1. A loan of $100,000 is being repaid by regular installments of $9,000 at the end of each year, and a smaller final payment made one year after the last regular payment. Interest is at the effective rate of 5%. What is the outstanding balance immediately following the eighth payment? (10 points)

\[
\text{OLB}_8 = (100,000)(1.05)^8 - (9000)(5.8705) \\
= 147,746 - 55,942 = 61,804
\]

In order to do this problem prospectively, we'd have to first solve for "S", the final smaller payment, and also determine "n", when that last pymt occurs.
2. A loan of $500,000 is scheduled to be repaid by payments at the end of each year for 25 years, at an effective interest rate of 10%. After 8 years of payments, the loan is re-financed: The new rate of interest is 8%; payments will be made at the end of each year for the next 10 years. What is the total amount of interest paid over the 18 year life of this loan? (10 points)

\[ L = 500,000 \]
\[ P_i = 55084 \]
\[ OLB = 441860 \]
\[ P_r = 65850 \]

\[ P_i = \frac{500,000}{a_{15.10}} = 55,084 \quad (P_{initial}) \]

\[ t = 8, \quad OLB = (500,000)(1.10)^8 - (P_i)(a_{5.10}) = 441,860 \]
\[ OLB = (P_i)(a_{17.10}) = 441,860 \]

\[ P_r = \frac{OLB}{a_{10.08}} = 65,850 \quad (P_{revised}) \]

Total payments = \( 8 \times P_i + 10 \times P_r = 1099174 \)

Total principal = 500,000

Total interest = 1099174 - 500,000 = 599,174
3. Bob and Charlie borrow $173,265 from the bank for their small business. The loan is to be repaid by the sinking fund approach, over \( n \) years. Interest is charged on the loan at 9.43%. The sinking fund payments accumulate at 4.05%. Bob agrees to pay the annual debt service on the loan, if Charlie will make the contributions to the sinking fund. (Both Bob and Charlie’s first payment is one year after the loan is made).

Find \( n \) if Bob and Charlie’s payments, rounded to the nearest whole dollar, are the same each year over the life of the loan. \( 10 \) points

\[
\text{Debt service} = (0.0943)(173,265) = 16,338.89
\]

Thus,
\[
(16,338.89)(S\overline{n}_{1.045}) = 173,265
\]

\[
(16,338.89)\frac{(1+i)^n - 1}{i} = 173,265
\]

\[
N = \ln \left[ \frac{(173,265)(0.045)}{(16,338.89)} + 1 \right] = 9 \text{ yrs}
\]
4. You are given two options for re-paying a $25,000 loan:
   a. Payments at the end of each year for 15 years, at an effective rate of interest of 7%
   b. Pay debt service of 6% per year, and accumulate the necessary principal over 15 years in a sinking fund which accumulates at an effective rate of interest of 4.5%

Which option do you prefer, and why? (10 points)

(a) \[ P_a = \frac{25,000}{a_{157.07}} = 2,745 \]

(b) \[ P_B = (0.06)(25,000) + S \]

where \( S(5_{157.045}) = 25,000 \)

\[ S = \frac{25,000}{S_{157.045}} = 1203 \]

\[ P_B = 2703 \]

I prefer option B, because it's less outlay for the same loan.
5. You are considering purchasing a 10-year $1000 bond with a 7% coupon rate (coupons are $35 payable every six months), redeemable at $980, and purchased to yield 6% convertible semiannually.

For the same price, the lender offers you a different bond: A 10-year zero coupon bond, which will be redeemable at $1900.

Which bond represents a better value to you, and why? (13 points)

\[
\begin{align*}
\text{1st Bond price} &= 35 \times \frac{1}{(1.03)^{10}} + \frac{980}{(1.03)^{20}} = 1063.31 \\
\text{2nd Bond same price } \Rightarrow (1063.31)(1+i)^{20} &= 1900 \\
i &= 0.0294
\end{align*}
\]

The second bond has a yield rate of 2.94% (expressed in six month periods), compared with the first bond's yield rate of 3%. I prefer the first bond, with the higher yield.

\[\text{Annualized, } 6.092 > 5.98%\]
**Bonus Question** *(up to 10 points applied to this quiz):*

Consider the amortization schedule of: \( $100 \left( Ia_{30}\right) = 13.4917\% \)

You are given that the first payment of $100 consists of interest of $754.99 and principal of -$654.99. Find \( n \) such that:

- \( 1 < n < 30 \)
- The \( n \)th payment includes $0 of principal (to the nearest dollar).

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The 18th pymt is almost 105th interest.

**** END OF QUIZ ****