1. You gave me $100 \ n \ \text{years ago. In exchange, I will give you}$ $1600 \ n \ \text{years from now. Assuming the same effective interest rate, what is the}$ $\text{value today of}$ $300 \ \text{to be received}$ $\frac{n}{2} \ \text{years from now, plus}$ $300 \ \text{to be received}$ $n \ \text{years from now, plus}$ $300 \ \text{to be received}$ $2n \ \text{years from}$ $\text{now?} \ (10 \ \text{points})$

\[
\begin{align*}
\text{(1+r)}^2n &= \frac{1600}{100} \Rightarrow (1+r)^n = 4 \Rightarrow V^n = \frac{1}{4} \\
\text{PV at } t = 0 &= 300 \ V^{\frac{n}{2}} + 300 \ V^n + 300 \ V^{2n} \\
&= 300 \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{16} \right) \\
&= \$243.75
\end{align*}
\]
2. Find the accumulated value of $4,000 at the end of 6 years and 7 months, assuming a nominal interest rate $d^{(2)}$ of 6%, and further assuming simple discount through the final fractional period. Express your answer to the nearest penny. (5 points)

$\text{Accumulated Value} = \frac{\$4000}{(1-0.03)^{13} (1- \frac{1}{6}(0.03))} = \$5,973.16$

$6 \text{ yrs, 7 mos} = 13 \text{ six month periods} + \frac{1}{6} \text{ six month period}$

$d^{(2)} = 6\% \approx d = 3\% \text{ per six month period}$
3. It costs $300 a year, payable on the first day of each year, to hire someone to clean the windows of your house. On January 1, 2007, you are considering buying a window-washing robot, which would clean your windows for 4 years, and then be sold for scrap metal on December 31, 2010. Its price as scrap metal will be $500. Assuming an annual effective interest rate of 5%, what is the most you would be willing to pay for the robot on January 1, 2007? (10 points)

\[
\begin{align*}
\text{Hire someone} & \rightarrow \quad \$300 \quad \$300 \quad \$300 \quad \$300 \\
\text{Robot} & \rightarrow \quad 4X \\
\end{align*}
\]

\[
0 \quad 1 \quad 2 \quad 3 \quad 4
\]

We want to be indifferent between hiring someone and buying the robot:

\[
300 \left[ 1 + v + v^2 + v^3 \right] = 4X - 500 v^4
\]

\[
4X = 300 \left[ 1 + v + v^2 + v^3 \right] + 500 v^4
\]

\[
= \$1528.33
\]
4. A famous actuary is telling you about his latest discovery – the Rule of 240. He explains that is similar to the Rule of 72, but instead of being useful for situations where we want to know how long until money doubles in value, this new discovery tells you how long until money grows to be “N” times its original value.

   a. What is N? (7 points)
   b. Does the Rule of 240 work better for $i=6\%$ or $i=8\%$? (3 points)
   c. Is the phrase “famous actuary” an oxymoron? (0 points)

\[
\frac{240}{100i} \approx N
\]

<table>
<thead>
<tr>
<th>$i$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>2%</td>
<td>10.765</td>
</tr>
<tr>
<td>4%</td>
<td>10.520</td>
</tr>
<tr>
<td>6%</td>
<td>10.286</td>
</tr>
<tr>
<td>8%</td>
<td>10.063</td>
</tr>
<tr>
<td>10%</td>
<td>9.850</td>
</tr>
<tr>
<td>12%</td>
<td>9.646</td>
</tr>
</tbody>
</table>

\[(b) \text{ Works better for } 8\% \text{ (see table above)}\]

\[(c) \text{ No!!} \]
5. $10,000 is invested for $n + k$ years at a nominal interest rate of $i^{(4)}$.
You are given:

a. $n$ is a positive integer

b. $0 < k < 1$

c. The following table is based upon an effective annual interest rate $i$ which is equivalent to $i^{(4)}$:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$V^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.978030991</td>
</tr>
<tr>
<td>$n$</td>
<td>0.818791922</td>
</tr>
<tr>
<td>$2n-1$</td>
<td>0.716620535</td>
</tr>
</tbody>
</table>

Calculate how much the $10,000 will accumulate to in $n + k$ years, assuming simple interest over the final fractional period. Please round your final answer to the nearest penny. (10 points)

\[
1 + i = \frac{V^{2n-1}}{(V^n)^2} = \frac{0.716620535}{(0.818791922)^2} = 1.068912486
\]

\[
i^{(4)} = 4 \left( (1 + i)^{\frac{1}{4}} - 1 \right) = 6.72\% \text{ (1.68\% per quarter)}
\]

\[
0.978030991 = 1.068912486^{-k} \Rightarrow k = \frac{1}{3} = 4 \text{ months}
\]

\[\therefore "n+k \text{ years} = "4n+1 \text{ quarters} + 1 \text{ month}\]

\[\text{Accum Value} = (10,000)(1.0168)^{4n+1} = (10,000)(1+i)^n(1.0168)(1.0056) = \$12,487.84\]

Note: could solve for $n = 3$, but it's not necessary
6. $1000 is deposited into a fund on November 7, 2006 (hey, that's today!), and will be withdrawn on January 11, 2007 (hey, that's the first day of C term!). (a) How many actual days is this? (3 points)

| Nov | 30 - 7 = 23 |
| Dec | 31 |
| Jan | 11 |
|     | \[ \frac{65}{365} \text{ days} \]

Compute the amount of interest earned assuming \( i = 7\% \) and using:

(b) Ordinary simple interest (3 points)

\[
(1000)(0.07)\left(\frac{64}{360}\right) = 12.44
\]

\*[360(2007-2006) + 30(1-11) + 11-7 = 64]

(c) Exact simple interest (3 points)

\[
(1000)(0.07)\left(\frac{65}{365}\right) = 12.47
\]

(d) The Banker's Rule (3 points)

\[
(1000)(0.07)\left(\frac{65}{360}\right) = 12.64
\]
7. At what annual effective interest rate is 150 received a year ago equivalent to 100 payable two years from now plus 100 received today? Use any method you like to do the calculation – but explain in words how you arrived at your answer. (10 points)

\[ 150 \]
\[ -1 \quad 0 \quad 1 \quad 2 \]
\[ 100 \quad 100 \]

\[ (150)(1+ti) = 100 + 100v^2 \]

\[ 100v^3 + 100v - 150 = 0 \]

\[ 2v^3 + 2v - 3 = 0 \]

A variety of possibilities exist to determine \( v = 0.8613 \)

from which we can determine \( i = 16.11\% \)

****END OF QUIZ****