Exercises for Lectures 7 and 8

Cardinality of Finite Sets:

Lectures 7 and 8 we discussed functions on sets, the cardinality of infinite sets and introduced formal logic.

If $X$ and $Y$ are finite sets, the number of functions from $X$ to $Y$, $f : X \rightarrow Y$ is $|Y|^{|X|}$. The number of one to one functions is $\frac{|Y|!}{(|Y| - |X|)!}$.

Two sets have the same cardinality if there is a one-to-one and onto function between them.

A set with the same cardinality as $\mathbb{N}$ is said to be countable.

We showed $\mathcal{P}(\mathbb{N})$ is uncountable.

We considered several examples. In general, countable unions and finite products of countable sets are countable. An infinite product of a finite sets is uncountable.

Formal logic concerns statements. A statement is either TRUE (1) or FALSE (0).

Statements can be formed from $\land$ (AND), $\lor$ (OR) and $\neg$ (NOT).

We stated the distributive laws:

$p \lor (q \land r) = (p \lor q) \land (p \lor r)$  \quad p \land (q \lor r) = (p \land q) \lor (p \land r)$

and Demorgan’s laws:

$\neg(p \lor q) = (\neg p) \land (\neg q)$  \quad $\neg(p \land q) = (\neg p) \lor (\neg q)$

1. find a one-to-one and onto function from $\mathcal{P}_2(\{1, 2, 3, 4, 5\})$ to $\mathcal{P}_3(\{1, 2, 3, 4, 5, 6\})$, or show that one does not exist.

2. find an onto function from $\mathcal{P}_2(\{1, 2, 3, 4, 5, 6\})$ to $\mathcal{P}(\{2, 4, 6\})$, or show that one does not exist.

3. find a one-to-one and onto function from $\mathcal{P}_2(\{1, 2, 3, 4\}) \cup \mathcal{P}_2(\{6, 7, 8, 9, 10\})$ to $\mathcal{P}(\{1, 2, 3, 4\})$, or show that one does not exist.

4. find a one-to-one and onto function from $\mathcal{P}_2(\{1, 2, 3, 4\}) \times \mathcal{P}_2(\{6, 7, 8, 9, 10\})$ to $\mathcal{P}(\{1, 2, 3, 4\})$, or show that one does not exist.

5. Is $\mathcal{P}_2(\mathbb{N})$ countable. Why or why not?

6. Is $\mathcal{P}_3(\mathbb{N})$ countable. Why or why not?

7. Is $\mathcal{P}_3(\mathbb{Q})$ countable. Why or why not?

8. Is $\mathcal{P}_3(\mathbb{Q} \times \mathbb{Q})$ countable. Why or why not?

9. Is the set of finite subsets of $\mathbb{N}$ countable? Why or why not?

10. Is the set of finite subsets of $\mathcal{P}(\mathbb{N})$ countable? Why or why not?
11. Suppose $p$ is TRUE and $q$ is FALSE, and $r$ is a statement. Label each of
the following as true or false, or undecidable:

- $(p \land q \land r)$
- $(p \lor q \lor r)$
- $p \land \neg (q \lor \neg q)$.
- $p \land (p \lor q) \land (p \lor q \lor r)$.
- $p \lor (p \land q) \lor (p \land q \land r)$.
- $p \lor \neg ((p \land q) \lor \neg (p \land q \land r))$.

12. Suppose $p \land (q \lor (p \land q))$ is TRUE. What can you conclude about the truth
of $p$ and $q$?