1. Example: A purely combinatorial Reciprocal

Here is what I think is a rather interesting example. There are two ingredients. The first ingredient is a framework in the plane and a specific resolvable self-stress. Figure 1 depicts the graph of a cube with diagonals,

in other words the complete bipartite graph $K_{4,4}$. The vertices are embedded in the plane at points of the integer grid. It has $|V| = 8$ and $|E| = 16$ so it is not a rigidity cycle. It is generically rigid, however the vertex embedding is singular so there may be an infinitesimal motion. The stresses on all the edges are $\pm 1$. In general the framework may have edge crossings. Since this is not a planar graph, we must must have edge crossings.

The second ingredient is the embedding of the graph of the framework into some surface. Figure 2 depicts the cube with diagonals embedded on the torus.

This embedding is to determine the the vertices of the reciprocal, and so is a true embedding - no edge crossings.

The dual graph on the torus also has eight vertices and 16 edges. In fact, we have chosen a self-dual embedding on the torus, however, that is not essential to the development.

We may now construct a reciprocal framework based on the toroidal dual of $K_{4,4}$. Figure 3 also possesses a resolvable stress whose values are the reciprocals
Figure 3. $K_{4,4}$ has a self-dual embedding in $T^2$.

Figure 4. The reciprocal of non-planar framework.

of the original stress. However, this cannot be called the reciprocal figure since it depends on the surface in which the graph is embedded.

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