1. **Modelling - Investments over time**

An investor has money-making activities A and B available at the beginning of each of the next 5 years (call them years 1 to 5). Each dollar invested in A at the beginning of year 1 returns $1.40 (a profit of $0.40) two years later (in time for immediate reinvestment). Each dollar invested in B at the beginning of year 1 returns $1.70 three years later. In addition, money-making activities C and D will each be available at one time in the future. Each dollar investment in C at the beginning of year 2 returns $1.90 at the end of year 5. Each dollar invested in D at the beginning of year 5 returns $1.30 at the end of year 5.

The investor begins with $50,000 and wishes to know which investment plan maximizes the amount of money that can be accumulated by the beginning of year six. Formulate the linear programming model for this problem.

2. **Modelling - Detergent Production**

The Rosseral Company is a small to medium size detergent manufacturing company. It is one of several companies having production facilities for a new, nonpolluting "washday whitener" which goes by the name of NPW. Rosseral can sell NPW to other detergent manufacturers for $0.80 per gallon. Rosseral itself manufactures detergent that uses NPW. This NPW can be purchased outside for $1.20 per gallon (shipping and handling charges have been added) or be obtained from Rosseral’s own production. Each gallon of detergent produced requires 0.1 gallons of NPW. Production costs for NPW and detergent are respectively $0.50 and $0.60 per gallon. For the detergent production this cost does not include any cost for the 0.1 gallons of NPW used in each gallon of detergent. Detergent can be sold for $0.70 per gallon. Production capacities at Rosseral are: NPW: 10,000 gallons per month; detergent: 120,000 gallons per month.

Formulate the problem of maximizing profit as a linear program and find the solution of the problem.

3. Vanderbei, 3.7

4. Vanderbei, Exercise 5.1. In addition, verify that the dual of the dual is the primal.
5. Given the linear programming problem:

\[
\begin{align*}
\text{maximize} & \quad x_1 + x_3 + 3x_4 \\
\text{subject to} & \quad -3x_1 + 2x_2 - 3x_3 - x_4 \geq -3 \\
& \quad x_1 - 3x_2 - 2x_3 - 2x_4 \geq -6 \\
& \quad 0 \leq x_1, x_2, x_3, x_4.
\end{align*}
\]

(a) Find the dual problem.

(b) Use the Strong Duality Theorem to show that \(\mathbf{x}^* = (0, 0, 0, 3)^T\) and \(\mathbf{y}^* = \left(\frac{5}{7}, \frac{8}{7}\right)^T\) are optimal solutions of the primal and dual problems, respectively.

(c) Verify the complementary slackness condition.

6. Consider the linear programming problem:

\[
\begin{align*}
\text{minimize} & \quad 2x_1 + 3x_2 + 5x_3 + 2x_4 + 3x_5 \\
\text{subject to} & \quad x_1 + x_2 + 2x_3 + x_4 + 3x_5 \geq 4 \\
& \quad 2x_1 - 2x_2 + 3x_3 + x_4 + x_5 \geq 3 \\
& \quad 0 \leq x_1, x_2, x_3, x_4, x_5.
\end{align*}
\]

(a) Find the dual problem.

(b) Solve the dual problem graphically.

(c) Use complementary slackness to deduce the optimal solution to the primal problem.

Note: No credit will be given for use of the simplex method in this question.