1. Write an expression for the current value at $t = 3$ of payments received as follows: $500$ received at $t = 0$; $700$ received at $t = 3$; four payments of $250$ each, received at $t = 4$, $t = 5$, $t = 8$, and $t = 11$, given that the effective rate of interest is $i$ from time $t = 0$ to time $t = 5$ and $j$ thereafter. (Note: You may wish to define $v_i = \frac{1}{1+i}$ and $v_j = \frac{1}{1+j}$, although this isn't required). 

(6 points)
2. You are given a choice between two investment funds:

**Fund A** credits a nominal rate of discount of 8%, convertible monthly.

**Fund B** credits a nominal rate of interest \(i^{(4)}\), convertible quarterly.

Determine \(i^{(4)}\) such that you will be equally happy with either fund. (6 points)

\[
A: \quad a(t) = \frac{1}{(1 - \frac{0.08}{12})^{12t}}
\]

\[
B: \quad a(t) = \left(1 + \frac{i^{(4)}}{4}\right)^{4t}
\]

\[
\left(1 + \frac{i^{(4)}}{4}\right)^{4t} = \frac{1}{(1 - \frac{0.08}{12})^{12t}}
\]

\[
1 + \frac{i^{(4)}}{4} = \frac{1}{(1 - \frac{0.08}{12})^{3}}
\]

\[
\frac{i^{(4)}}{4} = \frac{4}{(1 - \frac{0.08}{12})^{3}} - 4 = 8.1078638
\]

\[i^{(4)} = 8.11\%\]
3. Complete the following table. Please show both a formula and a numerical evaluation. No timeline required for this problem.

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Present value at $t=2$ of $1,000$ at $t=6$</th>
<th>Accumulated value at $t=6$ of $1000$ at $t=3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a^{-1}(t) = \frac{1}{1+it}$</td>
<td>$a(t) = 1 + it$</td>
</tr>
<tr>
<td>4% simple interest</td>
<td>$\frac{1000}{1+(4)(0.04)} = 862.07$</td>
<td>$1000(1+(3)(0.04)) = 1120$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6% simple discount</td>
<td>$a^{-1}(t) = 1-dt$</td>
<td>$a(t) = \frac{1}{1-dt}$</td>
</tr>
<tr>
<td></td>
<td>$1000 \left(1-(4)(0.06)\right) = 760.00$</td>
<td>$1050 \left(1-(3)(0.06)\right) = 1219.51$</td>
</tr>
<tr>
<td>8% nominal discount, compounded semiannually</td>
<td>$a^{-1}(t) = \left(1-\frac{d(m)}{m}\right)^{mt}$</td>
<td>$a(t) = \frac{1}{\left(1-\frac{d(m)}{2}\right)^{2t}}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1000}{1-(0.04)^2}(1-0.04) = 721.31$</td>
<td>$1050 \left(1-0.04\right)^{2(3)} = 1277.53$</td>
</tr>
</tbody>
</table>

Express your answers to the nearest penny. (6 points)
4. Express $i^{(12)}$ in terms of $d^{(4)}$ (5 points)

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$$

$$1 + \frac{i^{(12)}}{12} = \frac{1}{\sqrt[3]{\left(1 - \frac{d^{(4)}}{4}\right)}}$$

$$i^{(12)} = \frac{12}{\left(1 - \frac{d^{(4)}}{4}\right)^{\frac{1}{3}}} - 12$$

5. Given $a(t) = at^2 + b$ and also that $a(3) = 1.72$, find the accumulated value at time $t = 10$ of 100 invested at time $t = 5$. (6 points)

$$a(0) = 1 \Rightarrow b = 1$$

$$a(3) = 9a + 1 = 1.72 \Rightarrow a = 0.08$$

$$a(5) = (0.08)(5)^2 + 1 = 3$$

$$a(10) = (0.08)(10)^2 + 1 = 9$$

$3 @ t = 5$ becomes $9 @ t = 10$

Thus, $100 @ t = 5$ will become $300 @ t = 10$
6. You are given: 

\[ i^{(m)} = 0.0987654321 \]
\[ d^{(m)} = 0.0981949458 \]

Determine \( m \). (6 points)

\[
1 + \frac{i^{(m)}}{m} = \frac{1}{1 - \frac{d^{(m)}}{m}}
\]

\[
1 + \frac{i^{(m)}}{m} - \frac{d^{(m)}}{m} = \frac{i^{(m)} d^{(m)}}{m^2} = 1
\]

\[
\frac{i^{(m)}}{m} - \frac{d^{(m)}}{m} = \frac{i^{(m)} d^{(m)}}{m^2}
\]

\[
m = \frac{i^{(m)} d^{(m)}}{i^{(m)} - d^{(m)}}
\]

\[
m = \frac{(0.0987654321)(0.0981949458)}{0.0987654321 - 0.0981949458}
\]

\[ m = 17 \]

(Note: \( m = 17 \) in "real life")

(\( m = 17 \) would be very odd)
7. You are given \( \delta_t = \frac{2}{t-1} \) for \( 2 \leq t \leq 10 \). Calculate the equivalent nominal rate of discount compounded semiannually for the one year interval from \( t = 3 \) to \( t = 4 \). (8 points)

\[
\int_3^4 \frac{2}{r-1} \, dr = \frac{1}{\left(1 - \frac{d^{(2)}}{2}\right)^2}
\]

\[
2 \ln(r-1) \bigg|_3^4 = 2 \ln 3 - 2 \ln 2 = \ln \frac{9}{4}
\]

\[
\ln \frac{9}{4} = \frac{1}{\left(1 - \frac{d^{(2)}}{2}\right)^2} = \frac{9}{4}
\]

\[
1 - \frac{d^{(2)}}{2} = \frac{2}{3} \implies d^{(2)} = \frac{2}{3}
\]

**Note: In "real world" terms, this is an unusually large rate**

**** END OF QUIZ ****