Euclidean Algorithm

We studied the Euclidean Algorithm to
1. Compute gcd($n, m$)
2. Find $\lambda$ and $\mu$ so that $\lambda n + \mu m = \text{gcd}(n, m)$.

Exercises on the Euclidean Algorithm

1. Show that $\text{gcd}(n, m) = \text{gcd}(n + m, m)$.
2. Show that $\text{gcd}(n, m) = \text{gcd}(n - m, m)$.
3. Give an example to show that $\text{gcd}(n, m) = \text{gcd}(n + m, n - m)$ need not be true.
4. Suppose that $n$ is even and $\text{gcd}(n, m) = 5$. Show that $m$ is odd.
5. Find all numbers $k$ with $0 \leq k \leq 100$ such that $\text{gcd}(100, k) = 5$.
6. Find all numbers $k$ with $0 \leq k \leq 100$ such that $\text{gcd}(100, k) = 4$.
7. How many pairs of numbers $(n, m)$, with $0 \leq n, m \leq 100$ satisfy $\text{gcd}(n, m) = 5$.
8. How many pairs of numbers $(n, m)$, with $0 \leq n, m \leq 100$ satisfy $\text{gcd}(n, m) = 8$.
9. For each of the following pairs, apply the Euclidean Algorithm to compute the greatest common divisor. Then find $\lambda$ and $\mu$ so that $\lambda n + \mu m = \text{gcd}(n, m)$.
   a) $n = 24$, $m = 15$.
   b) $n = 60$, $m = 25$.
   c) $n = 144$, $m = 100$.
   d) $n = 162$, $m = 225$.
   e) $n = 101$, $m = 103$.
   f) $n = 101$, $m = 107$.
   g) $n = 1776$, $m = 2015$.
   h) $n = 1011$, $m = 1101$.
   i) $n = 1000$, $m = 888$.
   f) $n = 332211$, $m = 112233$. 