1. Show that the language $L$ consisting of strings of the form $a^n b^m$ with $n < m$ is not regular by the Pumping Lemma.

If $L$ is regular then it is recognized by an Automaton with $K$ states.

So, by the Pumping Lemma, any $x \in L$ with length($w$) $> K$ can be factored as $uvw$ with length($uv$) $\leq K$, length($v$) $> 0$, and $uv^i w \in L$ for all $i \geq 1$.

Take $x = a^{K+1}b^{K+2} \in L$. $x$ must factor as $x = uvw$, and since length($uv$) $\leq K$ we have $v = a^j$ for some $j \geq 1$.

$$x = [a^i][a^j][a^{K+1-i-j}b^{K+2}]$$

and we can “pump” $a^j$.

So $y = [a^i][a^j]^2[a^{K+1-i-j}b^{K+2}] = [a^i][a^{2j}][a^{K+1-i-j}b^{K+2}]$ should be in the language, but the power of $a$ is now too big, $i + 2j + K + 1 - i - j = K + 1 + j \geq K + 2$ since $j \geq 1$, a contradiction.

So $L$ is not regular.