1. (4 pts) Give a regular expression for the language $L \subseteq \{a, b\}^*$ in which the substring $bbb$ occurs exactly once.

$bbb$ occurs once.

After the $bbb$ the string is either empty, or starts with an $a$ and has at most two $b$'s in a row. So $(a \cup ab \cup ab^2)^*$.

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So the regular expression is $(a \cup ba \cup b^2a)^*(bbb)(a \cup ab \cup ab^2)^*$

2. (4 pts) Give a regular expression for the language $M \subseteq \{a, b, c\}^*$ consisting of all strings whose length is divisible by 3.

There are lots of ways to do this. $(a \cup b \cup c)^3$ is all strings of length exactly 3. So $M$ is given by the regular expression $((a \cup b \cup c)^3)^*$.

3. (2 pts) Give a regular expression for the language $L \cup M$, where $L$ and $M$ are described in the problems above.

The union, $\cup$, of languages is very easy in regular expressions.

$L \cup M$ is expressed by $((a \cup ba \cup b^2a)^*(bbb)(a \cup ab \cup ab^2)^*) \cup ((a \cup b \cup c)^3)^*$