This quiz contains eight questions, plus bonus as indicated with an assigned point value as shown below. When giving answers as percentages, please show four decimal places (so, 0.041 would be expressed as 4.1000%). No additional time will be given beyond the end of class to finish the quiz. You may earn partial credit on some questions, so be sure to show your work and/or explain things that you want me to be aware of!

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1. Given a nominal rate of discount, compounded quarterly, of 8%, find the equivalent nominal rate of interest, compounded semiannually.

\[ 1 + \frac{i^{(2)}}{2} = \left( 1 - \frac{d^{(4)}}{4} \right)^2 \]

\[ d^{(4)} = 0.08 \quad \Rightarrow \quad 8.2466\% \]

2. Given a nominal rate of interest, convertible monthly, of 9%, find the equivalent constant force of interest.

\[ e^{\frac{1}{12} \delta} = 1 + \frac{i^{(12)}}{12} \]

\[ i^{(12)} = 0.09 \quad \Rightarrow \quad 8.9664\% \]

3. Given a nominal rate of discount, compounded monthly, of 6%, find the equivalent effective discount rate.

\[ 1 - d = \left( 1 - \frac{d^{(12)}}{12} \right)^{12} \]

\[ d^{(12)} = 0.06 \quad \Rightarrow \quad 5.8377\% \]

4. Given a nominal rate of interest, compounded quarterly, of 8%, find the equivalent nominal rate of interest, compounded semiannually.

\[ 1 + \frac{i^{(2)}}{2} = \left( 1 + \frac{i^{(4)}}{4} \right)^2 \]

\[ i^{(4)} = 0.08 \quad \Rightarrow \quad 8.0800\% \]
5. Give an equation of value for each of the following situations. You do **not** have to calculate a numerical result for these problems.

**Example:** The value at \( t=3 \) of 100 invested at \( t=4 \).

**Answer**

\[
Value \text{ at } t=3 \text{ is } = 100v_i \text{ (or } 100v \text{)}
\]

a. The value at \( t=5 \) of 100 invested at \( t=-2 \), 200 invested at \( t=0 \), 300 invested at \( t=5 \), and 400 invested at \( t=7 \).

\[
\begin{align*}
100 & \quad 200 & \quad 300 & \quad 400 \\
& \quad \quad & \quad \quad & \quad \quad \\
-2 & \quad -1 & \quad 0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad 7 \\
\end{align*}
\]

**Answer**

\[
AVE \text{ at } t=5 \text{ is } = 100(1+i)^7 + 200(1+i)^5 + 300 + 400v^2
\]

b. 120 today is equivalent to 50 at each of times \( t=2 \), \( t=4 \), and \( t=n \), assuming a 3% effective interest rate between \( t=0 \) and \( t=3 \), and a 5% effective interest rate thereafter.

\[
\begin{align*}
50 & \quad 50 & \quad 50 \\
& \quad \quad & \quad \quad & \quad \quad \\
0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad n \\
\end{align*}
\]

**Answer**

\[
120 = 50v_{3\%}^2 + 50v_{3\%}^3v_{5\%} + 50v_{3\%}^3v_{5\%}^{n-3}
\]
6. Given a force of interest described by
\[ \delta_t = \frac{2}{t-1} \quad \text{for} \quad 2 \leq t \leq 10 \]
determine the equivalent nominal discount rate \( d^{(2)} \) for the one year period ending at \( t=5 \)

\[
\left(1 - \frac{d^{(2)}}{2}\right)^2 = e^{-\int_4^5 \delta_t \, dt} = e^{-\int_4^5 \frac{2}{t-1} \, dt}
\]

\[
\left(1 - \frac{d^{(2)}}{2}\right)^2 = e^{-2 \ln (t-1)} \bigg|_4^5
\]

\[
\left(1 - \frac{d^{(2)}}{2}\right)^2 = e^{-2[\ln 4 - \ln 3]} = \frac{9}{16}
\]

\[
1 - \frac{d^{(2)}}{2} = \frac{3}{4}
\]

\[ d^{(2)} = 50.0000\% \]

**ANSWER**

50.0000%
7. You have borrowed 1000 at $t = 0$ and have agreed to a simple rate of interest of 7% for a three year loan.

a. What is the annual effective rate of interest?

\[ 1 + 3i = (1 + j)^3 \]

\[ j = \text{annual effective rate} \]

\[ j = 1.21^{1/3} - 1 \]

**ANSWER**

6.5602%

b. What would the annual effective rate of interest be if you had agreed to a simple discount rate of 7% for this same loan?

\[ \frac{1}{1-3d} = (1 + j)^3 \]

\[ j = \text{annual effective rate} \]

\[ j = \frac{1}{0.79^{1/3}} - 1 \]

**ANSWER**

8.1744%
8. You invest 100 in Fund A that grows as follows:
   From $t=0$ to $t=1$, $i^{(2)} = 4\%$
   From $t=1$ to $t=4$, simple interest at 4\% is credited
   From $t=4$ to $t=10$, $\delta = 4\%$
   You also invest 100 in Fund B, which earns an annual effective rate of interest $j$. At the end of 10 years, your investment in Fund A equals your investment in Fund B. Calculate $j$.

   \[ A: \quad (100)(1.02)^2 = 104.04 \quad \text{time 1} \]
   \[ (104.04)(1 + 0.03)(1.04) = 116.5248 \quad \text{time 4} \]
   \[ 116.5248 \times 6.5 = 148.132053 \]

   \[ B: \quad (100)(1+j)^{10} = 148.132053 \]
   \[ j = 4.0076\% \]

**** END OF QUIZ ****
**BONUS:** Fund A uses simple interest, with \( i = 12\% \)

Fund B credits at a continuous force of interest \( \delta = 1/14 \)

You have 1000 to invest for ten years. You can invest in either Fund A or Fund B, and you may transfer money between funds at any time during the ten years.

a. Describe how to invest your money to achieve the highest possible rate of return on a ten-year investment.

b. Express your total return as an annual effective rate.

\[
\text{Fund A: } A(t) = 1 + it
\]

\[
S = \frac{A'(t)}{A(t)} = \frac{I}{1 + it} = \frac{0.12}{1 + 0.12t}
\]

\( S_A \) declines from 12\% at \( t = 0 \)

\[
S_A = S_B \text{ when } \frac{0.12}{1 + 0.12t} = \frac{\delta}{\lambda} = \frac{1}{14}
\]

\( \delta \) when \( t = 5 \frac{2}{3} \)

(a) \( \therefore \) Invest in Fund A from \( t = 0 \) to \( t = 5 \frac{2}{3} \), then move your money to Fund B

(b) \( a(10) = \left[1 + 0.12 \left(5 \frac{2}{3}\right)\right] e^{(4 \frac{1}{3})} \delta = 2.28946 \)

\( (1 + j)^{10} = a(10) \)

\[
j = 8.6359\%
\]