1. (4 pts) Give a regular grammar for the language \( L \subseteq \{a,b\}^* \) in which the substring \( bbb \) occurs exactly once.

We interpret the variables as follows:
- \( S \) – “The prefix has no \( bbb \) and doesn’t end in a \( b \).”
- \( A \) – “The prefix has no \( bbb \) and ends in \( b \).”
- \( B \) – “The prefix has no \( bbb \) and ends in \( b^2 \).”
- \( C \) – “The prefix has \( bbb \) and cannot add another \( b \).”
- \( D \) – “The prefix has \( bbb \) and can add another \( b \).”

\[
G : S \rightarrow aS | bA \\
A \rightarrow aS | bB \\
B \rightarrow aS | bC \\
C \rightarrow aD | \lambda \\
D \rightarrow aD | bC | \lambda
\]

2. (4 pts) Let \( G \) be the grammar

\[
G : S \rightarrow AB \\
A \rightarrow a^5A | a^4 | a^3 | a^2 | a \\
B \rightarrow Bb^3 | b^2b
\]

a) Provide a derivation tree for any element in the language of length 15.

\[
\begin{array}{c}
S \\
| \\
A \\
| \\
| \\
A \\
| \\
A \\
| \\
A \\
| \\
(a^5)
\end{array}
\]

There are lots of ways to do this.

b) Express \( L(G) \) in set notation.

Again, lots of ways of doing it, for example:

\[
\{a^{5i+j}b^{3k+l} | i, k \in \mathbb{Z}, 1 \leq j \leq 4, 1 \leq l \leq 2\}
\]

c) Give a regular expression for \( L(G) \).

\[
(a^5)^*(a \cup a^2 \cup a^3 \cup a^4)(b \cup b^2)(b^3)^*.
\]