For this quiz, you are given the following formulas:

\[
(Ia)_{n|} = \frac{\bar{a}_{n|} - ny^n}{i} \quad (Da)_{n|} = \frac{n-a_{n|}}{i}
\]

\[
(Is)_{n|} = \frac{\bar{s}_{n|} - n}{i} \quad (Ds)_{n|} = \frac{n(1+i)^n - s_{n|}}{i}
\]

Timeline. The magic word is TIMELINE.

1. A loan of $200,000 is scheduled to be repaid by payments at the end of each year for 30 years, at an effective interest rate of 7%. Immediately following the 7\textsuperscript{th} payment, the loan is re-financed: The new rate of interest is 5%; payments will be made at the end of each year for the next 15 years. What is the total amount of interest paid over the 22 year life of this loan? (10 points)

\[
OLB_7 = (200,000)(1.07)^7 - P \leq 717\% = 181,677,0081
\]

\[
Q = \frac{181,677,0081}{\alpha_{15}|5\%} \Rightarrow Q = 17503.1786
\]

Total payments = 7P + 15Q

Total principal = 200,000

Total interest = 7P + 15Q - 200,000 = $175,369
2. A loan of $10,000 is being repaid by regular installments of $845 at the **beginning** of each year, and a smaller final payment made one year after the last regular payment. Interest is at the effective rate of 4%.

   a. What is the outstanding loan balance immediately following the eighth payment? (7 points)

   \[
   \begin{array}{ccccccc}
   0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
   845 & 845 & 845 & 845 & 845 & 845 & 845 & \text{OLB}
   \end{array}
   \]

   \[
   \text{OLB} = (10,000)(1.04)^7 - 845 \times 874\%
   \]

   \[
   = 13,159.32 - 7,786.02
   \]

   \[
   = \$5,373.30
   \]

   b. How much principal is included in the fifth payment? (7 points)

   \[
   \begin{array}{ccccccc}
   0 & 1 & 2 & 3 & 4 & 5 \\
   845 & 845 & 845 & 845 & 845 & \text{OLB here}
   \end{array}
   \]

   \[
   \text{OLB} = (10,000)(1.04)^3 - 845 \times 574\%
   \]

   \[
   = 11,248.64 - 3,588.26
   \]

   \[
   = 7,660.38
   \]

   \[
   \text{int in 5th pymt} = (0.04)(7660.38) = 306.42
   \]

   \[
   \text{principal in 5th pymt} = 845 - \text{int} = \$538.58
   \]
3. Bob has borrowed $100,000 on which interest is charged at 8% effective. He is accumulating a sinking fund at 6% effective to repay the loan. At the end of six years the balance in the sinking fund is $40,000. At the end of the seventh year, Bob makes a total payment of $12,000. Answer the following questions (and show your work) (10 points total)

a. How much of the $12,000 pays interest currently on the loan?

\[
\text{debt service} = 0.08 \times 100,000 = \$8,000
\]

b. How much of the $12,000 goes into the sinking fund?

\[
12,000 - 8,000 = \$4,000
\]

c. How much of the $12,000 should be considered as interest?

- Sinking fund earns \(0.06 \times 40,000 = \$2,400\)
- Debt service - Sf interest = 8,000 - 2,400 = \$5,600

d. How much of the $12,000 should be considered as principal?

\[
12,000 - 5,600 = \$6,400
\]

e. What is the sinking fund balance at the end of the seventh year?

\[
40,000 + 6,400 = \$46,400
\]

or

\[
(40,000)(1.06) + 4000 = \$46,400
\]
4. You are running in a marathon with eight other runners, and you pass the last place runner. What place are you in now? (2 points)

Impossible to pass the last place runner

5. You are given two options for re-paying a $20,000 loan:
   a. Payments at the end of each year for 15 years, at an effective rate of interest of 10%
   b. Pay debt service of 8.75% per year, and accumulate the necessary principal over 15 years in a sinking fund which accumulates at an effective rate of interest of 5%

Which option do you prefer, and why? (10 points)

(a) annual payment = \[ \frac{20,000}{a_{15|10\%}} = \$2629.48 \]

(b) annual payment = debt service + sinking fund contribution

= \[(0.0875)(20,000) + \frac{20000}{s_{15|5\%}} \]

= \$1750 + 926.85

= \$2676.85

I prefer (a), since I can save almost $50/year in my payment
6. A loan of $X$ is to be repaid over $n$ years by level annual payments beginning at time $t=1$, calculated using an effective annual interest rate $i$. You are given:

a. $A =$ present value at $t=0$ of the $3^{rd}$ payment
b. $B =$ present value at $t=0$ of the $11^{th}$ payment
c. $C =$ principal portion of the $11^{th}$ payment
d. $A = 2B = 2C = $1000

Calculate $X$ \((10 \text{ points})\)

\[
\begin{align*}
A &= P \nu^3 \\
B &= P \nu^n \\
A &= 2B
\end{align*}
\]

\[
\begin{align*}
P \nu^3 &= 2P \nu^n \\
\nu^8 &= \frac{1}{2} \\
i &= 9.05077\% \\
OLB_{10} &= P a_{n-10} \\
\text{princ}_{11} &= P - (i)(0LB_{10}) = P - iPa_{n-10} = \nu^n \\
1 - (1 - \nu^{n-10}) &= \nu^n \\
\nu^{n-10} &= \nu^n \\
n &= 21
\end{align*}
\]

$P \nu^3 = 1000$

$P = 1296.84$

$X = P a_{n_i}$

$X = (1296.84) a_{21|9.05077\%}$

$X = $12,005.78

\[
\begin{align*}
X &= P a_{n_i} \\
X &= (1296.84) a_{21|9.05077\%} \\
X &= $12,005.78
\end{align*}
\]
**Bonus Question** *(up to 10 points):*

Allen repays a loan with ten annual payments that begin at $100 at time $t=1$, dropping to $90$ at time $t=2$, and so on, continuing to decrease by $10$ per year until his final payment of $10$ is made at time $t=10$.

Bob repays a different loan amount with $n$ level annual payments of $10$ beginning at time $t=1$.

Both loans are based on the same effective interest rate, $i=2.5365\%$.

Allen notices that the interest included in his $4^{th}$ payment is the same as the interest included in Bob's $4^{th}$ payment. (Allen doesn't have much of a social life, which is why he has time to figure things like this out).

What was Bob’s initial loan balance? What is $n$?

\[
\begin{align*}
\text{Allen:} & \quad OL_B^A = 10 \cdot (Da)_{7i} = OL_B^B \\
\text{Bob:} & \quad OL_B^B = 10 \cdot a_{n-31i} \\
& \quad (Da)_{7i} = a_{n-31i} \Rightarrow 7 - a_{7i} = 1 - v^{n-3} \Rightarrow n = 46
\end{align*}
\]

Bob's original loan = $10 \cdot a_{46i} = 269.69$

***END OF QUIZ***