ANSWER KEY

Name: ________________

MA 3211 - 2006 B Term  Theory of Interest
December 1, 2006  Quiz – Chapter 5

For this quiz, you are given the following formulas:

\[
(Ia)_n = \frac{\ddot{a}_n - nv^n}{i} \quad (Da)_n = \frac{n - a_n}{i}
\]

\[
(Is)_n = \frac{s_n - n}{i} \quad (Ds)_n = \frac{n(1+i)^n - s_n}{i}
\]

1. Fund A has a balance of $1,000,000 at time t=0 and a balance of $1,300,000 at time t=1. Contributions between t=0 and t=1 were $170,000.

Fund B also has a balance of $1,000,000 at time t=0, but its balance at time t=1 is $1,400,000.

The estimated one year yield rate for Fund A is equal to the estimated one year yield rate for Fund B. What were the total contributions to Fund B between time t=0 and time t=1? (10 points)

\[
I_A = B_A - A_A - C_A
\]

\[
I_A = 1,300,000 - 1,000,000 - 170,000
\]

\[
I_A = 130,000
\]

\[
I_A = \frac{2I_B}{A_B + B_B - I_B}
\]

\[
i_B = i_A \Rightarrow \frac{2I_B}{1,000,000 + 1,400,000 - I_B} = \frac{(2)(130,000)}{1,000,000 + 1,300,000 - 130,000}
\]

Solving, \[ I_B = 135,652.17 \]

\[
C_B = B_B - A_B - I_B = \$264,347.83
\]
2. You are running a marathon, and you pass the second place runner. What place are you in now? \( (2 \text{ points}) \)

\[ 2^{\text{nd}} \text{ place} \]

3. You invest $1000 at \( t=0 \). In exchange you receive $90 at \( t=2, t=3, t=4, t=5, t=6, t=7, \text{ and } t=8 \) (so, $630 in all); then, at time \( t=10 \), your $1000 is repaid. You are able to reinvest money at 5%. What is the IRR for your 10-year investment? \( (10 \text{ points}) \)

\[
\begin{array}{cccccccccc}
-1000 & 90 & 90 & 90 & 90 & 90 & 90 & 90 & 1000 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10
\end{array}
\]

\[
(1000)(1+\text{IRR})^{10} = (905.715\%)(1.05)^2 + 1000
\]

\[
= (90)(8.142008)(1.1025) + 1000
\]

\[
= 1807.89
\]

\[
(1+\text{IRR})^{10} = 1.80789
\]

\[
\text{IRR} = 6.10\%
\]
4. You make $100 payments into a fund each year until ten payments have been made, beginning at t=0. Annual interest payments are returned to you, calculated at 4% interest. Interest can be reinvested at 8%.

a. How much does your investment grow to by time t=10? (10 points)

\[\text{\$100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100} \]

\[\begin{array}{cccccccccc}
4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 & 40 \\
\end{array}\]

\[\text{at } t=10, \text{ fund } = 10 \times 100 + 4 (I5)_{10}0.08 \]

\[\text{fund } = 1000 + 4 \left[ \frac{5 - 10}{0.08} \right] = \$1282.27\]

b. What is your IRR for the ten year period t=0 to t=10? Explain how you determined (or would determine) this rate. (6 points)

\[\text{\$100 IRR} = 1282.27 \left< \text{solve for IRR} \right. \]

\[\text{\$10 IRR} = 12.8227 \]

\[\text{IRR} = 0.0447433 \]

\[\text{IRR} = 4.47\% \]
5. For a particular investment, you are given the following information.

<table>
<thead>
<tr>
<th>Time</th>
<th>Contributions</th>
<th>Returns</th>
<th>( R_t )</th>
<th>( PVCO )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100,000</td>
<td>0</td>
<td>-100,000</td>
<td>-100,000</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>40,000</td>
<td>20,000</td>
<td>-20,000</td>
<td>-17,781.47</td>
</tr>
<tr>
<td>3</td>
<td>30,000</td>
<td>30,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>X</td>
<td>X</td>
<td>( \frac{X}{(1.06055)^4} )</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>105,000</td>
<td>105,000</td>
<td>78,258.87</td>
</tr>
</tbody>
</table>

If the yield rate for this investment is 6.055\%, what is \( X \)? \( \text{(10 points)} \)

\[
P(\text{IRR}) = -100,000 - 17,781.47 + \frac{X}{(1.06055)^4} + 78,258.87 = 0
\]

\[
X = 50,000
\]
6. You loan me $1000 for 10 years at an 8% annual interest rate. You are given four choices of how the loan will be repaid:

   a. All principal and interest repaid at t=10
   
   b. Level annual payments equal to \( \frac{1000}{a_{10|8\%}} \) at t=1, 2, 3, ..., 10
   
   c. Interest paid annually; principal repaid at t=10
   
   d. Level annual payments equal to \( \frac{1000}{a_{10|8\%}} \) at t=0, 1, 2, ..., 9

Rank the above repayment methods in the order you would prefer if the reinvestment rate over the next 10 years is 11%. Explain your answer, in words! (10 points)

Since the reinvestment rate is higher than the rate on the loan, you prefer to receive as much money as you can sooner rather than later.

Option (d) pays principal + interest beginning immediately, so it's the best. Option (b) also pays principal + interest but begins one year later. Option (c) only pays interest, while option (a) gives you nothing to reinvest.

In order of preference,

\[ d > b > c > a \]
7. For a particular fund, you are given:

<table>
<thead>
<tr>
<th>Date</th>
<th>Contribution</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-1</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Feb-1</td>
<td>0</td>
<td>105</td>
</tr>
<tr>
<td>Mar-1</td>
<td>200</td>
<td>307</td>
</tr>
<tr>
<td>Apr-1</td>
<td>0</td>
<td>311</td>
</tr>
<tr>
<td>May-1</td>
<td>0</td>
<td>309</td>
</tr>
<tr>
<td>June-1</td>
<td>0</td>
<td>313</td>
</tr>
<tr>
<td>July-1</td>
<td>-100</td>
<td>215</td>
</tr>
<tr>
<td>Aug-1</td>
<td>0</td>
<td>220</td>
</tr>
<tr>
<td>Sep-1</td>
<td>0</td>
<td>224</td>
</tr>
<tr>
<td>Oct-1</td>
<td>100</td>
<td>330</td>
</tr>
<tr>
<td>Nov-1</td>
<td>0</td>
<td>335</td>
</tr>
<tr>
<td>Dec-1</td>
<td>0</td>
<td>340</td>
</tr>
<tr>
<td>Jan-1</td>
<td>0</td>
<td>320</td>
</tr>
</tbody>
</table>

For any given date, the balances shown are immediately following any contributions made on that date. What is the time weighted rate of return for this fund for the one year period shown in the table? (10 points)

\[
1 + \bar{r} = \frac{107}{100} \times \frac{315}{307} \times \frac{230}{215} \times \frac{320}{330} = 1.1388889
\]

\[
\bar{r} = 13.89\%
\]
**BONUS QUESTION**

You are considering paying $X$ at time $t=0$ for a five year investment opportunity, which will operate as follows: $500 will be invested at the beginning of each quarter (so, 20 payments of $500) beginning at t=0. The payments earn interest at a nominal rate $i^{(4)}=8\%$, which is paid each quarter as it is earned. The interest can be reinvested at a nominal rate of interest $i^{(4)}=6\%$. Find the purchase price $X$ for this investment which produces a yield rate of 9% effective for the five year period. (8 points)

\[
i^{(4)} = 8\% \Rightarrow 2\% \text{ per 4 tr} \\
i^{(4)} = 6\% \Rightarrow 1.5\% \text{ per 4 tr}
\]

\[
(X)(1.09)^5 = 20 \times 500 + 10 \left( \overline{I_{5}} \right)_{20} 1.5\
\]

\[
X = \frac{10,000 + 10 \left( 231.36814 \right)_{20}}{(1.09)^5}
\]

\[
X = \$8603.05
\]

**** END OF QUIZ ****