Name: ________________________

MA 3211 - 2005 B Term  Theory of Interest
November 1, 2005  Quiz – Chapter 1

1. Write an expression for the current value at \( t=6 \) of payments received as follows: $800 received at \( t=2 \); $700 received at \( t=3 \); four payments of $200 each, received at \( t=6, t=8, t=10, \) and \( t=15 \).  

\[
@ \ t=6, \ PV = 800(1+i)^4 + 700(1+i)^3 + 200 \left(1 + i^2 + i^4 + i^9\right)
\]

2. You are given a choice between two investment funds:

**Fund A** has as its accumulation function:  
\[ a(t) = 1 - \frac{1}{25} t^2 + \frac{2}{5} t \]

**Fund B** credits a nominal rate of interest \( i^{(4)} \), convertible quarterly.

a. Determine \( i^{(4)} \) such that you will be equally happy with either fund, assuming you plan to invest your money for exactly three years.  

(5 points)

**Fund A:**  
\[ a(3) = 1 - \frac{9}{25} + \frac{6}{5} = \frac{46}{25} \]

**Fund B:**  
\[ a(3) = \left(1 + \frac{i^{(4)}}{4}\right)^{12} \]

Set the accumulation funds at \( t=3 \) equal to each other; solve for \( i^{(4)} \)

\[ i^{(4)} = 4 \left(\frac{46}{25}\right)^{\frac{1}{12}} - 4 = 0.2085 \approx 20.85\% \]
b. You have $10000 to invest for six years. Suppose that at time 0, you elect to put your money into either Fund A or Fund B. Then, at either time $t=1$, $t=2$, $t=3$, $t=4$, or $t=5$, you are allowed to move all of your money from one fund to the other, if you so desire. (You get at most one movement of money between funds, and this movement must occur at either $t=1$, $t=2$, $t=3$, $t=4$, or $t=5$. It's also OK to leave the money in one fund the entire time.) Describe how you would invest your money to achieve the highest possible rate of return on a six-year investment. Express that return as an annual effective rate. (6 points)

<table>
<thead>
<tr>
<th>$t$</th>
<th><strong>Fund A</strong> return</th>
<th><strong>Fund B</strong> return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.36 36.06%</td>
<td>22.54%</td>
</tr>
<tr>
<td>2</td>
<td>1.64 20.59%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.84 12.20%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.96 6.52%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.00 2.04%</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.96 -2.00%</td>
<td></td>
</tr>
</tbody>
</table>

\[
\left(1 + \frac{0.2085}{4}\right)^4 - 1
\]

**NOTE:** Fund A has a 36% return in the first year, 20.59% in the second year, and so forth. Fund B has a 22.54% return in every year.

Therefore, the best strategy is to put your money in Fund A for one year, then move it to Fund B at $t=1$ and leave it there until $t=6$.

\[
\text{Annual return} = \left[ (1.36)(1.2254)^5 \right]^\frac{1}{6} - 1 \approx 24.68\%
\]
3. It is known the present value of $10,000 payable one year from now plus $5,000 payable two years from now is $13,432. What is the sum of the present value of four payments of $2,500 each which will occur at the end of 1 year, 2 years, 3 years, and 4 years? 

\[ 10,000V + 5,000V^2 = 13,432 \Rightarrow 5000V^2 + 10000V - 13432 = 0 \]

\[ V = \frac{-10,000 \pm \sqrt{10,000^2 + (4)(5000)(13432)}}{10,000} \]

\[ \begin{align*}
V &= 0.92 \\
V &= -2.92 \text{ reject}
\end{align*} \]

So, \( 2500(v + v^2 + v^3 + v^4) = 8154 \)

4. Complete the following table:

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Present value at ( t=3 ) of $1,000 at ( t=8 )</th>
<th>Accumulated value at ( t=5 ) of $1000 at ( t=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% simple interest</td>
<td>( \Delta t=5, i=5% ) ( \frac{1000}{1+5(0.05)} = 800.00 )</td>
<td>( \Delta t=4, i=5% ) ( (1000)(1+4(0.05)) = 1200.00 )</td>
</tr>
<tr>
<td>8% simple discount</td>
<td>( \Delta t=5, d=8% ) ( 1000(1-5(0.08)) = 600.00 )</td>
<td>( \Delta t=4, d=8% ) ( \frac{1000}{1-4(0.08)} = 1470.59 )</td>
</tr>
<tr>
<td>6% nominal discount, compounded monthly</td>
<td>( \Delta t=60 \text{ months}, d=0.5% \text{ per month} ) ( 1000(1-0.005)^{60} = 740.26 )</td>
<td>( \Delta t=480 \text{ months}, d=0.5% \text{ per month} ) ( \frac{1000}{1-0.005^{48}} = 1272.01 )</td>
</tr>
</tbody>
</table>

Express your answers to the nearest penny. (6 points)
5. Calculate the nominal rate of discount convertible monthly that is equivalent to a nominal rate of interest of 18.9% per year convertible monthly (Question 19 from the May 2005 SOA FM exam; 5 points)

\[ \frac{i}{1+i} \Rightarrow \frac{d^{(m)}}{m} = \frac{i^{(m)}}{1 + \frac{i^{(m)}}{m}} \Rightarrow d^{(m)} = 1 + \frac{i^{(m)}}{m} \]

\[ d^{(12)} = \frac{1}{1 + \frac{i^{(12)}}{12}} = \frac{0.189}{1 + 0.189} = 0.186 \]

\[ d^{(12)} \approx 18.6\% \]

6. At a nominal rate of interest of \( i \) convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year. Calculate \( i \).

(Question 13 from the May 2005 SOA FM exam; 5 points)

\[ 1000 + 1500v^2 = 2600v^2 \]
\[ 2600v^2 - 1500v - 1000 = 0 \]

\[ v = \frac{1500 \pm \sqrt{1500^2 + 4(2600)(1000)}}{5200} \]

\[ 0.97244 \ (OK) \]
\[ 0.3956 \ (reject) \]

\[ \Delta \left( \text{effective} \right) = \frac{1}{v} - 1 = 2.8342\% \]

\[ 1 + \left( \text{effective} \right) = (1 + \frac{i^{(\text{nominal})}}{2})^2 \]

\[ i^{(\text{nominal})} = 2.814\% \]

**** END OF QUIZ ****