1. Calculate the value at t=0.00 of 40 payments of $20 each, made at quarterly intervals beginning at t=0.25, given \( i^{(2)} \) equals 6% (10 points)

\[ j = \text{quarterly rate} = \left(1 + \frac{i^{(2)}}{2}\right)^{\frac{1}{3}} - 1 = 1.4889157\% \]

\[ 20 \cdot a_{\overline{40}|j} = 20 \cdot (29.97646178) \]

\[ = \$599.53 \]

2. Write an expression for the annuity in the previous problem, using standard actuarial notation. (4 points)

\[ (4) \]

\[ 80 \cdot a_{\overline{10}|0.0609} \]

\[ \begin{align*}
\dot{i} &= 6.09\% \\
\dot{i}^{(4)} &= 5.955663\% 
\end{align*} \]

\[ (80) \left(1 - V^{10}_{\dot{i}^{(4)}}\right) = \$599.53 \]

If we wanted to evaluate \( i \)
3. Calculate the value at time 0 of an annuity due which pays $100 every three years if $i^{(4)}$ equals 8% and if twelve payments are made in total. **(10 points)**

\[ j = "3\text{-yr int}" = (1 + \frac{i^{(4)}}{4})^{12} - 1 = 26.8241795\%
\]

\[ 100 \ddot{a}_{12}\overline{j} = 100 \left( \frac{1 - \left(\frac{1}{1+j}\right)^{12}}{\frac{j}{1+j}} \right) = \$4,454.49 \]

4. Give a numerical value for \((Ia)_{\infty}\) if the present value of the tenth payment is equal to the present value of the eleventh payment. **(12 points)**

\[ 10v^{10} = 11v^{11} \Rightarrow v = \frac{10}{11} \Rightarrow i = \frac{1}{10} \Rightarrow d = \frac{1}{11} \]

\[ (Ia)_{\infty} = \lim_{n \to \infty} \frac{\ddot{a}_n - nv^n}{i} = \frac{\ddot{a}_{\infty}}{i} = \frac{1}{id} \]

\[ (Ia)_{\infty} = \frac{1}{\left(\frac{1}{10}\right)\left(\frac{1}{11}\right)} = \$110 \]
5. If $\frac{a_{41}}{a_{21}} = 1.81873$, what is $S_{10}^{(12)}$? (11 points)

\[ \frac{1 - \nu^4}{\delta} = 1.81873 \Rightarrow 1 + \nu^2 = 1.81873 \]

\[ \nu = 0.904837 \]

\[ \iota = 10.5171426\% \]

\[ S_{10}^{(12)} = \frac{(1 + i)^{10} - 1}{\iota^{(12)}} \]

\[ \iota^{(12)} = 12 \left[ (1 + i)^\frac{1}{12} - 1 \right] = 10.041829\% \]

\[ S_{10}^{(12)} = \frac{\left(1.105171426\right)^{10} - 1}{0.10641829} = 17.1114 \]
6. Calculate the value at time 0 of an annuity which pays $300 every six months starting at time 1/2 and continuing for 16 payments if \( i^{(12)} \) equals 12%. (10 points)

\[
\bar{a}_{\overline{16}|j} = 300 \times 300.37
\]

\[
j = 6\text{-month rate} = (1 + \frac{i^{(12)}}{12})^{8} - 1 = 6.1520151\%
\]

7. Write an expression for the annuity in the previous problem, using standard actuarial notation. (4 points)

\[
(2) \quad PV = 600 \times a_{\overline{8|0.126825}}
\]

\[
i = (1 + \frac{i^{(12)}}{12})^{12} - 1 = 12.6825\%
\]

\[
R_{S_{670.01}} = 300
\]

\[
PV = R \times a_{\overline{96|0.01}}
\]

\[
PV = \frac{300 \times a_{\overline{96|0.01}}}{5_{670.01}}
\]
8. Calculate the value at time 8 of an annuity which pays $100 per month starting at time 0 and continuing for 120 payments if \( i^{(4)} \) equals 5%. (10 points)

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
0 & 1 & 2 & 3 & \ldots & 8 & 9 & 10 \\
\hline
\end{array}
\]

\[
\dot{\bar{a}}_{sj} = \text{monthly int rate} = \left(1 + \frac{i^{(4)}}{4}\right)^\frac{1}{3} - 1
\]

\[
\dot{\bar{a}}_{sj} = 0.4149425\%
\]

\( t = 8 \text{ yrs} \) [96 months]

\[
PV = 100 \cdot \bar{a}_{96|j} + 100 \cdot \dot{\bar{a}}_{24|j}
\]

\[
= 100 \left[ 118.1262348 + 22.89428042 \right]
\]

\[
= \$14,102.05
\]
BONUS QUESTION  (up to 6 points; quiz score cannot exceed 100%)

Use the "ratio technique" of chapter section 4.3 to write an expression for the present value of 10 payments which are made every three years beginning immediately, given a nominal rate of interest convertible quarterly. Be sure to define R, n, k, m, i, j, and whatever other variables you use – also, draw a timeline, and make sure your answer is in the \( \frac{a_{12}\, a_{1}}{a_{1}} \) or \( \frac{a_{12}\, a_{1}}{s_{1}} \) format!

\[ j = \frac{6\%}{4} \]

\[ RA_{121\, j} = 1 \]

\[ PV = RA_{120\, j} = \frac{a_{120\, j}}{a_{12\, j}} \]

\[ n = \text{annuity period in interest conversion periods} = 10 \times 3 \times 4 = 120 \]

\[ k = \text{int conversion periods between payments} = 12 \]

**** END OF QUIZ ****