1. Calculate the value at $t=5.00$ of 32 payments of $10 each, made at quarterly intervals beginning at $t=0.25$, given $i^{(4)}$ equals 8%.

VALUE = \(10 S_{20|0.02} + 10 A_{12|0.02}\)

\[\begin{align*}
\text{VALUE} &= (10) \left[ \frac{(1.02)^{20}-1}{0.02} + \frac{1}{(1.02)^{12}} \right] \\
&= (10) \left[ 24.29737 + 10.57534 \right] \\
&= \$ 348.73
\end{align*}\]
2. You are given an annuity-immediate which pays $100 every six months for 12 years, with an nominal interest rate compounded monthly of 9%.

   a. Calculate the present value of this annuity at $t=0$ \( (10 \text{ points}) \)

   \[ i^{(12)} = 9\% \implies j = \text{monthly rate} = (1 + \frac{0.09}{12})^{-1} = 0.4852235\% \]

   \[ PV = 100 \ a_{\overline{24}|j} = \frac{1 - \left(\frac{1}{1+j}\right)^{24}}{j} \times 1000 \]

   \[ PV = \$ 1,437.30 \]

   b. Write an expression for the present value at $t=0$ of this annuity, using standard actuarial notation. \( (5 \text{ points}) \)

   \[ \begin{align*}
   100 & \left[ \frac{1}{2} \right] \quad \Rightarrow \quad 100 = R \ s_{\overline{6}|0.0675} \\
   0 & \left[ \frac{1}{2} \right] \quad \Rightarrow \quad PV = R \ a_{\overline{14.4}|0.0075} = \frac{100 a_{\overline{14.4}|0.0075}}{s_{\overline{6}|0.0675}}
   \end{align*} \]

   \[ PV = \frac{100 \ a_{\overline{14.4}|0.0075}}{s_{6|0.0675}} \]

   \[ i = \text{annual effective rate} = 1.0075^{12} - 1 = 9.3807\% \]

   \[ PV = 200 \ a_{\overline{12}|i} \]
3. You are given an annuity-immediate which pays $100 every month for 8 years, with an effective interest rate of 6%.

a. Calculate the present value of this annuity at t=0 \( (10 \text{ points}) \)

\[ j = \text{monthly rate} = 1.06^{\frac{1}{12}} - 1 = 0.04867581\% \]

\[ \text{PV} = 100 \cdot \overline{a}_{\overline{96}|j} \]

\[ = 100 \left[ \frac{1 - \frac{1}{1 + j}^{96}}{j} \right] = \$7,654.52 \]

b. Write an expression for the present value at t=0 of this annuity, using standard actuarial notation. \( (5 \text{ points}) \)

\[ \overline{a}_{\overline{8}|0.06} \]
4. Evaluate $(Ia)_{n|} + (Da)_{n|}$, given $n=50$ and $a_{n|} = 20.59306131$ (10 points)

\[ (Ia)_{50|} \quad 1 \quad 2 \quad 3 \quad 4 \quad 48 \quad 49 \quad 50 \]
\[ (Da)_{49|} \quad 49 \quad 48 \quad 47 \quad 46 \quad 2 \quad 1 \quad - \]
\[ \Sigma \Rightarrow \quad 50 \quad 50 \quad 50 \quad 50 \quad 50 \quad 50 \quad 50 \]

From the timeline, we see:

\[(Ia)_{50|} + (Da)_{49|} = 50 \cdot a_{50|} \]

\[= (50)(20.59306131) \]

\[= \$1,029.65 \]
5. You and your twin have inherited an increasing perpetuity immediate, with an annual effective interest rate of 5%.

You suggest splitting the perpetuity so that you will receive the 1st, 3rd, 5th, 7th, etc. payments, and your twin can have the 2nd, 4th, 6th, 8th, etc. payments.

You point out that each of your twin’s payments will be bigger than yours ($2 > $1, $4 > $3, etc.), and you suggest that this makes the arrangement a “good deal” for your twin.

Your twin points out that you get your payments a year ahead of his payments, so maybe your share of the perpetuity will be greater. (Apparently he has heard of “the time value of money”).

What is the total value of the perpetuity? How much is your share of the perpetuity worth? Who is the evil twin? \( \text{10 points} \)

\[
(Ia)_{\overline{\infty}} = \lim_{n \to \infty} \frac{\ddot{a}_n - n \dot{v}^n}{i} = \frac{\ddot{a}_n}{i} = \frac{1}{i d}
\]

\[
(Ia)_{\overline{\infty}} = \frac{1 + i}{i^2} = \frac{1.05}{(0.05)^2} = 420
\]

Twin’s share = \(2v^2 + 4v^4 + 6v^6 + 8v^8 + \cdots = S\)

\(2v^4 + 4v^6 + 6v^8 + \cdots = v^2S\)

\((1 - v^2)S = \frac{2v^2 + 2v^4 + 2v^6 + \cdots}{1 - v^2} = \frac{2v^2}{1 - v^2}\)

\(S = \frac{2v^2}{(1 - v^2)^2} = 209.88\)

Your share = \(420 - 209.88 = $210.12\) \(\text{You are the evil one!}\)
BONUS QUESTION  (up to 10 points; quiz score cannot exceed 100%) 

Give a numerical value for \((Da)_{50}\) if the present value of the tenth payment is equal to the sum of the present value of the eighteenth payment plus the present value of the twenty sixth payment.

\[
\begin{array}{cccccccc}
50 & 49 & 51-1 & 2 & 1 \\
0 & 1 & 2 & \cdots & t & 49 & 50 \\
\end{array}
\]

\[\text{PV of } t^{th} \text{ pymt} = (51-t) V^t\]

\[(51-10)V^{10} = (51-18)V^{18} + (51-26)V^{26}\]

\[25V^{16} + 33V^8 - 41 = 0\]

Quadratic in \(V^8\)

\[V^8 = \frac{-33 + \sqrt{33^2 + (4)(25)(41)}}{50}\]

\[i = 3.1430277\%\]

\[(Da)_{50} = \frac{50 - a_{50}}{i} = \frac{50 - 25.04537}{0.031430277}\]

\[(Da)_{50} = \$793.97\]

**** END OF QUIZ ****