1. You gave me $100 \ 2n \text{ years ago. In exchange, I will give you $800 \ n} \text{ years from now.}
   
   a. Use the Rule of 72 to estimate the annual effective interest rate associated with this transaction. Express your answer giving the interest rate \( i \) as a function of \( n \). Evaluate for \( n=6, 8, \text{ and } 12. \) (6 points)

   \[
   \text{Rule of } : \quad i = \frac{72}{n}
   \]

   $100 \text{ grows to } $800 \text{ in } 3n \text{ years.}
   
   In other words, money doubles every \( n \) years.

   \begin{align*}
   n=6 & \quad \Rightarrow \quad i = 12\% \\
   n=8 & \quad \Rightarrow \quad i = 9\% \\
   n=12 & \quad \Rightarrow \quad i = 6\%
   \end{align*}

   b. Now assume the transaction is based upon a nominal discount rate, compounded monthly. Express \( d^{(12)} \) as a function of \( n \). Evaluate for \( n=6, 8, \text{ and } 12. \) (4 points)

   \[
   \left[ 1 - \frac{1}{d^{(12)}^{12}} \right]^n = 2 \quad \Rightarrow \quad d^{(12)} = \left( 12 \right) \left( 1 - 2^{\frac{1}{12n}} \right)
   \]

   \begin{align*}
   n=6 & \quad \Rightarrow \quad d^{(12)} = 11.497\% \\
   n=8 & \quad \Rightarrow \quad d^{(12)} = 8.633\% \\
   n=12 & \quad \Rightarrow \quad d^{(12)} = 5.762\%
   \end{align*}

   c. Assuming the same nominal discount rate \( d^{(12)} \), what is the value today of $300 \ n/2 \text{ years from now, plus $300 \ n \text{ years from now, plus $300 \ 2n \text{ years from now?}} \) (4 points)

   \[
   \text{any } d^{(12)} \text{ we know } v^n = \frac{1}{2}
   \]

   \[
   \begin{align*}
   PV & = 300 \left( \sqrt{\frac{a}{2}} + v^n + v^{2n} \right) \\
   & = 300 \left( \sqrt{\frac{1}{2}} + \frac{1}{2} + \frac{1}{4} \right) \\
   & = 437.13
   \end{align*}
   \]
2. Find the accumulated value of $5,000 at the end of 7 years and 5 months, assuming a nominal interest rate $i^{(4)} = 8\%$, and further assuming simple interest through the final fractional period. Express your answer to the nearest penny. (7 points)

$$7 \text{ years } 5 \text{ months} = 89 \text{ months}$$
$$= 29 \text{ quarters } + \frac{2}{3} \text{ quarter}$$
$$i^{(4)} = 8\% \implies 2\% \text{ per quarter}$$

$$(5000)(0.02)^{29}(1+\left(\frac{2}{3}\right)(0.02)) = 8,997.61$$

↑

Compound interest over the "complete" quarters

↑

Simple interest over the final fractional quarter
3. $1000 is deposited into a fund on November 8, 2005 (hey, that’s today!), and will be withdrawn on January 12, 2006 (hey, that’s the first day of C term!). (a) How many actual days is this? (3 points)

Nov 8 to Nov 30 = 22 days
Dec 1 to Dec 31 = 31 days
Jan 1 to Jan 12 = 12 days
\[ \text{Total days} = 65 \]

Don’t count the first day but do count the last day.

(b) Ordinary simple interest (3 points)

\[ \text{DAYS} = 360(2005-2004) + 30(1-11) + 12-8 = 64 \]

\[ \text{ordinary simple interest} = (1000)(0.07)\left(\frac{64}{360}\right) = 12.44 \]

(c) Exact simple interest (3 points)

\[ \text{DAYS} = 65 \text{ (see above)} \]

\[ \text{exact simple interest} = (1000)(0.07)\left(\frac{65}{365}\right) = 12.47 \]

(d) The Banker’s Rule (3 points)

\[ \text{Banker’s Rule} = (1000)(0.07)\left(\frac{65}{360}\right) = 12.64 \]
4. You are given a table of "equations of value" (see next page). The expressions down the left are equivalent to the expressions across the top, at the effective interest rate shown in the intersecting row and column. Note: the cashflows in each row are not necessarily the same!

As an example of how to read the table, \(100v^2 + 200v^3 + 500v^5\) (third row) is equivalent to \(700(1+i)^3\) (third column) at an annual effective interest rate of 1.8952% (see rate circled in table).

**Use the table** to answer these questions. **Explain how you used the table.**

a. You are told that \(100\) at \(t=1\) plus \(200\) at \(t=3\) plus \(500\) at \(t=5\) is \(700\). What is the annual effective interest rate? *(6 points)*

\[
\text{Given: } 100v + 200v^3 + 500v^5 = 700 \text{ at } t=0
\]

Note that at \(t=3\):
\[
100(1+i)^2 + 200 + 500v^2 = 700(1+i)^3
\]

This equation of value corresponds to row 4, column 3, of the table; therefore
\[
i = 3.4244\%
\]

b. You are told that \(200\) at \(t=2\) plus \(500\) at \(t=4\) is \(600\). What is the annual effective interest rate? *(6 points)*

\[
\text{Given: } 200v^2 + 500v^4 = 600 \text{ at } t=0
\]

Note that:
\[
100 + 200v^2 + 500v^4 = 700 \text{ at } t=0
\]

Further note:
\[
100v + 200v^3 + 500v^5 = 700v \text{ at } t=-1
\]

This last equation of value corresponds to row 2, column 1 of the table; therefore
\[
i = 4.6244\%
\]
### Equations of value

<table>
<thead>
<tr>
<th>Expression</th>
<th>$700v$</th>
<th>$700(1+i)$</th>
<th>$700(1+i)^2$</th>
<th>$700(1+i)^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100v^3 + 200v^5 + 500v^7$</td>
<td>2.7216%</td>
<td>1.9313%</td>
<td>1.4973%</td>
<td>1.3461%</td>
</tr>
<tr>
<td>$100v + 200v^3 + 500v^5$</td>
<td>4.6244%</td>
<td>2.7216%</td>
<td>1.9313%</td>
<td>1.6867%</td>
</tr>
<tr>
<td>$100v^2 + 200v^3 + 500v^5$</td>
<td>4.4084%</td>
<td>2.6491%</td>
<td>1.8952%</td>
<td>1.6594%</td>
</tr>
<tr>
<td>$100(1+i)^2 + 200 + 500v^2$</td>
<td>NA</td>
<td>7.1699%</td>
<td>3.4244%</td>
<td>2.7216%</td>
</tr>
<tr>
<td>$100v^2 + 200v^3 + 500v^6$</td>
<td>3.6739%</td>
<td>2.3626%</td>
<td>1.7432%</td>
<td>1.5414%</td>
</tr>
<tr>
<td>$100(1+i)^4 + 200(1+i)^2 + 500$</td>
<td>-6.6737%</td>
<td>NA</td>
<td>7.1699%</td>
<td>4.6244%</td>
</tr>
</tbody>
</table>
At what annual effective interest rate is 150 received 1 year ago equivalent to 100 payable two years from now plus 100 received today? Use any method you like to do the calculation—but explain in words how you arrived at your answer. (8 points)

Many possible approaches:
"guess and check"
iterative interpolation
Newton's method
fancy calculator

\[ 150(1+i) = 100v^2 + 100 \]

all leading to

\[ i = 16.114\% \]

**** END OF QUIZ ****