1. \textbf{Calculate} each of the following.
   
a. Given a nominal rate of interest convertible quarterly of 6%, calculate the nominal rate of interest convertible monthly.
   \[
   \dot{i}^{(4)} = 6\%
   \]
   \[
   \left(1 + \frac{i^{(12)}}{12}\right)^3 = 1 + \frac{i^{(4)}}{4}
   \]
   \[
   i^{(12)} = 5.9702\%
   \]

b. Given an annual effective rate of discount of 6%, calculate the nominal rate of discount convertible monthly.
   \[
   d = 6\%
   \]
   \[
   \left(1 - \frac{1 - d^{(12)}}{12}\right)^{12} = 1 - d
   \]
   \[
   d^{(12)} = 6.1716\%
   \]

c. Given $d^{(4)} = 6\%$ find $\delta$
   \[
   d^{(4)} = 6\%
   \]
   \[
   e^{-\delta} = \left(1 - \frac{d^{(4)}}{4}\right)^4
   \]
   \[
   \delta = 6.0455\%
   \]
2. Given a force of interest \( \delta_t = 0.07 - 0.01t \), find the equivalent rate of simple interest for money invested from time \( t=0 \) to time \( t=4 \)

\[
a(4) = e \int_0^4 (0.07 - 0.01t) \, dt
\]

\[
a(4) = e \left[ \frac{0.07t - 0.005t^2}{0} \right]_0^4
\]

\[
= e^{0.2}
\]

\[
= e \approx 1.2214
\]

\[1 + 4i = 1.2214\]

\( i = 5.5351\% \)
3. Calculate

\[
\left( \frac{Ia}{s_{5|i}^{j|k}} \right)
\]

if \(i=16.89595\%\), \(j=16.32671\%\), \(k=5\%

\[
S_{510.1689595} = 7
\]

\[
a_{710.1632671} = 4
\]

\[
(Ia)_{75\%} = 1 + 2v^2 + 3v^3 + 4v^4 = 8.6488
\]

4. A ten-year 8\% bond with a 100 face amount is priced at 118.20 to yield an annual nominal rate of 6\% convertible semiannually. Calculate the redemption value of the bond.

\[
4 \ a_{20|3\%} + \frac{K}{(1.03)^{20}} = 118.20
\]

\[
K = 106
\]
5. Calculate each of the following.

a. \[ 7 \left| I \ddot{a}_{20|1} \right| = 6 \left| I a_{20} \right| = \sqrt{6} \left| I a_{20} \right| \]
   \[ = \sqrt{6} \left( \ddot{a}_{20} - 20v_0^2 \right) \]

   answer: 49.7254

b. \[ \left( Ia_{19|5\%} \right) + \left( Da_{19|5\%} \right) \]
   \[ = 20a_{19} \]
   \[ = 20 \left( \frac{1 - V_{19}}{i} \right) \]

   answer: 241.7064

c. \[ v^n + \delta a_{n} \]
   \[ \text{given } \frac{S_n}{1 + \alpha_n} = 3 \]

   answer: 1.0000

   Always 1!  
   The proverbial red herring
6. You are repaying a $10,000 loan made at \( t=0 \) over ten years via a sinking fund approach. Each year, beginning at \( t=1 \), interest is paid to the lender at 6%, and a contribution is made to the sinking fund. The sinking fund, which earns interest at 3%, will be used to pay off the full amount of the loan at time \( t=10 \). At \( t=6 \), you decide to pay off the loan completely, so you take all the money in the sinking fund and add enough to that to pay off the loan. What is your internal rate of return for the six year period?

Sinking fund:

\[
\begin{align*}
P &= \frac{10,000}{0.0739} \\
&= 872.31
\end{align*}
\]

\( e^{k} \) at \( t=6 \):

\[
P = \frac{4,770.12}{0.0739} = 65,229.88
\]

balance = \( 10,000 - 4,770.12 = 5,229.88 \)

<table>
<thead>
<tr>
<th>Time</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Debt ( \downarrow ) Sinking Fund ( \downarrow ) 10,000</td>
</tr>
<tr>
<td>1</td>
<td>( -600 - 872.31 = -1472.31 )</td>
</tr>
<tr>
<td>2</td>
<td>( -600 - 872.31 = -1472.31 )</td>
</tr>
<tr>
<td>3</td>
<td>( -600 - 872.31 = -1472.31 )</td>
</tr>
<tr>
<td>4</td>
<td>( -600 - 872.31 = -1472.31 )</td>
</tr>
<tr>
<td>5</td>
<td>( -600 - 872.31 = -1472.31 )</td>
</tr>
<tr>
<td>6</td>
<td>( -600 - 5229.88 = -5829.88 )</td>
</tr>
</tbody>
</table>

IRR is 6.7942%
7. Compute each of the following. Please show your work, and express your answers to four decimal places.

a. The value at time $t=10$ of 15 payments of 100 made every year, first payment four years from today, given $i=6\%$

\[
\left(100 \, a_{15\mid} \right) \left(1+i\right)^7
\]

\[\text{Value at } t=3 \text{ to } t=10\]

\[\text{answer: } 1460.36\]

b. The present value of annual payments of 400, 500, ..., 1300 (ten payments in total), first payment three years from today, given $i=8\%$

\[
\left(300 \, a_{10\mid 8\%} + 100 \left(\bar{a}_{10\mid 8\%}\right) \left(1 + \frac{1}{2}\right)\right)
\]

\[\text{Value at } t=2 \text{ to } t=0\]

\[\text{Discount to } t=0\]

\[\text{answer: } 4528.22\]
8. You pay 1000 for a bond which pays annual coupons of 70 at the end of each year for 12 years, and then repays the 1000 at t=12. You are able to reinvest the coupon payments into a bank account which earns a 6% annual effective interest rate. What is your yield rate on this investment?

\[
(1000)(1+j)^{12} = 70 \times \overline{S}_{12|6\%} + 1000
\]

\[
(1+j)^{12} = \frac{2180.90}{1000}
\]

\[
j = 6.7136\%
\]
9. A loan of 100,000 is being paid off over 5 years at an annual effective interest rate of 6%. Assuming DECREASING payments of 5X, 4X, 3X, 2X, and X at the end of years 1, 2, 3, 4, and 5, respectively, construct the amortization schedule (you may round each entry to the nearest dollar). Be sure to calculate X!

<table>
<thead>
<tr>
<th>t</th>
<th>OB$_t$</th>
<th>pymt</th>
<th>I$_t$</th>
<th>PR$_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67,911</td>
<td>38,088.65</td>
<td>6,000.00</td>
<td>32,088.65</td>
</tr>
<tr>
<td>2</td>
<td>41,515</td>
<td>30,470.92</td>
<td>4,074.68</td>
<td>26,396.24</td>
</tr>
<tr>
<td>3</td>
<td>21,153</td>
<td>22,853.19</td>
<td>2,490.91</td>
<td>20,362.28</td>
</tr>
<tr>
<td>4</td>
<td>7,187</td>
<td>15,235.46</td>
<td>1,269.17</td>
<td>13,966.29</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>7,617.73</td>
<td>431.19</td>
<td>7,186.54</td>
</tr>
</tbody>
</table>

100,000 = X (Da)\text{5\%}.

\[ X = 7617.73 \]
10. Find the present value of a annual perpetuity whose first payment occurs 1 year from today, if the payments following this pattern:
1, 2, 3, 5, 7, 9, 12, 15, 18, 22, 26, 30, etc. and i=6%

\[ S = v + 2v^2 + 3v^3 + 5v^4 + 7v^5 + 9v^6 + 12v^7 + 15v^8 + 18v^9 + \ldots \]

\[ v^3 S = v^4 + 2v^5 + 3v^6 + 5v^7 + 7v^8 + 9v^9 + \ldots \]

\[ (1-v^3) S = v + 2v^2 + 3v^3 + 4v^4 + 5v^5 + 6v^6 + 7v^7 + 8v^8 + \ldots \]

\[ S = \frac{(Ia)_{\infty}}{1-v^3} = \left( \frac{1}{i} \right) \left( \frac{1}{1-v^3} \right) \]

\[ S = 1835.91 \]
11. Give an expression for the value at $t=0$ for each set of cashflows shown below, using only multiples of the following symbols. Pay attention to what is and isn’t shown, and don’t use a symbol that doesn’t appear below!

$$(Ia)_{\bar{m}} , (Da)_{\bar{m}} , (I\ddot{a})_{\bar{m}} , (D\ddot{a})_{\bar{m}} , (Is)_{\bar{m}} , (Ds)_{\bar{m}} , (I\ddot{s})_{\bar{m}} , (D\ddot{s})_{\bar{m}} , a_{\bar{m}} , \ddot{a}_{\bar{m}} , s_{\bar{m}} , \ddot{s}_{\bar{m}}$$

<table>
<thead>
<tr>
<th>Problem A</th>
<th>Problem B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>time</strong></td>
<td><strong>cashflow</strong></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
</tr>
<tr>
<td>6</td>
<td>400</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

**Answer A**  
Many answers possible!

$$PV_A = 200(Da)_{\bar{7}} - 800(\ddot{a})_{\bar{7}} - 200(Da)_{\bar{3}}$$

**Answer B**

$$PV_B = 2100(\ddot{a})_{\bar{7}} - 400(D\ddot{a})_{\bar{5}}$$
12. You borrow $100,000 today at an interest rate of 6%, and agree to pay off this loan over twenty years on the following plan: 8 annual payments of $5,000 beginning one year from now, followed by a series of annual payments that are $X$ at time 9, and then the payments in years 10 to 20 are each 90% of the previous year’s payment.

Immediately following the fourth payment, you are able to refinance the outstanding loan balance at 3% interest. Your revised payments will be level for 10 more years, the last payment coming at $t=14$.

What is your internal rate of return on this transaction?

\[ OB_4 = (100,000)(1.06)^4 - 5000 \cdot 5.76\% \]

\[ = 104,374.62 \]

\[ OB_4 = P \cdot a_{10\%|3\%} \]

\[ P = 12,235.89 \]

<table>
<thead>
<tr>
<th>$t$</th>
<th>$CF_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100,000</td>
</tr>
<tr>
<td>1</td>
<td>-5,000</td>
</tr>
<tr>
<td>2</td>
<td>-5,000</td>
</tr>
<tr>
<td>3</td>
<td>-5,000</td>
</tr>
<tr>
<td>4</td>
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<td>6</td>
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<td>-12,236</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>14</td>
<td>-12,236</td>
</tr>
</tbody>
</table>

\[ IRR \text{ is } 4.3845\% \]
13. The dollar amount of interest earned on $X$ for **three** years is $4,000, while the equivalent amount of discount is $3,000. Calculate $X$.

\[
\frac{x+4000}{x} = \frac{x}{x-3000}
\]

\[
x^2 + 1000x - 12,000,000 = x^2
\]

\[
x = \frac{12,000,000}{1000}
\]

\[
x = 12,000
\]
14. Answer each of the following:

a. Use the Rule of 72 to estimate how long it will take $100 invested at 12% to grow to $800.

100 → 200 → 400 → 800
Money doubles three times

@ 12%, money doubles in \( \frac{72}{12} = 6 \) years
Doubling three times takes 18 years

b. What is the one big idea of this course?

You'd better get this one right!!!
A loan is repaid with level annual payments based on an annual effective interest rate of 7%. The 8th payment consists of 789 of interest and 211 of principal. Calculate the amount of interest paid in the 18th payment.

\[ \text{Level pymt} = PR_8 + I_8 = 211 + 789 = 1000 \]

\[ PR_{18} = (1+j)^{10} \times PR_8 \]
\[ = (1.07)^{10} \times (211) \]
\[ = 415.07 \]

\[ I_{18} = 1000 - PR_{18} \]

\[ I_{18} = 584.93 \]

****** END OF FINAL EXAM ******