For this quiz, you are given:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
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<tbody>
<tr>
<td>((Ia)_{n</td>
<td>l} = \frac{\ddot{a}_{n</td>
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<tr>
<td>((Da)_{n</td>
<td>l} = \frac{n - a_{n</td>
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<tr>
<td>((Is)_{n</td>
<td>l} = \frac{s_{n</td>
</tr>
<tr>
<td>((Ds)_{n</td>
<td>l} = \frac{n(1+i)^n - s_{n</td>
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- Please write your name on the **first** and **last** page of this quiz.
- For all questions, **show your work**. No credit will be given if you don’t show your work. If you are confused by something, write down what you know or think about the problem – the more you share with me how you are approaching the problems, the easier it is for me to assess whether you understand the concepts.
- Time lines, whether horizontal or vertical (tabular form) are quite helpful, for you to think about and for me to see!
- Please express interest rates (including force of interest) as percentages, with four decimal places (so 0.0123456 would be shown as 1.2346%).
- Annuity factors should be expressed with four decimal places, and dollar amounts shown to two decimal places.
- A few questions may say something like: [SET UP BUT DO NOT EVALUATE] – pay attention to these instructions – all you need is an equation of value that could be solved for the desired item. You could say something like, “Solve the expression \(v + v^2 + v^{10} = 2.3\) for \(i\), for example, and that would be enough.
1. Calculate each of the following.
   
a. Given a nominal rate of interest convertible monthly of 8%, calculate the equivalent nominal rate of discount convertible quarterly.

   \[ 1 - \frac{d^{(4)}}{4} = \frac{1}{\left(1 + \frac{i^{(12)}}{12}\right)^3} \]

   \[ i^{(12)} = 8\% \]

   \[ d^{(4)} = 4 \left[ 1 - \frac{1}{\left(1 + \frac{0.08}{12}\right)^3} \right] = 7.8945\% \]

   b. Given a force of interest \( \delta(t) = 0.01t \) for \( 0 \leq t \leq 10 \), calculate the annual effective rate of interest for the time period between \( t=3 \) and \( t=6 \).

   \[ (1+i)^3 = e^{\int_3^6 \delta(t) dt} \]

   \[ \int_3^6 0.01t dt = \frac{1}{3} \left[ 0.005t^2 \right]_3^6 \]

   \[ i = e^{\frac{1}{3} \left[ 0.005t^2 \right]_3^6} - 1 \]

   \[ \hat{i} = e^{\frac{1}{3} \left[ 0.005(6)^2 \right]_3^6} - 1 \]

   \[ \hat{i} = e^{0.045} - 1 = 4.6028\% \]
2. Calculate each of the following.

a. \((I\bar{s})_{10|i=6\%}\)
   \[(I\bar{s})_{10} = \frac{S_{11}-11}{0.06}\]
   \[(I\bar{s})_{10} \approx [1.06] \left( \frac{\frac{1.06^{11}-1}{0.06}}{0.06} \right)\]
   answer: 70.1657

b. \(a^{(2)}_{15|i^{(2)}=6\%}\)
   \(a^{(2)}_{15} = \left( \frac{1}{2} \right) a^{(2)}_{30} l_{15}^{(2)}\)
   \[= \left( \frac{1}{2} \right) \left( \frac{1 - \left( \frac{1}{1.03} \right)^{30}}{0.03} \right)\]
   answer: 9.8002

c. \(\overline{a}_{\infty|i=7\%}\)
   \(\overline{a}_{\infty} = \frac{1}{\delta} = \frac{1}{\ln(1+i)} = \frac{1}{\ln(1.07)}\)
   answer: 14.7801
3. Give an expression for each of the following, but do not evaluate. Please use standard actuarial symbols.

a) $100 payable at the end of every month for six years, valued one month after the last payment, given \( i^{(12)} = 12\% \)

\[ \text{value} = 100 \cdot \ddot{s}_{\overline{72}|j} = \left( 100 \cdot \ddot{s}_{\overline{72}|1\%} \right) \]

b) The value on January 1, 2011, of payments at the rate of $100 per year beginning July 1, 2013, and occurring every six months thereafter until 14 payments have been made, given \( i^{(2)} = 8\% \)

\[ \text{value} = \left( 50 \cdot a_{\overline{14}|4\%} \right) \to \left( 50 \cdot a_{\overline{14}|4\%} \right) \]

c) The value at time 10 of payments of $40 each at time 4 through time 24 (twenty one payments in all), given an annual effective interest rate of \( i \).

\[ \text{value \ at \ t=10 \ is: } 40 \cdot s_{\overline{7|i}} + 40 \cdot a_{\overline{14|i}} \]
4. What is the value at time $t=0$ of a perpetuity-immediate with annual payments which follow the pattern of 1, 1, 2, 2, 3, 3, etc., for the first 48 payments, with payments at time 49 and beyond equal to 25, given an effective rate of interest of 6%?

\[ \Sigma = 1 + 1 + 2 + 2 + 3 + 24 + 24 + 25 + 25 + 25 + 25 + 25 \]

The above is equivalent to buying a perpetuity due at $t = 1, t = 3, t = 5, \ldots$, and $t = 49$.

\[
PV = \ddot{a}_\infty \left( v + v^3 + v^5 + \ldots + v^{49} \right) = \ddot{a}_\infty \frac{1 - v^{50}}{i + d} = 143.2850
\]
5. What is the internal rate of return associated with the cashflows described below? If you can’t work this out on your calculator, you can receive 80% of the points on this problem for explaining how you could solve this (i.e., show the equation of value you would “solve” to find the answer). The cashflows are: You invest 2000 today, and another 1000 two years from now. You receive repayments of 400 at time t=3, t=5, and t=7, plus 3000 at t=10. **Be sure to show your work!**

\[
\begin{array}{c|c}
\text{time} & \text{cash flow} \\
0 & -2000 \\
1 & 0 \\
2 & -1000 \\
3 & 400 \\
4 & 0 \\
5 & 400 \\
6 & 0 \\
7 & 400 \\
8 & 0 \\
9 & 3000 \\
10 & 0 \\
\end{array}
\]

Cashflows described above

\[
\text{IRR is the rate } i \text{ which solves: }
2000 + 1000v^2 = 400v^3 + 400v^5 + 400v^7 + 3000v^{10}
\]

Using a financial calculator
\[
i = 4.4132\%
\]

\[\uparrow \quad 8 \text{ points} \quad \downarrow \]

\[\leftarrow 2 \text{ points} \quad \rightarrow \]
6. Give an expression for the PV of each of the following cashflows.

<table>
<thead>
<tr>
<th>time</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>400</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>700</td>
<td>600</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>1000</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>1300</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>1600</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>1900</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Grading for this problem will be somewhat subjective, and here’s a hint: Saying “100 (v^2 + v^3 + v^4 + v^5 + v^6)” for $PV_A$ will probably not earn you full points (even though it is “correct”!)

$$PV_A = 100 \frac{a_{\overline{7}|}}{a_{\overline{7}|}} - 100 \cdot a_{\overline{7}|}$$

or

$$PV_A = 100 \cdot \ddot{a}_{\overline{7}|} - 100 \cdot \dddot{a}_{\overline{7}|}$$

$$PV_B = 100 \cdot \dddot{a}_{\overline{7}|} + 300 (Ia)_6$$

or

$$300 \cdot (I \ddot{a})_{\overline{7}|} - 200 \cdot \dddot{a}_{\overline{7}|}$$

$$PB_C = 200 (Da)_{\overline{4}|}$$
7. A loan is made with an effective interest rate of 5.7051%. The loan is to be repaid by payments of $1,000 at the end of each year for 40 years, first payment one year from today.

If the original loan amount were reduced by $1,000, then the two payments scheduled for time $t=k$ and $t=k+1$ could be skipped, with the other 38 payments remaining at $1,000. What is $k$?

The PV of the "skipped" pymts is $1000:

$$1000 v^k + 1000 v^{k+1} = 1000$$

$$v^k = \frac{1}{1+v} = \frac{1+i}{2+i}$$

$$k \ln v = \ln (1+i) - \ln (2+i)$$

$$k = \frac{\ln (2+i)}{\ln (1+i)} - 1$$

$$k = 12$$
8. Using the Rule of 72, do each of the following:

a. Given that 100 invested today grows to 800 by time 12, estimate the annual effective interest rate.

\[
i \approx \frac{72}{4} = 18\%\]

100 \rightarrow 800 means doubling every 4 years

b. Given that 100 invested today grows to 400 by time 12, estimate the annual effective interest rate.

\[
i \approx \frac{72}{6} = 12\%\]

100 \rightarrow 400 means doubling every 6 years
9. Al, Bob, and Charlie agree to split a perpetuity-due evenly, such that each person’s share has the same present value. Al will receive $k\%$ of the first 12 payments, Bob will receive $k\%$ of all payments after the first 12 payments, while Charlie will receive $1-k\%$ of every payment. What is the effective rate of interest?

\[
A = B = C = \frac{1}{3} \ddot{a}_{\infty} \]

\[
k \ddot{a}_{12} = k \left( \left| 12 \right| \ddot{a}_{\infty} \right) = (1-k) \ddot{a}_{12} = \frac{1}{3} \ddot{a}_{12}
\]

\[
k = \frac{2}{3}
\]

\[
B = C
\]

\[
\frac{2}{3} V^{12} \ddot{a}_{\infty} = \frac{1}{3} \ddot{a}_{12} \Rightarrow V^{12} = \frac{1}{2}
\]

Solve for $i$

\[
i = 2^{\frac{1}{12}} - 1
\]

\[
i = 5.9463\%
\]