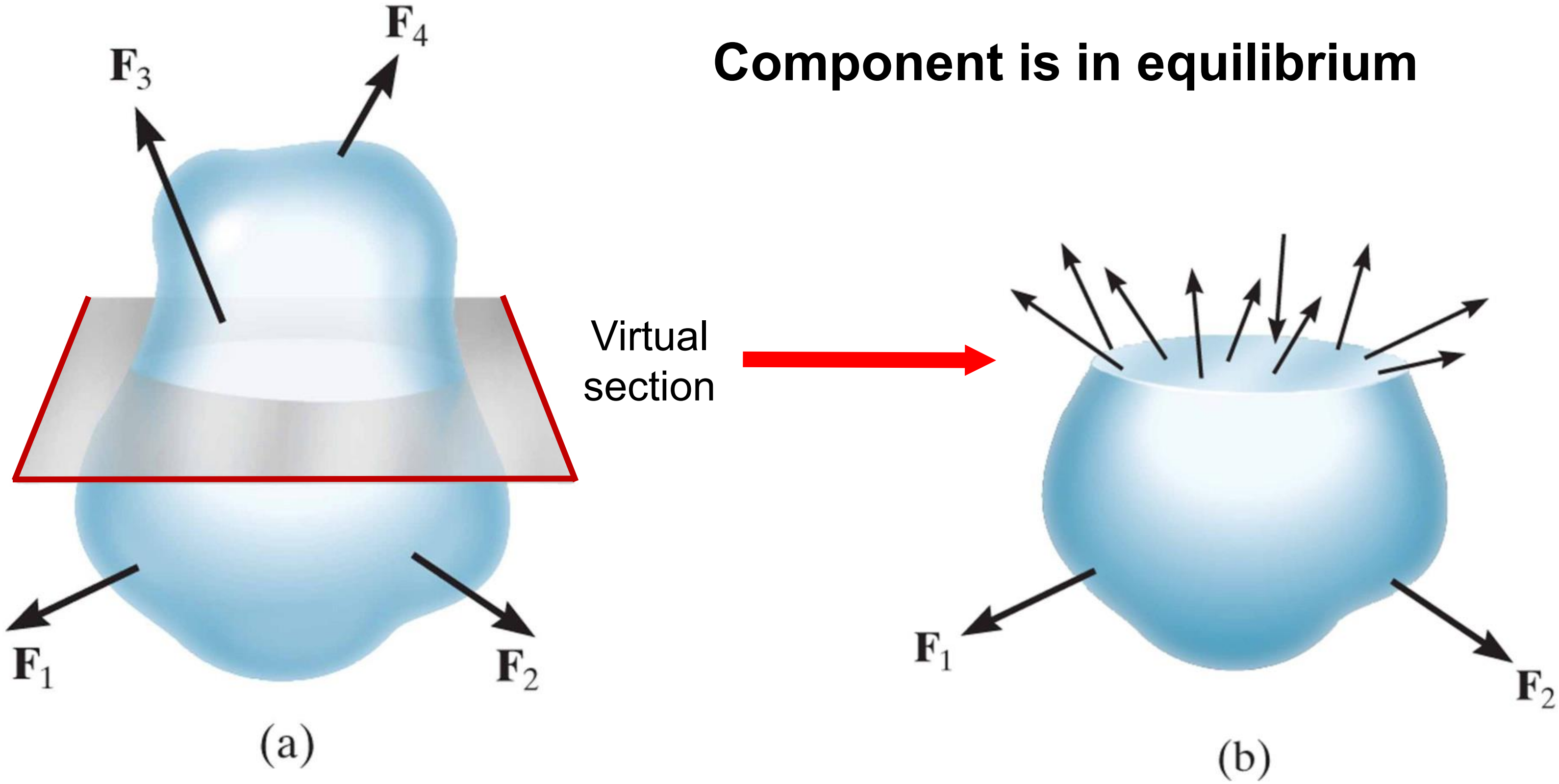


Individual FBD's

Internal resultant loading

Arbitrary component under load

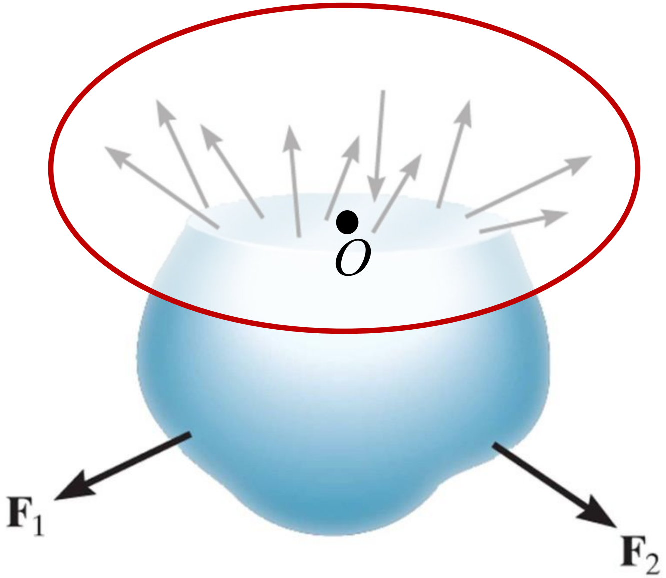
Component is in equilibrium



Internal resultant loading

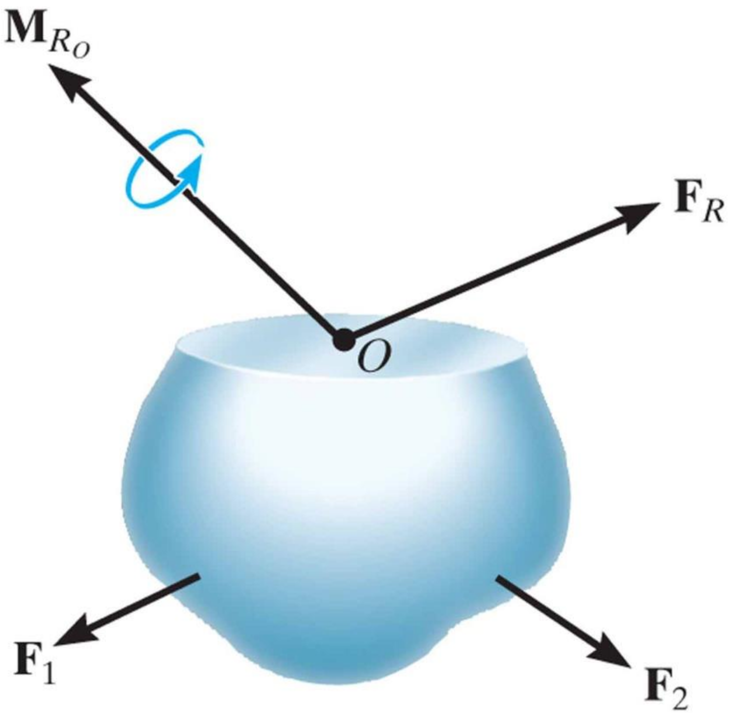
$$\mathbf{F}_R = \sum \mathbf{F}$$

$$\mathbf{M}_{R_O} = \sum \mathbf{M}_O$$



(b)

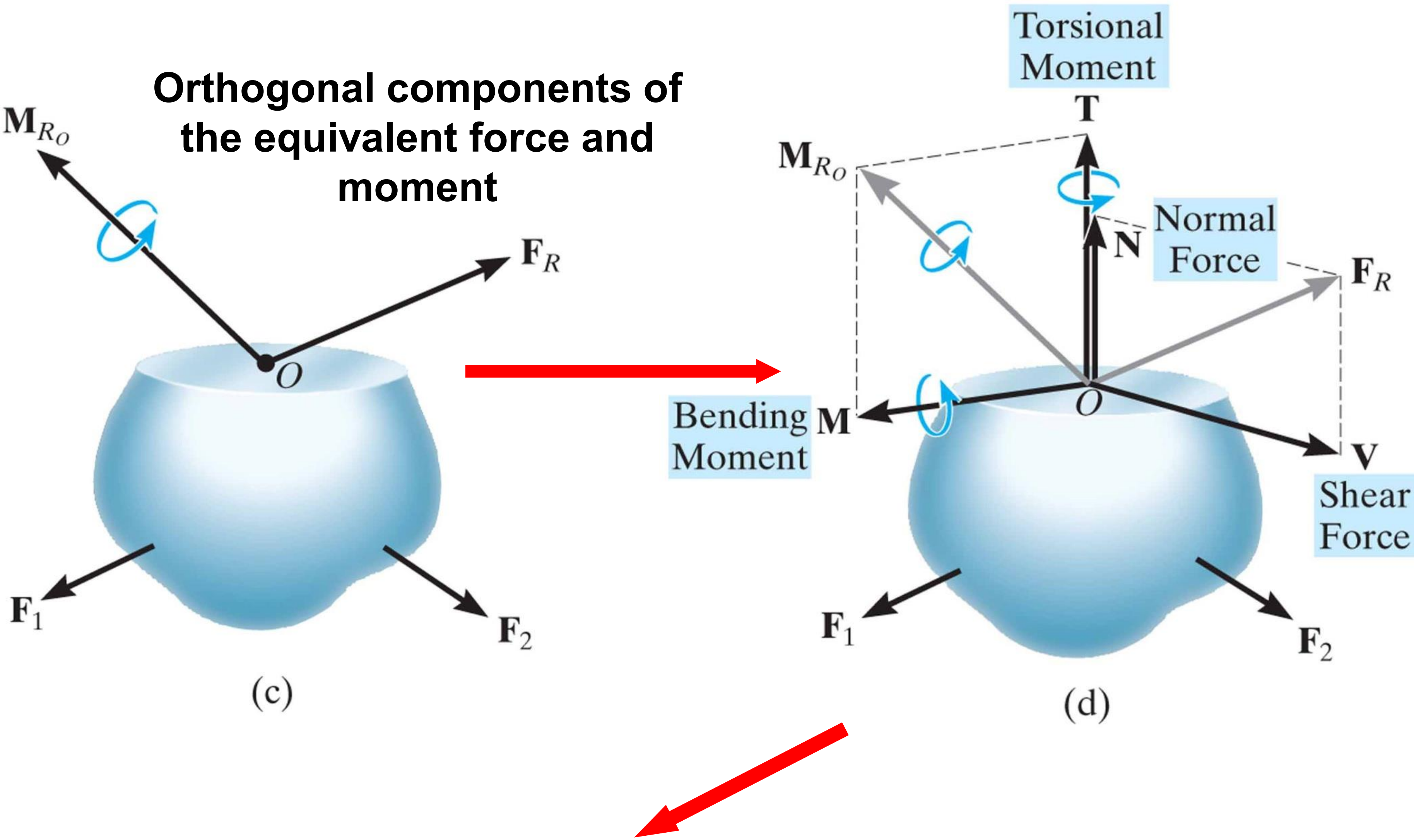
Equivalent force and moment at the section



(c)

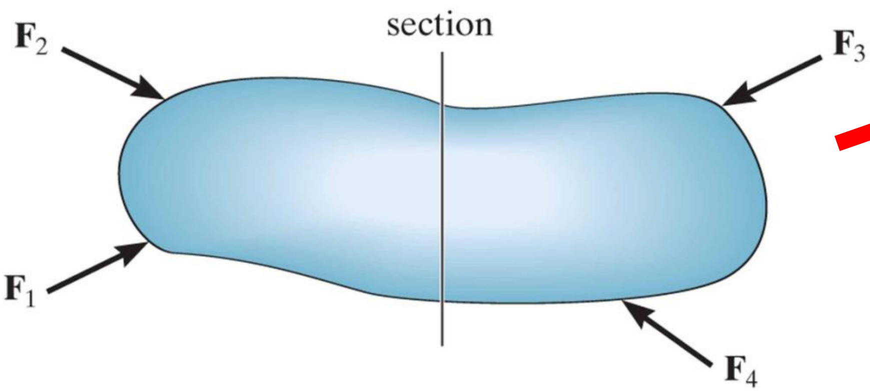
Internal resultant loading

Orthogonal components of the equivalent force and moment



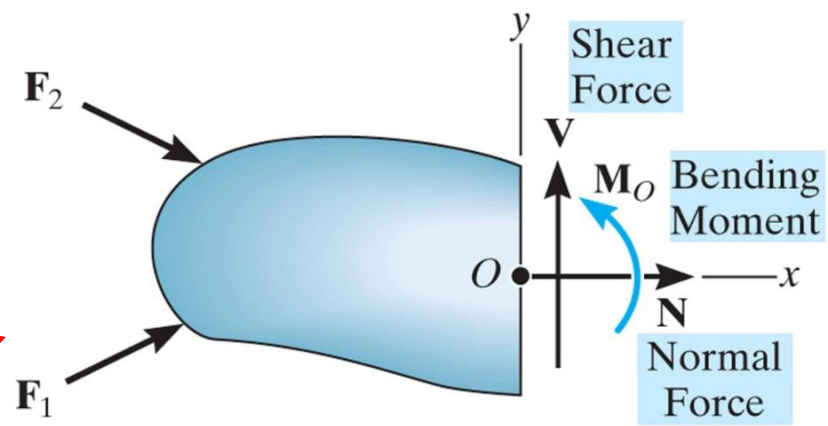
Internal resultant loading

Component is in equilibrium



(a)

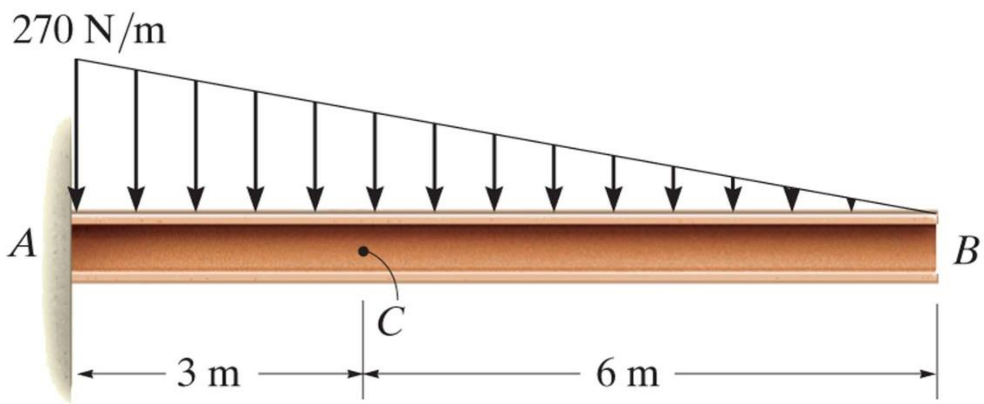
Equivalent forces and moments at the section



(b)

Internal resultant loading: example A

Determine resultant internal loading acting on the cross section at C of the cantilever shown:

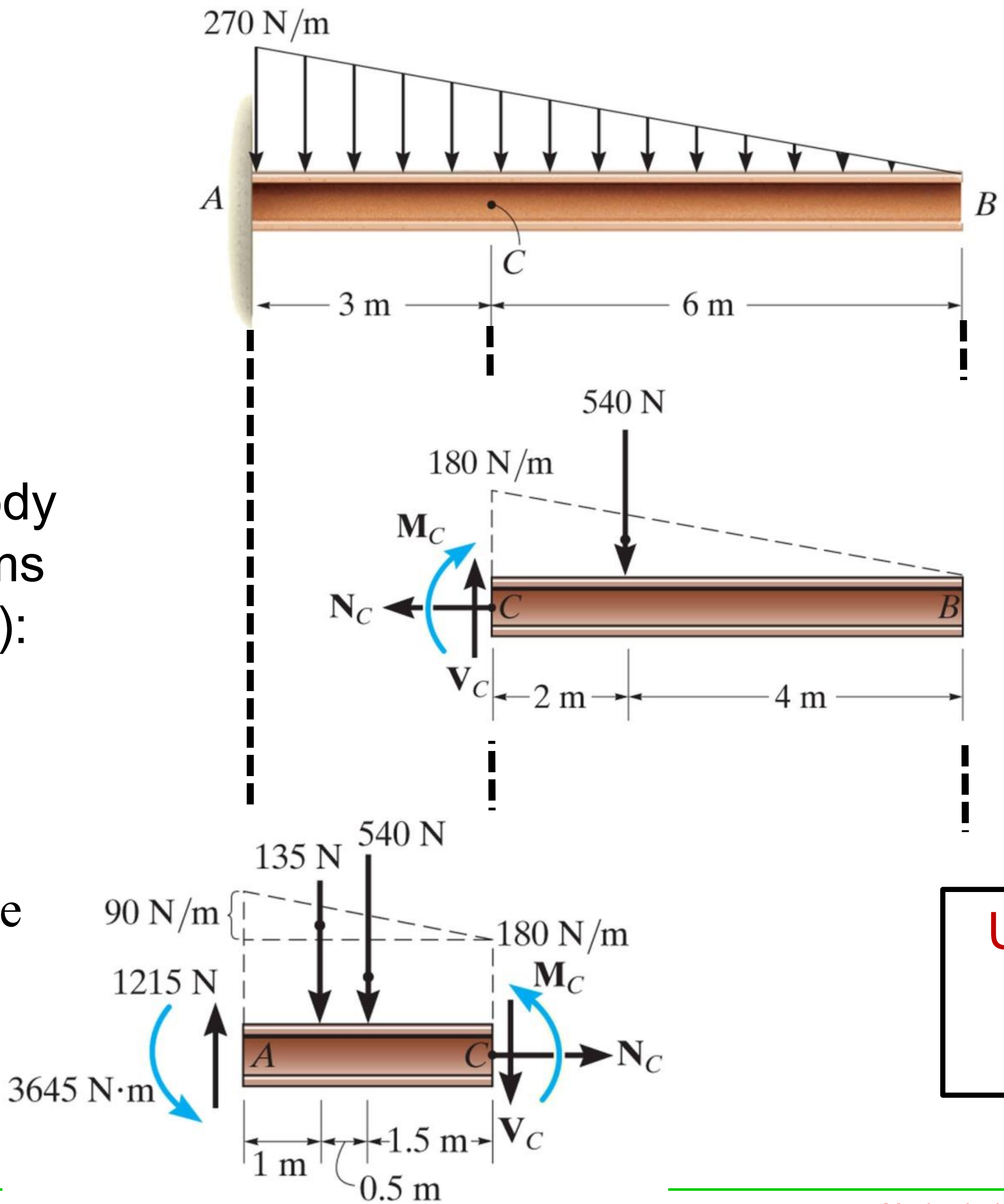


(a)

Approach:

1) Define free-body diagrams
2) Apply equilibrium equations

Internal resultant loading: example A



Free-body diagrams (FBDs):

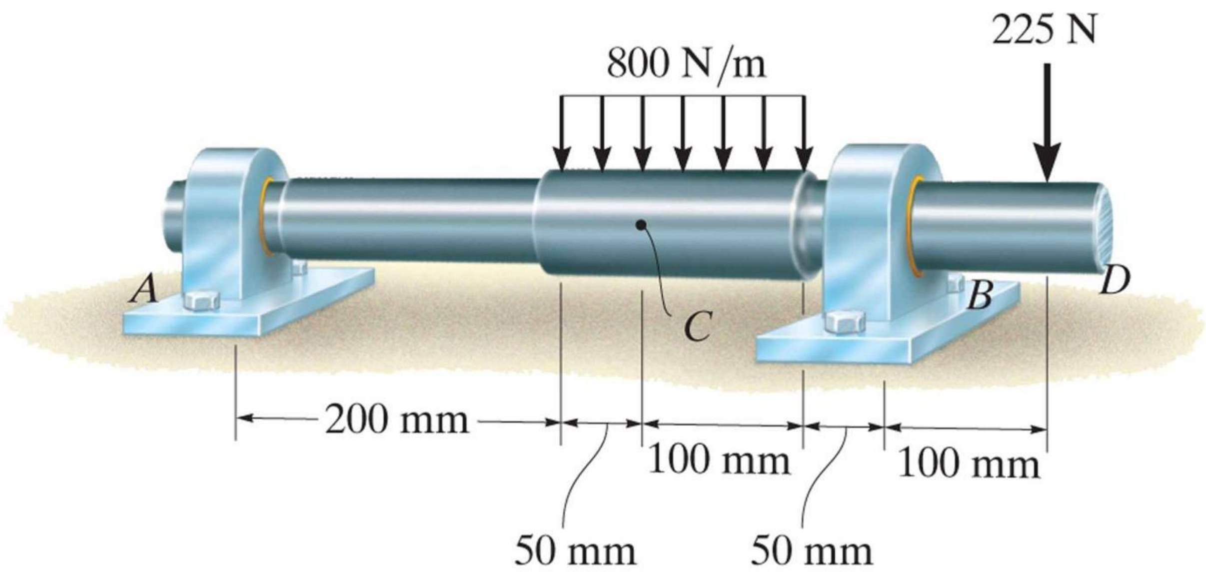
Section to the right of *C*

Section to the left of *C*

Use either section to determine internal loadings

Internal resultant loading: example B

Determine resultant internal loading acting on the cross section at C of the machine shaft shown. *Shaft is supported by bearings at A and B, which only exert radial forces on the shaft*



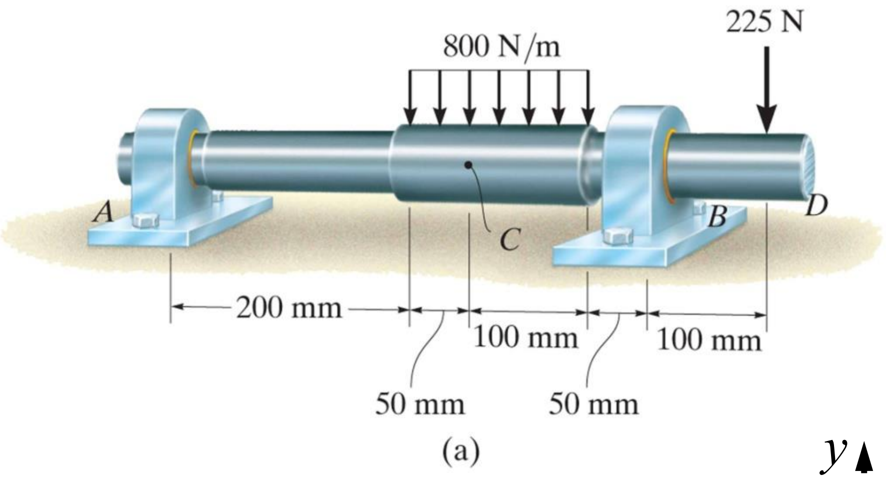
(a)

Approach:

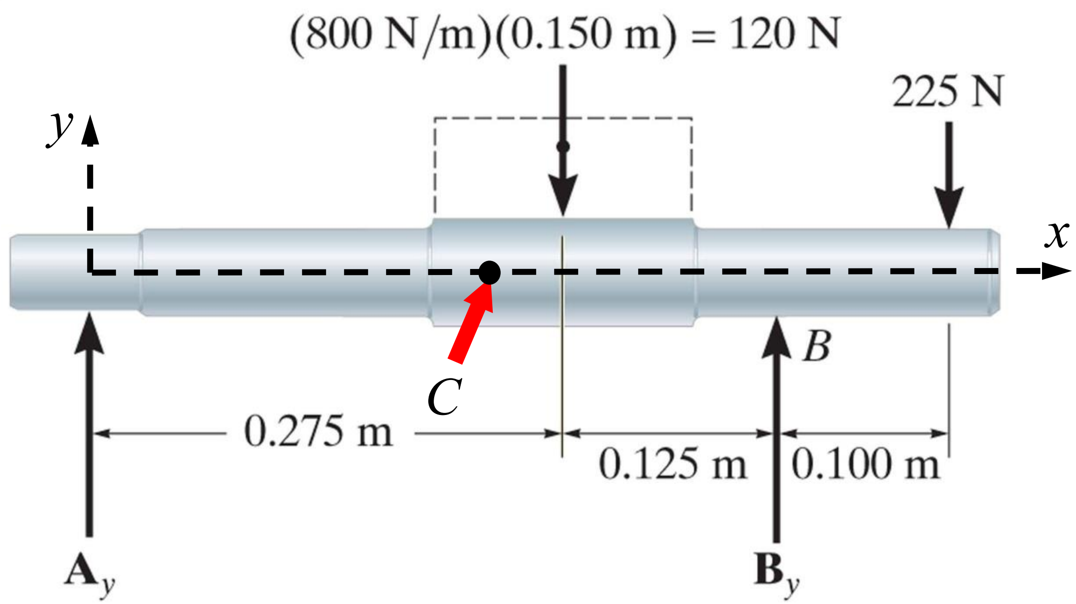
1) Define free-body diagrams
2) Apply equilibrium equations: reactions at bearings
3) Apply equilibrium equations: internal loading

Internal resultant loading: example B

Overall free-body diagram
(FBD)

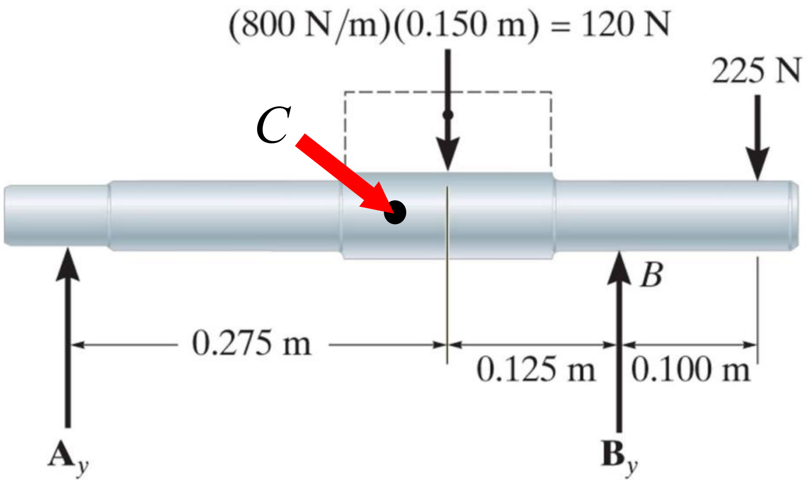


FBD is in2D, why?

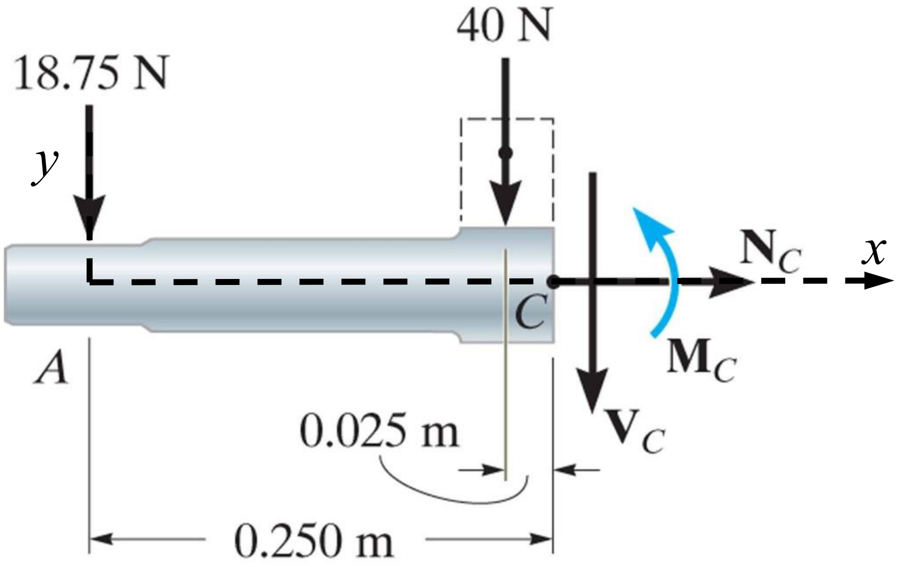


Internal resultant loading: example B

Select and define free-body diagram of section (FBD)

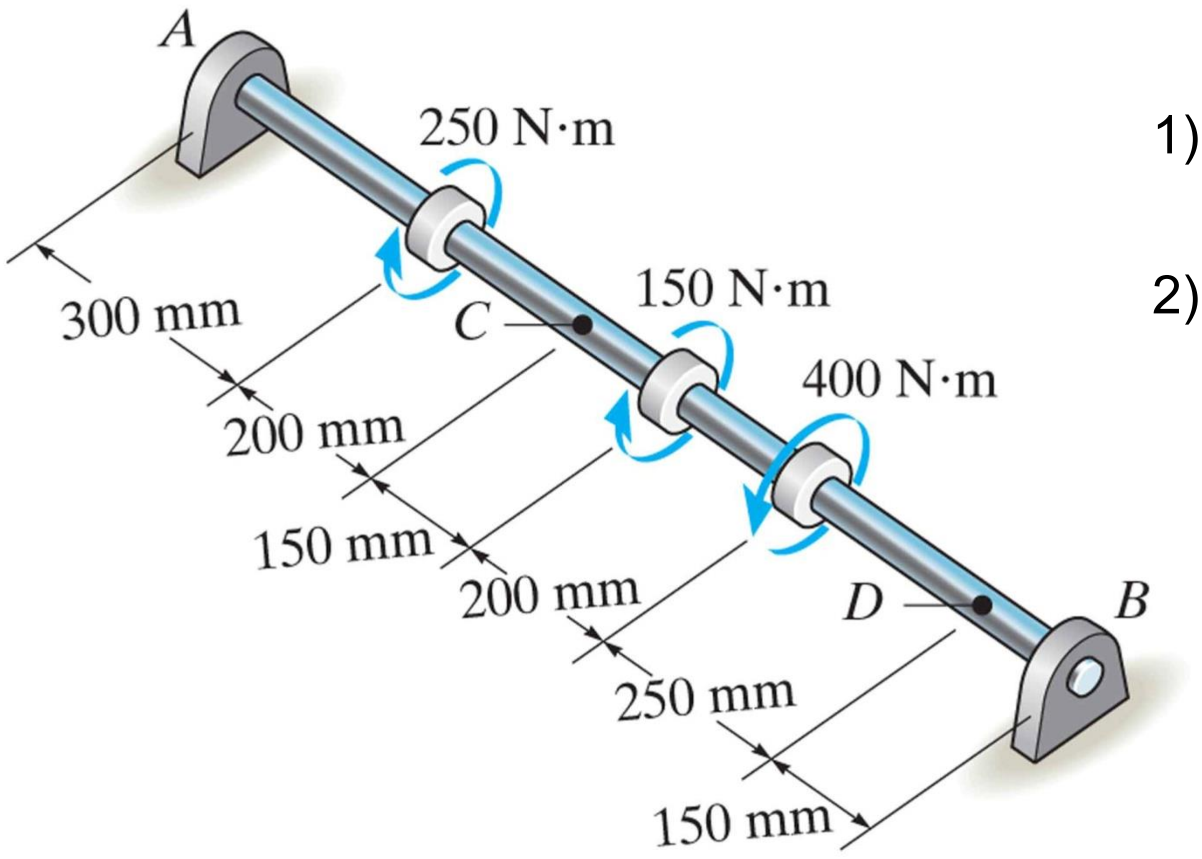


Compute internal loading



Internal resultant loading: example C

Determine the resultant internal torque acting on the cross sections through points C and D. Supports A and B allow free turning of the shaft

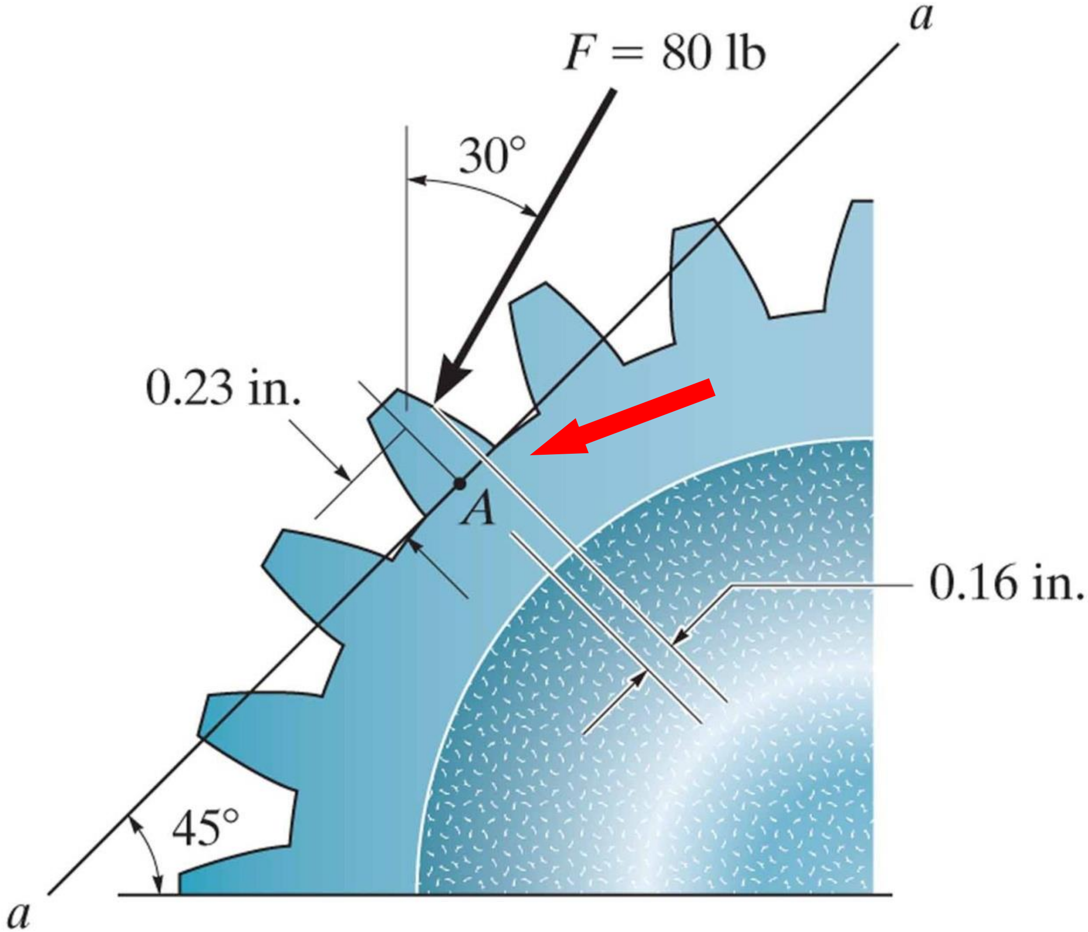


Approach:

1) Define free-body diagram
2) Apply equilibrium equations

Internal resultant loading: example D

The force $F = 80\ lb_f$ acts on the gear tooth. Determine the resultant internal loadings on the root of the tooth, i.e., at the centroid point A of section a-a



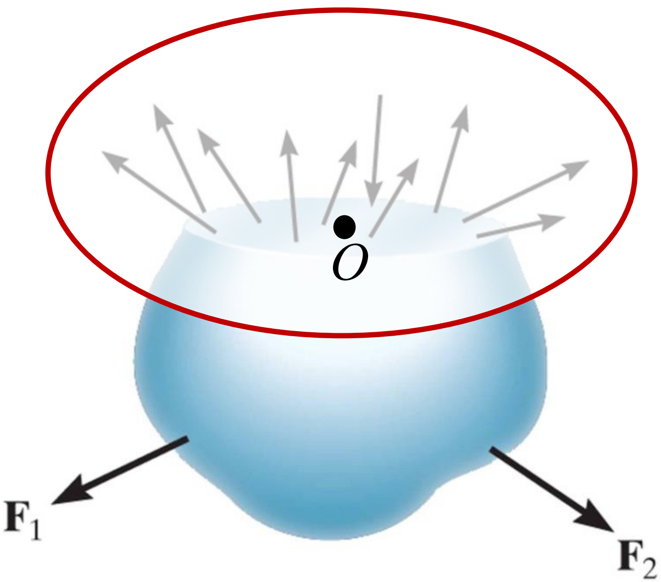
Approach:

1) Define free-body diagram
2) Apply equilibrium equations

Stress. Definition: intensity of internal force: acting on a specific plane passing through a point

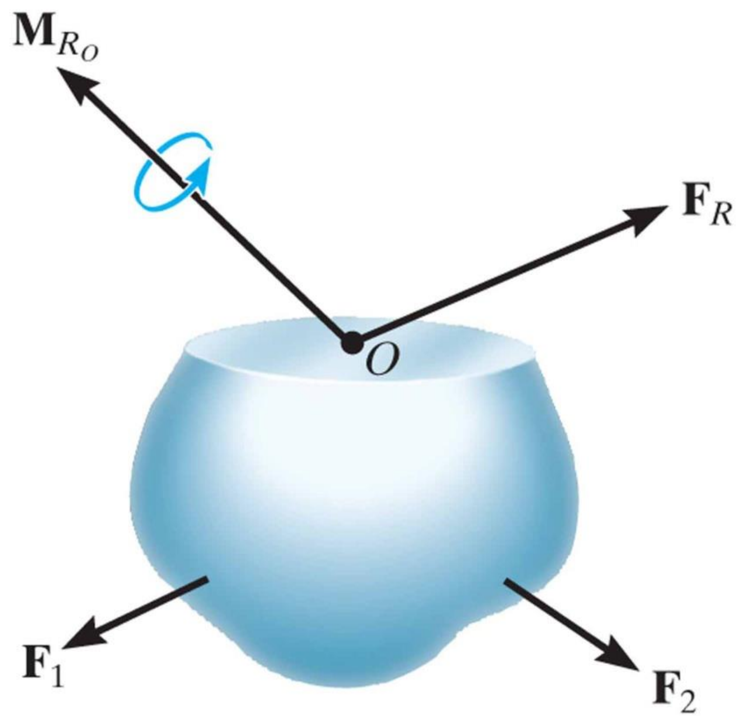
$$\mathbf{F}_R = \sum \mathbf{F}$$

$$\mathbf{M}_{R_O} = \sum \mathbf{M}_O$$



(b)

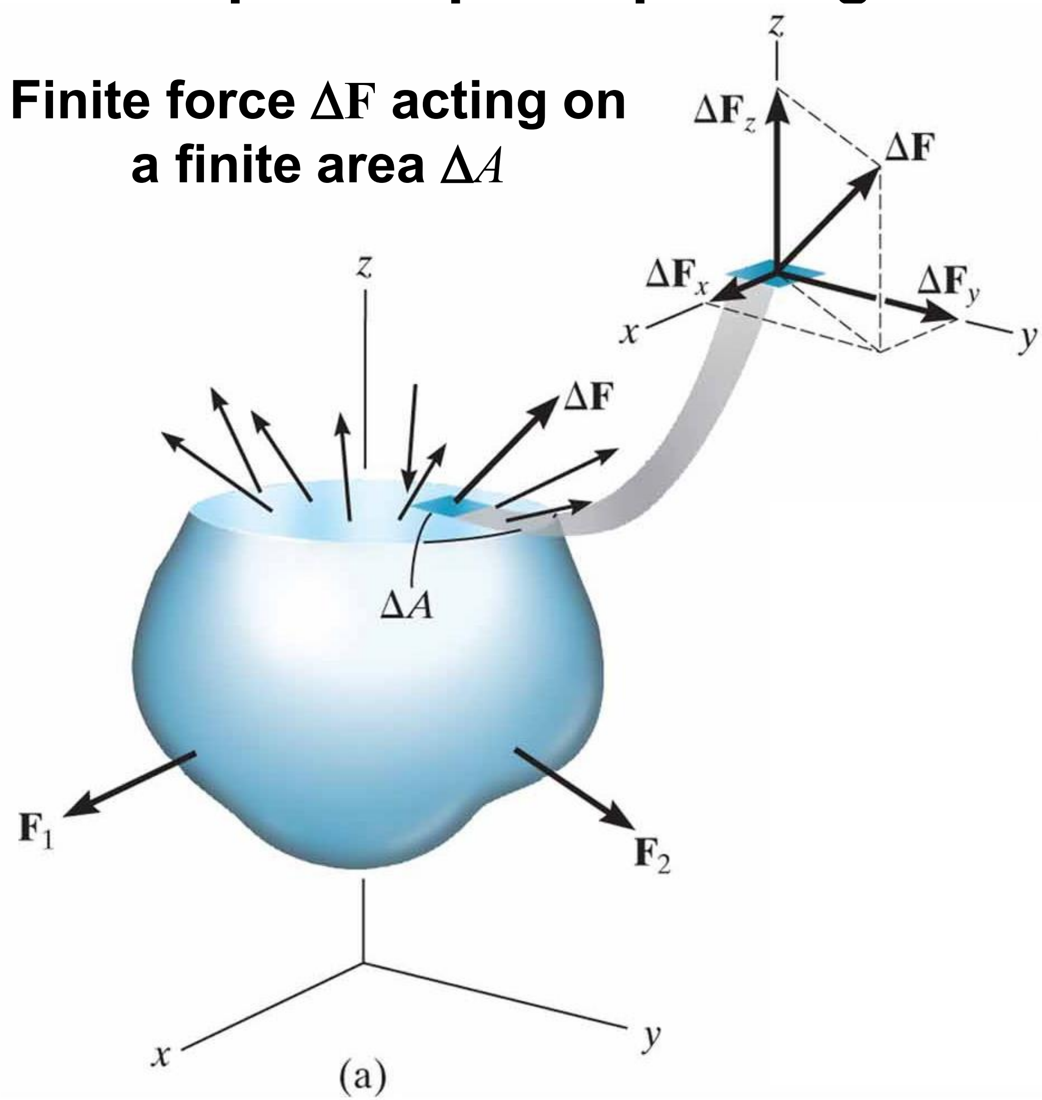
Equivalent force and moment at the section



(c)

Stress. Definition: intensity of internal force: acting on a specific plane passing through a point

Finite force $\Delta\mathbf{F}$ acting on a finite area ΔA



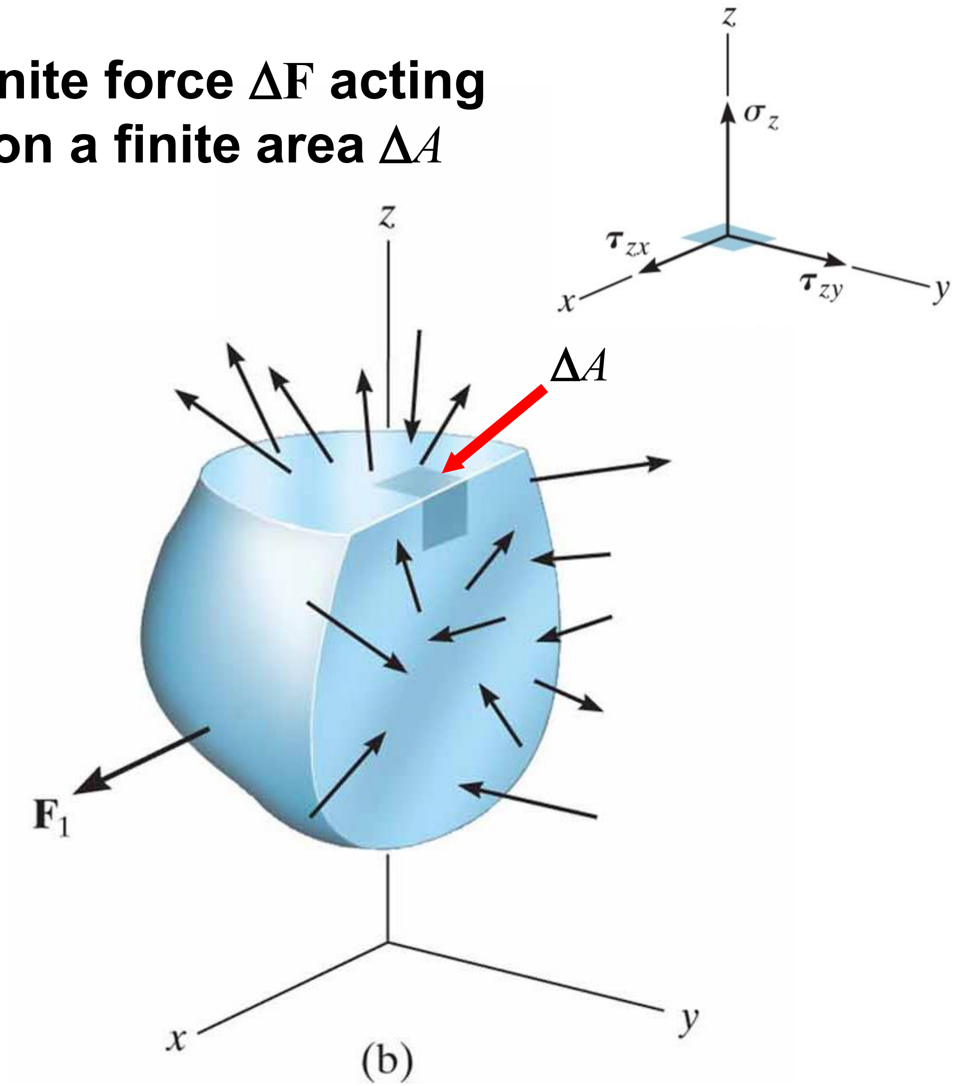
Definition

Normal stress:

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

Stress. Definition: intensity of internal force: acting on a specific plane passing through a point

Finite force ΔF acting on a finite area ΔA



Definitions

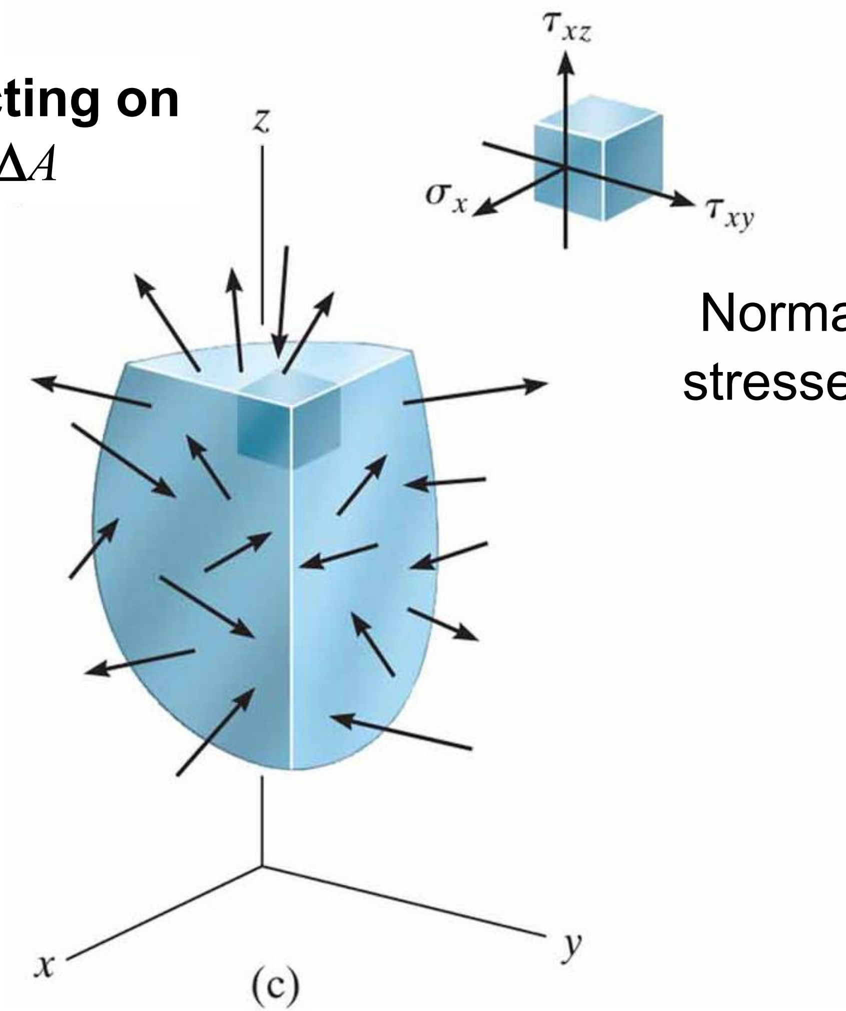
Shear stresses:

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

Stress. Definition: intensity of internal force: acting on a specific plane passing through a point

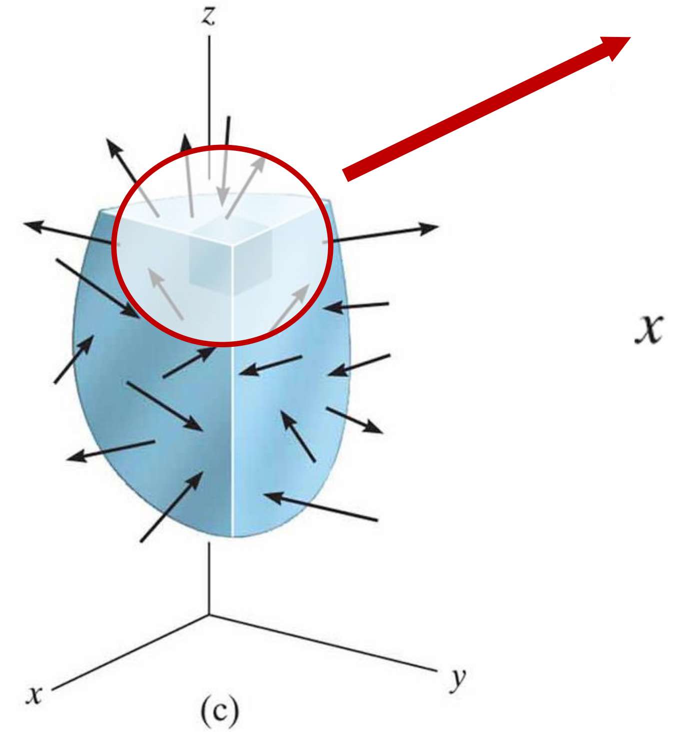
Finite force ΔF acting on a finite area Δ*A*

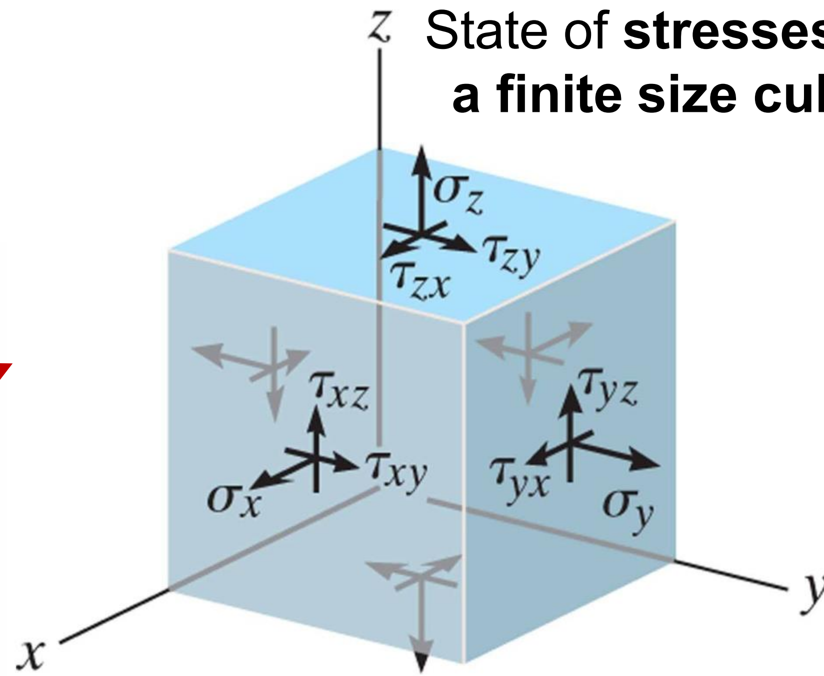


Normal and shear stresses on plane *x*

General state of stresses

Further sectioning leads to a **"stress cube"**





State of **stresses of a finite size cube**

Average normal stress in an axially loaded bar

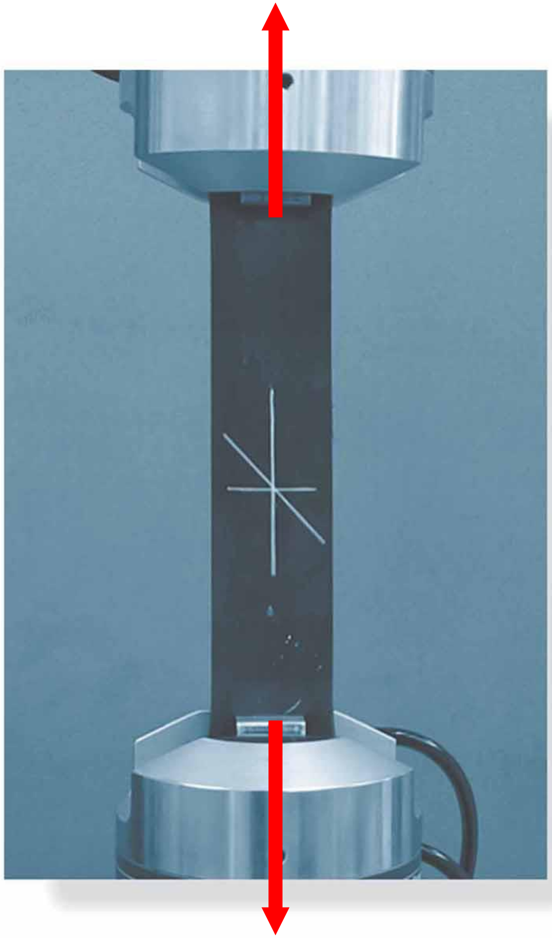


Figure: 02-01-A-UN

Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

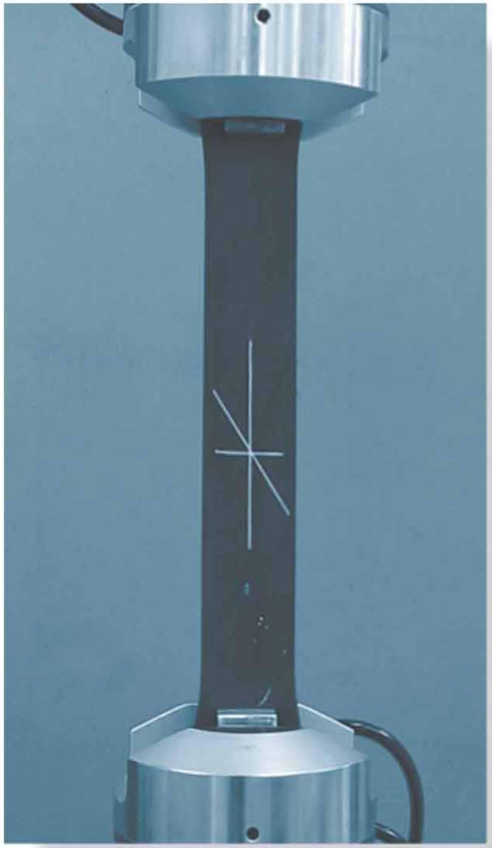
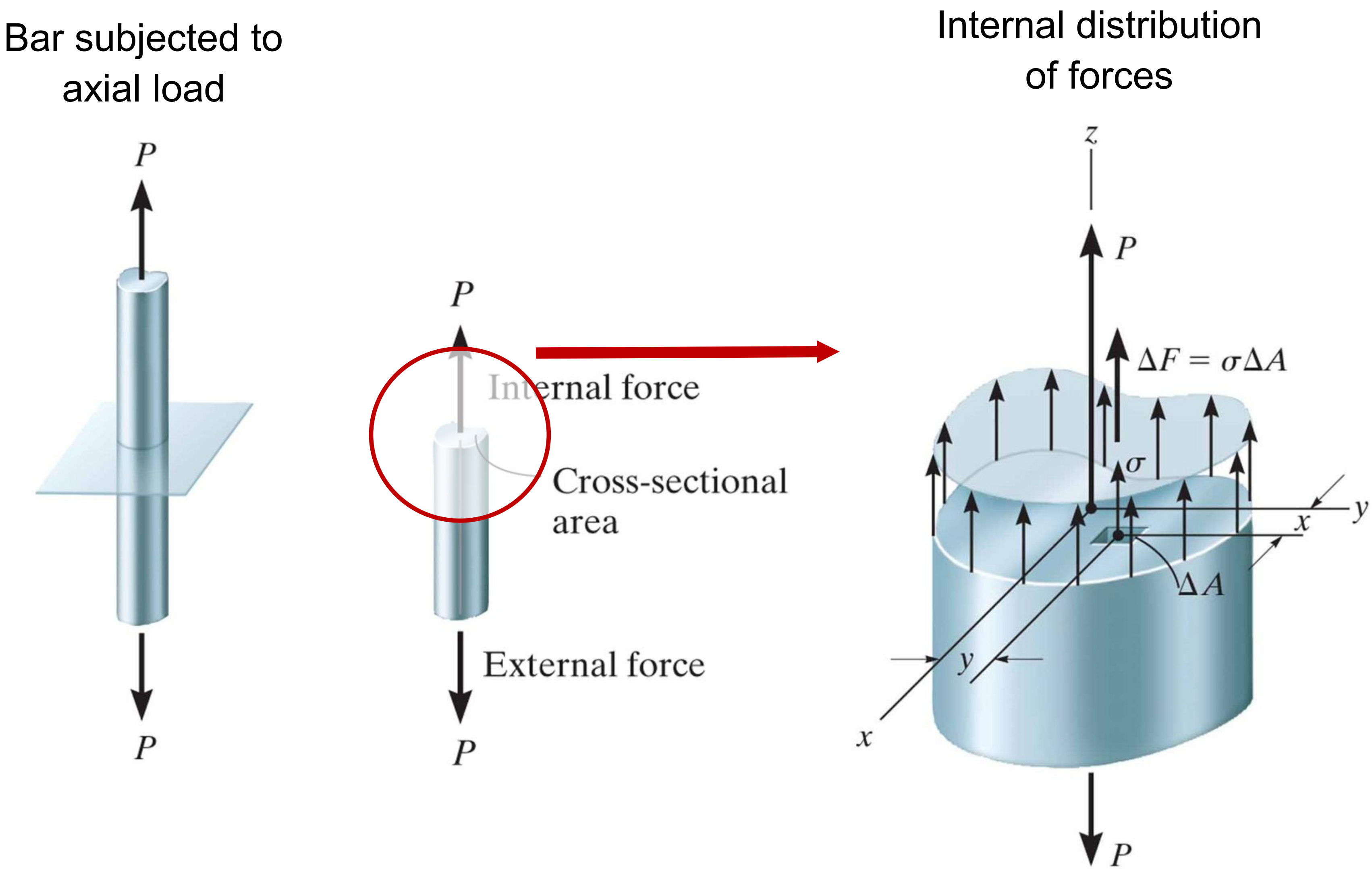


Figure: 02-01-B-UN

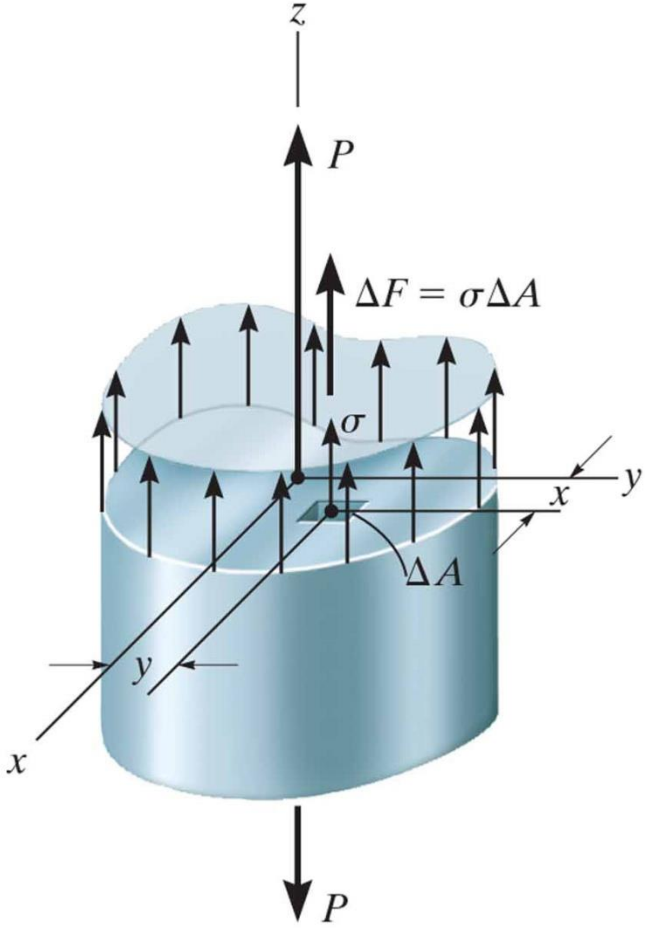
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension.The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Average normal stress in an axially loaded bar



Average normal stress in an axially loaded bar

Internal distribution of forces



$$+\uparrow F_{Rz} = \sum F_z$$

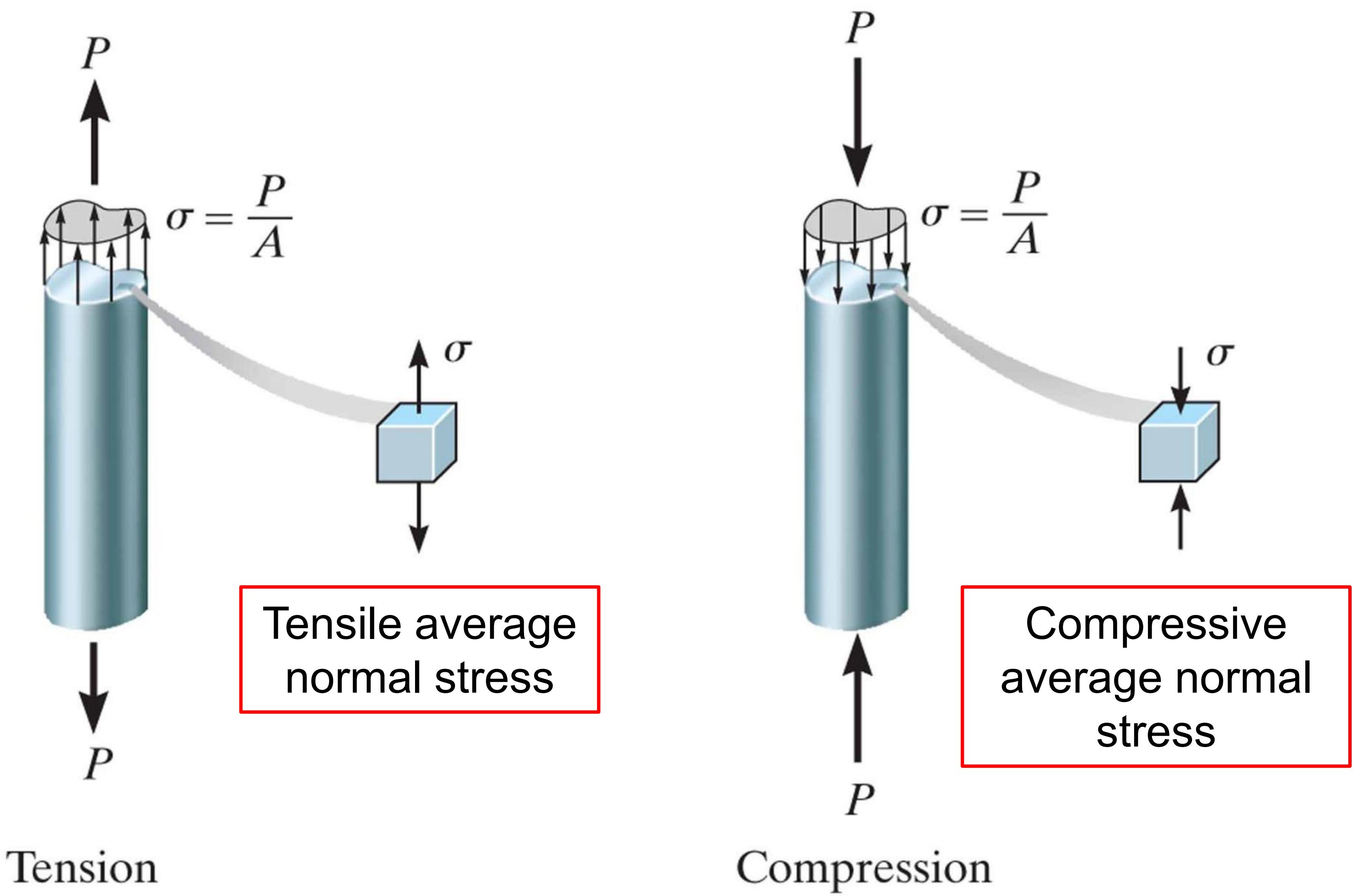
$$\int dF = \int_A \sigma \, dA$$

$$P = \sigma A$$

Average normal stress:

$$\sigma = \frac{P}{A}$$

Average normal stress in an axially loaded bar



Reading assignment

- Chapter 1 of textbook
- Review notes and text: ES2001, ES2501

Homework assignment

- **As indicated on webpage of our course**