MA3233 Assignment 1

DUE DATE: Thursday, November 6, by 5:00pm.

Please carefully read the presentation rules on the back of this sheet. Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.

Please provide carefully written solutions to the following five problems:

1.) Apply the greedy algorithm to the graph $G_1$ on the attached sheet. Submit not only the attached sheet with your chosen spanning tree highlighted, but also a table showing in which order the edges were considered, with "ACCEPT" or "REJECT" written next to each one. Be sure to terminate when the number of accepted edges reaches the number of vertices, minus one. [NOTATIONAL SUGGESTION: Rather than making up names for all 49 edges, use [A, F] etc.]

2.) A cotree in a connected graph $G$ is a subset of the edges of $G$ whose deletion leaves behind a connected graph with no cycles (i.e., as edge sets, the cotrees of $G$ are exactly the complements of the spanning trees of $G$).

(a) Devise a greedy algorithm for the minimum weight cotree problem. As with Kruskal’s Algorithm, you should first sort the edges by increasing weight.

(b) Prove that, for any connected graph $G$, the greedy algorithm you specified in part (a) always finds a minimum weight cotree.

3.) A proper vertex coloring of a graph is an assignment of integers (or “colors”) $1, 2, 3, \ldots$ to the vertices in such a way that no two adjacent vertices receive the same color. A graph $G$ is $k$-colorable if $G$ admits a proper vertex coloring using only colors $1, 2, \ldots, k$. 

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(a) Devise a simple greedy algorithm for vertex coloring. Do not include any preprocessing.

(b) Show that, for each $k > 2$, there exists a bipartite\(^1\) graph $G$ and an ordering of the vertices of $G$ under which your greedy algorithm uses $k$ or more colors. (Will a cycle suffice?)

4.) Consider the two graphs $G_{2,0}$ and $G_{2,1}$ on the attached sheet, each with 12 vertices and 22 edges. One of these graphs has the property that it contains two edge-disjoint spanning trees. Determine which of the two graphs contains two edge-disjoint spanning trees and exhibit the solution. (You may also be curious enough to explain why the other graph does not have this property.)

5.) In the digraph $G_3$ on the attached sheet, find a minimum weight strongly connected spanning subgraph. You need not describe your method; simply highlight the edges you chose. [GRADING: 10 points minus the difference between the weight of your solution and the optimal weight.]

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BASIC RULES FOR MA3233 ASSIGNMENTS

I) Each student must compose his/her assignments independently. However, rough work may be done in groups;

II) Write legibly and use only one side of each sheet of paper;

III) Show your work. Explain your answers using FULL SENTENCES;

IV) Late assignments will be accepted for credit only under very rare circumstances.

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\(^1\)A graph $G$ is bipartite if its vertices can be properly colored using only two colors.