Lectures 10 and 11

Exercises Inclusion and Exclusion

1. A small town has 500 men in its population, and 500 college graduates, and 100 people who are both men and college graduates. How many adults are either male or college graduates?

2. The license plates of Corruptica consist of 6 characters satisfying any of the following conditions:
   - c1: 3 capital letters followed by 3 digits.
   - c2: a string of capital letters or digits starting with ‘CP’.
   - c3: a string of capital letters.
   How many legal licence plates are possible in Corruptia.

3. A passport number in Kleptonia is any string of 12 digits satisfying one of the following conditions:
   - k1: The number starts with 111 and ends in 000.
   - k2: The number is evenly divisible by 5.
   - k3: The number or ends in 404 ends in 404.
   How many legal licence plates are possible in Kleptonia.

4. A prison id Putinella is any string of 5 letters satisfying one of the following conditions:
   - p1: The word starts with ‘kgb’.
   - p2: The word ends in three identical characters.
   - p3: The word contains no vowels (\{a, e, i, o, u\}).
   What is the maximum number of prisoners in Putinella.

5. The website www.weharvetidsandpasswordsandsecurityquestions.ru provides registered users free software downloads and irregular email entertainment. The site assigns each user an identification tag consisting of 6 case sensitive letters satisfying all of the following conditions:
   - x1: The string starts with A.
   - x2: The string is all caps.
   - x3: The string reads the same backwards and forwards.
   - x4: The string is a pair of distinct characters repeated three times.
   How many tags are there?
6. The website www.weharvetidsandpasswordsandsecurityquestionstoo.ru provides registered users free software and game downloads and constant email entertainment. The site assigns each user an identification tag consisting of 6 case sensitive letters satisfying any of the following conditions:

y1: The string starts with A.
y2: The string is all caps.
y3: The string reads the same backwards and forwards.
y4: The string is a pair of distinct characters repeated three times.

How many tags are there?

Exercises on functions

1. List all the one-to-one functions with domain \( A = \{a, b, c\} \) and target \( B = \{1, 2, 3, 4, 5\} \).
2. List all the onto functions with domain \( A = \{1, 2, 3, 4\} \) and target \( B = \{a, b, c\} \).
3. Let \( A, B \) and \( C \) be disjoint sets with \( |A| = 15, |B| = 11, \) and \( |C| = 11 \).
   a) How many functions \( A \rightarrow B \) are there?
   b) How many functions \( B \rightarrow C \) are there?
   c) How many onto functions \( B \rightarrow C \) are there?
   d) How many one-to-one functions \( B \rightarrow C \) are there?
   e) How many one-to-one functions \( A \cup B \rightarrow C \) are there?
   f) How many one-to-one functions \( B \rightarrow A \cup C \) are there?

Exercises on the Binomial Theorem

For each of the following compute the sum:

1. \( \sum_{k=0}^{25} \binom{25}{k} 3^k \)
2. \( \sum_{k=1}^{10} \binom{11}{k} 9^k \)
3. \( \sum_{k=0}^{5} \binom{5}{5-k} 2^{5-k} 3^k \)
4. \( \sum_{k=0}^{10} \binom{10}{k} (-1)^k 2^{2k} 3^{5-k} \)