1. Give a regular expression for the strings on $\Sigma = \{a, b, c\}$ such that every $b$ is immediately followed by at least one $c$ or at least two $a$’s.

There are many ways to analysis this. Here is a simple one:

$$(a \cup bc \cup baa \cup c)^*$$

2. For the following Context Free Grammar, use set notation to describe the language generated.

$$S \rightarrow aSb | A$$

$$A \rightarrow cAd | CBd$$

$$B \rightarrow aBd | ab$$

From the rules on $B$ we have $B \xrightarrow{n} a^n b^n$, with $n \geq 1$.

From the rules on $A$ we have $A \xrightarrow{m} a^m Be^m$, with $m \geq 1$.

From the rules on $S$ we have $S \xrightarrow{s+1} a^s Ab^s$, with $s \geq 0$.

So

$$S \xrightarrow{s+1} a^s Ab^s \xrightarrow{m} a^s a^m Be^m b^s \xrightarrow{n} a^s a^m a^n b^n c^m b^n$$

In set notation we write

$$L = \{a^s a^m a^n b^n c^m b^n | n \geq 1, m \geq 1, s \geq 0\}$$

Since we have produced a derivation sequence, we have proved $L \subseteq L(G)$, which was not asked.

We might as well prove $L(G) \subseteq L$.

What we will show is the follow. The sentential forms $L(G)$ are of one of the following forms:

- $a^s S b^s$ with $s \geq 0$,
- $a^s a^m A c^m b^s$ with $s \geq 0$ and $m \geq 0$,
- $a^s a^m A d^m c^m b^s$ with $s \geq 0$, $m \geq 1$ and $n \geq 0$,
- or
- $a^s a^m a^n d^n e^n c^m b^s$ with $s \geq 0$, $m \geq 1$ and $n \geq 0$

To show this, first note $S$ is one of these forms. Then observe that this set is closed under the operations of the rules.

Since the only language elements are the last form, all elements of the language $L(G)$ are in that form and so $L(G) \subseteq L$. 

1 of 2