1. You are saving for a new car. You wish to accumulate sufficient funds by making sixty monthly deposits into a savings account, beginning today. Interest is a nominal 6%, compounded monthly.

   You have two goals:
   a. Have $18,000 five years from today to buy a car
   b. Have enough money left over to provide for 48 monthly payments of $100 to operate the car (gas, maintenance, etc.).

Assume you will need the first $100 on the day you buy the car, and that the nominal interest rate is 6%, compounded monthly.

How much does your monthly deposit need to be? (10 points)

\[
\$x \ddot{s}_{\overline{60}\rvert 0.06}^{\frac{12}{12}} = 18,000 + 100 \ddot{a}_{\overline{48}\rvert 0.06}^{\frac{12}{12}}
\]

\[
i = \frac{0.06}{12} = 0.005 \quad d = \frac{i}{1+i} = \frac{0.005}{1.005}
\]

\[
\ddot{s}_{\overline{60}\rvert i} = \frac{(1.005)^{60}-1}{(0.005)(1.005)} = 70.11888
\]

\[
\ddot{a}_{\overline{48}\rvert i} = \frac{1-(\frac{1}{1.005})^{48}}{(0.005)(1.005)} = 42.79322
\]

\[
\$x = \frac{(18000) + (100)(42.79322)}{(70.11888)} = 317.74
\]
2. $1000$ five years from now plus $1000$ ten years from today is known to be equivalent to a series of ten annual payments of $X$ commencing six years from today. If $i=6\%$, what is $X$? \(\text{8 points}\)

\[
\begin{array}{c|cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
\hline
1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 & 1000 \\
\end{array}
\]

\[e^{t=5}: \quad 1000 + 1000v^5 = X \cdot a_{10\,|\,0.06} \]

\[X = \frac{1000 + 1000v^5}{a_{10\,|\,0.06}} = \frac{231.65}{a_{10\,|\,0.06}}\]

3. Evaluate $300\bar{s}_8^{\cdot\cdot}$ assuming $d=8\%$. \(\text{7 points}\)

\[d = 8\% \Rightarrow v = 0.92 \Rightarrow 1+i = \frac{1}{0.92} \]

\[300 \bar{s}_8^{\cdot\cdot} = 300 \frac{(1+i)^8-1}{d} \]

\[(300) \left[ \frac{(1)(0.92)^8-1}{0.08} \right] = \frac{3556.82}{0.08} \]
4. At time \( t=6 \), the current value of \( A + B \) is equivalent to the current value of \( C + D \), where:

- \( A \) is a ten year annuity-certain of $X$ per year, first payment occurring at time \( t=1 \)
- \( B \) is $5000$ payable at time \( t=5 \)
- \( C \) is an annual perpetuity with first payment of $600$ at time \( t=7 \)
- \( D \) is $10,000$ payable at time \( t=10 \)

If \( i=8\% \), what is $X$? \((10\text{ points})\)

\[ X = \frac{6000 \times 6^6 a_{\overline{10}|0.08}}{a_{\overline{10}|0.08}} + 10000 \times 10 - 5000 \times 5 \]

\[ X = 2,548 \]
5. A perpetuity-due which pays $1,200 per year and costs $13,200 is known to be equivalent to a perpetuity of $X per year which has its first payment five years from today. What is $X$? (10 points)

\[
\begin{align*}
1200 & \quad 1200 \quad 1200 \quad 1200 \quad 1200 \quad 1200 \quad 1200 \quad X \\
0 & \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7
\end{align*}
\]

\[
1200 \, a_{\overline{\infty}} = 13,200 = X \cdot v^5 \cdot a_{\overline{\infty}}
\]

\[
\frac{1200}{d} = 13200 \Rightarrow d = \frac{1}{11} \Rightarrow v = \frac{10}{11}
\]

\[
1200 = X \cdot v^5 \Rightarrow X = \frac{1200}{v^5} = \left(\frac{11}{10}\right)^5 (1200)
\]

\[
X = 1932.61
\]

6. A, B, and C are dividing the proceeds from the sale of their company. Each person will receive an equal share of the proceeds. The company has been sold for $X. Instead of a single payment, however, the buyer has offered 30 years of annual $30,000 payments, beginning one year from today. From these payments, A has asked to receive $15,000 per year for $n$ years and nothing thereafter. B and C have agreed to this. Assuming $i^{(4)} = 7.68\%$, what is $n$? (10 points)

\[
15000 \, a_{\overline{n}} = 10000 \, a_{\overline{30}}
\]

\[
\frac{1-v^n}{i} = \left(\frac{2}{3}\right) \left(\frac{1-v^{30}}{i}\right) \Rightarrow n = \frac{\ln\left[1 - \frac{3}{2}(1-v^{30})\right]}{\ln[v]}
\]

\[
i^{(4)} = 7.68\% \Rightarrow i = \left(1 + \frac{0.0768}{4}\right)^4 - 1 = 0.07904
\]

\[
n = 12
\]
A special annual perpetuity due pays $1 for the first two payments, $2 for the next two payments, $3 for the next two payments, and so on. What is the present value of this perpetuity, if \( i = 8\% \)?

\[
S = 1 + v + 2v^2 + 2v^3 + 3v^4 + 3v^5 + 4v^6 + 4v^7 + \ldots
\]

\[
v^2 S = v^2 + v^3 + 2v^4 + 2v^5 + 3v^6 + 3v^7 + \ldots
\]

\[
(1-v^2) S = 1 + v + v^2 + v^3 + v^4 + v^5 + v^6 + v^7 + \ldots
\]

\[
S = \frac{a_\infty}{1-v^2} = \frac{1}{d(1-v^2)}
\]

\[
S = \frac{1}{(\frac{0.08}{1.08})(1-(\frac{1}{1.08})^2)} = 94.63
\]

**** END OF QUIZ ****