Hybrid computational and experimental approach for the study and optimization of mechanical components

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Abstract. Increased demands on the performance and efficiency of mechanical components impose challenges on their engineering design and optimization, especially when new and more demanding applications must be developed in relatively short periods of time while satisfying design objectives, as well as cost and manufacturability. In addition, reliability and durability must be taken into consideration. As a consequence, effective quantitative methodologies, computational and experimental, should be applied in the study and optimization of mechanical components. Computational investigations enable parametric studies and the determination of critical engineering design conditions, while experimental investigations, especially those using optical techniques, provide qualitative and quantitative information on the actual response of the structure of interest to the applied load and boundary conditions. We discuss a hybrid experimental and computational approach for investigation and optimization of mechanical components. The approach is based on analytical, computational, and experimental solutions (ACES) methodologies in the form of computational, noninvasive optical techniques, and fringe prediction (FP) analysis tools. Practical application of the hybrid approach is illustrated with representative examples that demonstrate the viability of the approach as an effective engineering tool for analysis and optimization.© 1998 Society of Photo-Optical Instrumentation Engineers. [S0091-3286(98)00405-X]

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1 Introduction
Computational methodologies have recently reached levels at which a mechanical component can be completely dimensioned and analyzed.1,2 With these methodologies, parametric studies of the effects of different boundary conditions, loadings, and material properties can be investigated. In addition, structural optimization can be performed through a computationally intensive iterative procedure using methods such as the finite or boundary element combined with mathematical programming techniques.3 Computational solutions depend on appropriate selection of modeling parameters, such as material constants, boundary conditions, and loads. Therefore, solutions may lack any significance if modeling parameters are not selected properly.

Experimental noninvasive optical measuring methodologies, such as electro-optic holography (EOH), are available to perform studies of mechanical components. EOH can provide qualitative information in real time through interferograms and quantitative information can be obtained by processing the interferograms.4 Quantitative information includes displacements and or deformations obtained after the object of interest has been subjected to specific boundary and loading conditions. In addition, it is possible to perform geometrical measurements of mechanical components using optical contouring. Results obtained with noninvasive optical techniques are valuable information since they are a measure of the behavior of the actual object of interest subjected to realistic operating conditions.

An effective study and structural optimization of mechanical components should include the use of both computational and experimental methodologies. Computational investigations enable parametric studies and the determination of critical engineering design conditions, while experimental investigations, especially those using optical techniques, provide quantitative information on the actual response of the structure of interest to the applied load conditions.5

2 Hybrid Computational-Experimental Procedure

2.1 Computational Investigations
Solutions of the partial differential equations (PDE) used for modeling different physical phenomena either cannot always be found using analytical considerations, do not exist, or are limited to specific domains. For practical purposes, however, special techniques capable of solving these PDE have been developed for a wide range of different domains and boundary conditions. These techniques comprise the computational methodologies. Computational
methodologies are computer-aided mathematical techniques for obtaining approximate numerical solutions to the PDE that predict the response of physical systems subjected to external influences. Their applications extend to many areas that include solid mechanics, heat transfer, fluid mechanics, acoustics, and electromagnetism, as well as a coupled interaction of these phenomena. Three computational methodologies have been used extensively for solving the PDE generated in many engineering and science fields. These include the finite difference (FD), finite element (FE), and boundary element (BE) methods. The fundamental principle of these methodologies is based on the reduction of governing PDE to an approximation in terms of algebraic equations. This reduction replaces continuous PDE, whose solution space is generally infinite dimensional, with a finite set of algebraic equations whose solution space is finite dimensional. In general, the reduction of continuous PDE into a set of discrete algebraic equations is performed by identifying a finite number of discrete points within the domain of interest. These points are called nodes and it is at these locations that approximations to the true solution are computed. For instance, consider the FE method in which the domain of interest is fully discretized into finite elements (Fig. 1). In the case of structural problems and a displacement formulation, the deformation field on each element is written in terms of known interpolating functions and unknown nodal displacements. By considering element \( e \) in the domain \( D \) of Fig. 1, coordinates \((x,y)\) and displacements \((u,v)\) within the element can be written, using 2-D isoparametric elements, as

\[
x = \sum_{i=1}^{NN} N_i x_i, \quad y = \sum_{i=1}^{NN} N_i y_i,
\]

and

\[
u = \sum_{i=1}^{NN} N_i u_i, \quad v = \sum_{i=1}^{NN} N_i v_i,
\]

where \( NN \) is the number of nodes in the element; \((x_i,y_i)\) and \((u_i,v_i)\) are the nodal coordinates and unknown nodal displacements, respectively; and \( N_i \) represents the interpolating or shape functions, which for a linear approximation can be written as

\[N_i = \frac{1}{4} (1 + \xi_i)(1 + \eta_i),\]

where \( \xi \) and \( \eta \) are the natural coordinates \( \in \{[-1,1], [-1,1]\} \), and \( \xi_i \) and \( \eta_i \) are the nodal coordinates in the unit parametric space. Therefore, the response of the entire domain is obtained by solving a system of equations that takes into consideration the effects of all the elements and has as unknowns the nodal, or discrete, displacements instead of a continuous displacement function.

### 2.2 Experimental Investigations: EOH

EOH has been successfully applied to different fields of nondestructive testing of mechanical components subjected to static and dynamic loading conditions. Being non-invasive and providing qualitative and quantitative full-field information are some of the main advantages of EOH over other experimental techniques. In addition, it requires much less mechanical stability than that required in conventional holographic interferometry, which makes it very suitable for on-site investigations. EOH is based on a combined use of speckle interferometry, phase stepping, and image acquisition and processing techniques. Figure 2 depicts a typical experimental setup for generation of interferograms using the EOH technique. In this system, the laser output is divided into two separate beams by means of a beamsplitter (BS). One of the beams is directed toward a beamspander (BE) to illuminate the object (OB) uniformly. This beam, modulated by interaction with an object is transmitted by the object imaging lens (IL) to the object input of the speckle interferometer (SI). The other beam is directed toward a microscope objective (MO) and then to a single-mode fiber optic cable (FO), which is connected to the reference input of the SI. The object and reference beams are combined at the SI, which is capable of projecting the combined signal onto the charge-coupled device (CCD) chip of the camera. The signal generated at the CCD camera, due to the interaction of object and reference beams, is directed to a pipeline image processor computer.
IP processing images at video rates. This computer is also controlling phase shifter PSI for introduction of phase steps between consecutive captured frames.

2.2.1 Static investigations

In EOH, information is extracted from the interference pattern of object and reference beams having complex light fields $F_o$ and $F_r$. After the BS, and considering phase stepping, irradiance $I_n$ of the combined wavefronts, as recorded by the $n$th video frame, can be described by

$$I_n = (F_o + F_r)(F_o + F_r)^*$$

$$= |A_o|^2 + |A_r|^2 + 2A_oA_r \cos [(\phi_o - \phi_r) + \Delta \theta_n],$$  \hspace{1cm} (4)

where $A_o$ and $A_r$ are the amplitudes of the object and reference beams, respectively; $\phi_o$ is the phase of the object beam; and $\phi_r$ is the phase of the reference beam, and $\Delta \theta$ is the known phase step introduced between the frames.\textsuperscript{4,10} To facilitate static investigations, the argument of the periodic term of Eq. (4) is modified to include the phase change due to static deformations of the object of interest subjected to specific loading and boundary conditions. This phase change is characterized by the fringe-locus function $\Omega$, whose constant values define fringe loci on the surface of the object as

$$\Omega(x,y) = 2\pi n(x,y) = K(x,y) \cdot L(x,y),$$  \hspace{1cm} (5)

where $n(x,y)$ is the interferometric fringe order at the $(x,y)$ location in the image space, $K$ is the sensitivity vector, and $L$ is the displacement vector. Therefore, the irradiance from a deformed object can be described as

$$I_n = I_o + I_r + 2A_oA_r \cos (\Delta \phi + \Omega + \Delta \theta_n),$$  \hspace{1cm} (6)

where $\Delta \phi = \phi_o - \phi_r$; and $I_o$ and $I_r$ represent the intensities of the object and reference beams (i.e., amplitudes squared), respectively. In Eq. (6), $I_o$ is assumed to remain constant, and the $(x,y)$ arguments were omitted for clarity. Since it is $\Omega$ that carries information pertaining to mechanical displacements and/or deformations, the EOH’s video frame processing algorithm eliminates $\Delta \phi$ from the argument of the periodic function of the irradiance distributions given in Eqs. (4) and (6), yielding an image that has intensity modulated by a periodic function with $\Omega$ as the argument.\textsuperscript{4}

The EOH can work in display and data mode. In display mode, interference patterns are observed at video rate speed and are modulated by a sinusoidal function of the form

$$8A_oA_r \cos \left( \frac{\Omega}{2} \right),$$  \hspace{1cm} (7)

which is obtained by performing specific algebraic operations between frames acquired at the undeformed and deformed states, described by Eqs. (4) and (6), respectively. This mode is used to adjust the EOH system and for qualitative investigations. Data mode is used to perform quantitative investigations. In the data mode, two images are generated: a sinusoidal image,

\begin{equation}
D = 64A_o^2A_r^2 \cos (\Omega),
\end{equation}

and a sinusoidal image,

\begin{equation}
N = 64A_o^2A_r^2 \sin (\Omega),
\end{equation}

which are processed simultaneously to produce quantitative results by computing

$$\Omega = \tan^{-1} \left( \frac{N}{D} \right).$$  \hspace{1cm} (10)

Figure 3(a) depicts a typical interferogram obtained with the EOH system functioning in static display mode.

2.2.2 Dynamic investigations

To perform dynamic investigations, or modal analysis, using the EOH, it is necessary to take into consideration a time-varying fringe-locus function $\Omega(x,y,t)$, which is related to periodic motion of the object under investigation.\textsuperscript{4,5} Using Eq. (6), it is possible to write

$$I_t(x,y,t) = I_o(x,y) + I_r(x,y) + 2A_o(x,y)A_r(x,y)$$

$$\times \cos \left[ \Delta \phi(x,y) + \Omega(x,y,t) \right].$$  \hspace{1cm} (11)

Since the CCD camera registers average intensity at the video rate characterized by the period $\Delta t$, which, in the EOH system used in this study is equal to 1/30 s, the intensity that is observed can be expressed mathematically as

$$I(x,y) = \frac{I}{\Delta t} \int_I^{I+\Delta t} I_t(x,y,t) \, dt,$$  \hspace{1cm} (12)

and, using phase stepping, the resultant intensity distribution for the $n$th frame can be written as

$$I_n(x,y) = I_o(x,y) + I_r(x,y) + 2A_o(x,y)A_r(x,y)$$

$$\times \cos \left[ \Delta \phi(x,y) + \Delta \theta_n \right] M[\Omega(x,y)],$$  \hspace{1cm} (13)

where $M[\Omega(x,y)]$ is known as the characteristic function determined by the temporal motion of the object. For the case of sinusoidal vibrations with a period much shorter than the video framing time,
where \( J_0[\Omega(x,y)] \) is the zero-order Bessel function of the first kind. Equation (13) contains four unknowns: irradiances \( I_x \) and \( I_y \), phase difference \( \Delta \phi \), and the fringe-locus function \( \Omega(x,y) \). The EOH’s video frame processing algorithm eliminates \( \Delta \phi \) from the argument of the irradiance function given by Eq. (13) using a set of four equations obtained at a specific step size value of \( \Delta \theta \).

For time-averaged investigations, the EOH can work in display and data modes. In the display mode, interference patterns are observed at video rate and are modulated by a function of the form

\[
4A_xA_y[M\Omega(x,y)].
\]

This mode is used to adjust the EOH system and for qualitative investigations. Figure 3(b) shows a typical interferogram of a sinusoidally excited plate, as recorded by the EOH system functioning in the time-averaged display mode. The data mode\(^4,5,11\) is used to perform quantitative investigations. In the data mode, images of the form

\[
16I_xI_y[M^2\Omega(x,y)],
\]

are generated for quantitative processing and extraction of \( \Omega(x,y) \).

### 2.2.3 Shape measurements using EOH

With the EOH, it is possible to perform noninvasive static and dynamic investigations of mechanical components subjected to a large variety of loading conditions.\(^4\) In addition, it is also possible to measure the shape of mechanical components using optical contouring.\(^12\) The combination of these capabilities makes the EOH a powerful engineering tool that can be utilized to study and optimize mechanical components. Recent technological advances in computer and fiber optic technologies can be applied to the EOH system. These advances dramatically increase the versatility of the EOH method and add the possibility of using it in on-site investigations to study and diagnose problems in industrial environments.

Figure 4 depicts a currently operational EOH system used for static, dynamic, and shape measurement investigations. This system is based on a single illumination and single observation directions. The light source is a low-power laser diode (LD) with thermoelectric cooling capabilities driven by the controller (LDD), which provides the LD with adequate current to generate light emission and temperature control. The output of the LD is directed through a Faraday optical isolator (OI), which reduces the magnitude of back reflected light into the LD, to a microscope objective (MO) coupling it into a single-mode fiber optic directional coupler (DC). This directional coupler splits the light power into a 1:9 ratio. The higher power beam is used to illuminate the object of interest and the lower power beam is used as a reference beam. Both beams are recombined in the interferometer (SI) and the acquired irradiances are transmitted to an image processing computer (IP) using a CCD camera. PZT1 is a piezoelectric fiber optic modulator for phase stepping, and PZT2 is similar to PZT1 but is used to introduce a bias signal for quantitative analysis of time-averaged interferograms.\(^5\) This setup occupies a minimum of space and can be readily rearranged to obtain different experimental configurations.

The EOH system shown in Fig. 4 uses the two-wavelength technique to generate depth contours related to the geometry of the object under investigation. The wavelength of the LD used can be controlled either by modulating its injection current or by modifying the temperature of the thermally controlled stage of the diode. Contouring can be performed by using the EOH in static mode. This is done by acquiring a set of interferograms using wavelength \( \lambda_1 \), representing a reference state, or undeformed state, and then acquiring the second set of interferograms after the wavelength has been changed to \( \lambda_2 \), representing a modified state, or deformed state. The phase change related to depth contours \( \gamma(x,y) \) obtained after performing this operation is now equivalent to the fringe-locus function \( \Omega(x,y) \) in static deformations, so Eqs. (6) to (10) hold for \( \gamma(x,y) \). Figure 5 depicts some typical results obtained after contouring a 3-D mechanical component: a jet engine turbine blade. For this particular application, collimated retroreflective conditions were utilized to obtain the fringe separation of

\[
h = \frac{1}{2} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}.
\]

### 2.3 Fringe Prediction

Fringe prediction (FP) is an important part of the hybrid, experimental and computational, approach presented in this paper. FP is used to interact between information provided by computational and EOH investigations. This interaction can provide valuable information that can be utilized to increase the degree of correlation between the computational and the experimental results.

FP can be used to specify the optimum EOH configuration for a specific application, before making the actual measurements. This is of particular importance, especially for complicated mechanical components, since the magni-
tude and location of the experimentally applied loads is not easy to determine. The FP objective, in this case, is to specify loading conditions such that at areas of interest on the object, phase change can be measured accurately and reliably. This means that phase changes must be large with respect to the sensitivity of the EOH system and small enough to prevent speckle decorrelation from reducing phase measurements quality, since there is the requirement of a minimum number of pixels per fringe that are necessary for successful phase unwrapping. Figures 6 to 8 show representative applications of the FP.

Figure 6 shows a 15×15-mm, 110-μm-thick AL-2024 plate constrained along all four edges vibrating at its first fundamental mode. This figure depicts overloaded [Fig. 6(a)] and appropriately loaded [Fig. 6(b)] condition levels for quantitative EOH investigations.

FP can also be utilized to simulate different experimental arrangements characterized by the sensitivity vector K [see Eq. (5)], and, in this way, can lead to optimizing the setup to facilitate specific applications of the EOH. In addition, FP is useful for effective comparison of experimental and computational results because it takes into consideration the actual experimental illumination/observation conditions while interpreting (displaying) computational data. Figure 7 shows typical interferograms recorded using two different experimental setups, each characterized by a different sensitivity vector, of the same component subjected to similar loading conditions (for each recording). These interferograms show substantially different fringe patterns (for each setup), the interpretation of which, without taking into consideration of the specific recording geometry, will lead to erroneous results, and clearly demonstrate the importance of accurate knowledge of the illumination/observation conditions when performing comparisons between experimental and computational results.

FP can be performed by

1. theoretical/computational modeling using theoretically/experimentally obtained boundary conditions and/or loads, theoretically/experimentally obtained geometry, and experimentally obtained material properties
2. use of computational nodal information at the object boundary
3. orientation of theoretical/computational geometry to match experimental conditions
4. evaluation of the fringe-locus function Ω, or evaluation of the time varying fringe-locus function Ω,
5. use of the cosine or J₀ irradiance distributions while considering optical configuration of the specific EOH setup in which characteristics of optical components affecting laser beam profile (e.g., Gaussian) are taken into consideration.

![Fig. 5 Contouring the tip of a turbine blade with Δλ<0.1 nm: (a) object as used in the experimental setup, (b) wrapped phase in the area of interest, and (c) wireframe representation of the unwrapped phase in the area of interest.](image)

![Fig. 6 Interferograms showing (a) specimen overloaded (difficult to perform quantitative interpretation) and (b) specimen loaded at an appropriate level (quantitative interpretation can be readily performed).](image)

![Fig. 7 Interferograms showing the effects of different sensitivity vectors for the same loading conditions: (a) retroreflective illumination/observation condition and (b) oblique illumination/observation condition.](image)
Figure 8 depicts FP on a computational cantilever plate model, simulating static bending conditions and a double-exposure hologram.

### 2.3.1 Use of computational-EOH-FP as a hybrid computational-experimental tool

Computational techniques are capable of predicting surface and interior displacements of an object of interest subjected to specific loading and boundary conditions. With these techniques, it is possible to perform parametric investigations. However, computational investigations are only reliable if proper geometry, boundary conditions, loading, and material properties are provided. Unexpected defects in the object of interest may cause computational simulations to deviate from the actual behavior of the object.

Shape measurements using the EOH can be useful for the description of the actual geometry of an object under investigation. In addition, qualitative and quantitative information obtained in static and dynamic EOH modes can be utilized as boundary conditions in a computational model. Also, a computational model can be used to predict the interferometric fringes, which can be useful to determine appropriate experimental parameters and setup as well as a tool for verifying that realistic boundary conditions have been applied accurately.

Figure 9 depicts the interaction of computational, EOH, and FP in hybrid experimental-computational investigations. Quantitative and qualitative results provided by the EOH methodology are utilized by, or compared with, the computational predictions. FP is used as a complementary step for measuring the degree of correlation between computational and EOH results and to provide feedback to either method to improve the results of the hybrid analysis.

**Fig. 8** Fringe prediction based on computational simulations of a cantilever plate loaded under static bending conditions: (a) FE solutions for displacement and stress and (b) predicted continuous (unwrapped) phase map corresponding to a double-exposure hologram assuming collimated retroreflective illumination/observation conditions and the resulting fringe pattern.

**Fig. 9** Interaction of computational methods (FEM/BEM), EOH, and FP in the hybrid, experimental, and computational investigations.
2.4 Hybrid Computational and Experimental Procedure

Figure 10 depicts a flowchart of the hybrid, computational-experimental procedure used in this paper. According to Fig. 10, computational investigations are made on the initial design to obtain the response of the structure before its structural optimization is performed and the results compared with the response obtained from the EOH studies of an equivalent initial design model. The comparisons between experimental and computational results are based on FP. When discrepancies between computational and experimental results are encountered, it is necessary to resolve them. We have found that one of the most effective ways to achieve such a resolution is by using experimentally obtained boundary conditions in the computational model. Verifications also include characterization of material constants, geometric modeling accuracy, and mechanical and optical setups. The results of these comparisons provide information on the accuracy of the computational analyses with respect to experimental modeling of the behavior of the structure. When an acceptable degree of accuracy is obtained, the computational model is applied to perform sensitivity analyses of the objective function with respect to the specified design variables as well as shape optimization of the initial design. The geometry obtained from the shape optimization analyses is used to manufacture a prototype, which is experimentally tested to obtain computational and experimental correlation through the FP operation.

3 Conclusions and Recommendations

Computational techniques are very powerful methods for analysis of structural components and parametric investigations for determination of critical conditions. However, these techniques may lack any significance if incorrect input parameters, e.g., boundary conditions, loads, and material properties, are utilized. Experimental studies using electro-optic techniques have the characteristic of being able to obtain noninvasive measurements of the actual behavior of mechanical components subjected to specific loads and boundary conditions. In this paper, computational investigations, EOH, and fringe prediction were discussed as essential ingredients of a hybrid approach for an effective study and structural optimization of mechanical components. Practical application of the hybrid approach is illustrated by representative examples, which demonstrate the viability of this approach as an engineering tool for structural analysis and optimization.

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**Ryszard J. Pryputniewicz:** Biography and photograph appear with the paper “Holographic microscope for measuring displacements of vibrating microbeams using time-averaged, electro-optic holography” in this issue.