1. (6 pts) The plane \( \mathcal{P} \) contains the points \((0, 3, 2), (2, 3, 1), \text{ and } (6, 1, 0)\).
   
a) Find a vector equation for \( \mathcal{P} \).
   
   To find a normal vector, take the cross product of the displacement vectors \((2, 3, 1) - (0, 3, 2) = (2, 0, -1)\) and \((6, 1, 0) - (0, 3, 2) = (6, -2, -2)\) giving
   
   \[
   \begin{vmatrix}
   i & j & k \\
   2 & 0 & -1 \\
   6 & -2 & -2 \\
   \end{vmatrix} = (-2, -2, -4)
   \]
   
   which is a scalar multiple of \((1, 1, 2)\). Taking the dot product of \((1, 1, 2)\) with any of the given vectors gives 7, so the equation of the plane is
   
   \[ \langle x, y, z \rangle \cdot \langle 1, 1, 2 \rangle = 7. \]

   b) What is the distance of \( \mathcal{P} \) from the origin.
   
   The easiest way is to scale the equation of the plane so that the normal vector is a unit vector. \(|(1, 1, 2)| = \sqrt{6}\), so \(\langle x, y, z \rangle \cdot (1/\sqrt{6}, 1/\sqrt{6}, 2/\sqrt{6}) = 7/\sqrt{6}\) is the equation of the plane and \(7/\sqrt{6}\) is the distance to the plane.

   c) Find the \( x \), \( y \) and \( z \) intercepts of \( \mathcal{P} \).

   We can normalize. Or you can simply look for them. The \( x \) intercept is of the form \((a, 0, 0)\), which is on the plane if \(1a = 7\), or \(a = 1/7\).

   The \( y \) intercept is of the form \((0, b, 0)\), which is on the plane if \(1b = 7\), or \(b = 1/7\).

   The \( y \) intercept is of the form \((0, 0, c)\), which is on the plane if \(2c = 7\), or \(b = 2/7\).

   So they are \((1/7, 0, 0)\), \((0, 1/7, 0)\), and \((0, 0, 2/7)\).

2. (4 pts) Let \( A \), \( B \), and \( C \), be three 3-dimensional vectors all of whose magnitudes are 2 and such that \( A \cdot B = A \cdot C = B \cdot C = 2 \).

   Circle each of the following statements which are true.

   a) \( C = A \times B \)

   False: \( A \times B \) is perpendicular to \( A \) and \( B \), so \((A \times B) \cdot A = (A \times B) \cdot B = 0\), but \( C \) has dot product 2 with \( A \) and \( B \).

   b) The solutions of \((A \times \langle x, y, z \rangle) + (B \times C) = 1\) form a plane.

   False: There are no solutions since the sum of two vectors cannot be a scalar.

   c) The solutions of \((A + \langle x, y, z \rangle) \cdot (B \times C) = 0\) form a plane.

   True: By the distributive law, the equation is equivalent to

   \[ (B \times C) \cdot \langle x, y, z \rangle = -A \cdot (B \times C) \]

   so the normal vector is \( B \times C \) and the distance to the origin is

   \[ \frac{|A \cdot (B \times C)|}{|B \times C|} \]

   d) \((A + B + C) \cdot (A + B + C) = 24\).

   True: Since the magnitudes are all two, We have \( A \cdot A = B \cdot B = C \cdot C = 4 \), and using the distributive law \((A + B + C) \cdot (A + B + C) = 4 + 2 + 2 + 2 + 4 + 2 + 2 + 2 + 4 = 24\).